Computational investigation of third-order first-harmonic forces on a horizontal cylinder through phase-manipulated wave and body motion

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HIGHLIGHTS

- Third-order first-harmonic forces on a horizontal semi-immersed cylinder are investigated.
- Two-input phase manipulation is demonstrated computationally.
- Isolated higher-order first-harmonic forces contribute meaningful fractions of the total forcing.

1 INTRODUCTION

Phase manipulation has been employed effectively to study the higher-order super- and sub-harmonic wave forces which are typically relevant to rigid and soft-moored floating offshore structures respectively (e.g. [1]). Standard two- or four-phase combinations directly decompose the Stokes-like harmonics for the target response, typically through controlling the phase of incident waves. Recent work has proposed an expanded phase-manipulation with multiple controlled inputs, such as wave and body motion [2]. Two-input phase manipulation allows third-order first-harmonic cross terms to be isolated explicitly, unlike the wave (or motion) self-interaction terms which cannot be separated using phase only. This expanded phase manipulation is of particular importance for wave energy converters (WECs), as they are typically designed to resonate with the primary linear wave field. Furthermore, WECs' relatively large body motions, even in mild sea states, make nonlinear effects arising from body motions and wave-motion cross terms highly relevant.

In this work, we applied controlled phase-manipulated wave and heave body motions to numerically investigate nonlinear first-harmonic hydrodynamic forces on a heaving semi-immersed horizontal cylinder. Phase-manipulated focused wave groups and imposed body motions were used in a series of radiation, diffraction and combined tests conducted using the CFD software OpenFOAM. The measured forces are combined to successfully isolate third-order first-harmonic cross terms. Amplitude scaled body motions allowed for the isolation of further third-order first harmonics.

2 THEORY

For a weakly nonlinear wave-structure interaction described by potential flow theory, the hydrodynamic force can be written as

$$F(t) = A(t) f_1 \cos(\theta + \varphi_1) + A^2(t) \left(f_2^- + f_2^+ \cos(2\theta + \varphi_2^+) \right) + \mathcal{O}(A^3(t)) \tag{1}$$

where we have expressed the incident wave in a narrow-banded form such that $A(t)\cos(\theta) = A(t)\cos(\omega t)$ represents the linear content of the incident waves with $A(t)$ the slowly varying envelope relative to the peak frequency ω. For the remainder of this paper the time dependence is dropped. The amplitudes $f_n^{\pm}$ and phase $\varphi_n^{\pm}$ describe force transfer functions at order n in the incident wave amplitude A. To separate harmonics in the frequency domain, two incident wave groups separated in phase by π may be combined to isolate even and odd harmonics. Additional phase combinations can be used to achieve further separation. To consider two-input phase combinations we may first express forces on a body in an incident wave field undergoing imposed motion as

$$F(t) = F_W + F_M + F^{+}_{WW} + F^{-}_{WW} + F^{+}_{MM} + F^{-}_{MM} + F^{+}_{WM} + F^{-}_{WM} + \mathcal{O}(A^3) \quad (2)$$

where the subscript denotes motion or wave amplitude dependence, the superscript sum or difference frequency contributions, and explicit phase and amplitude is omitted for brevity. To isolate odd-harmonic terms that include the first-harmonic in wave, four combined tests may be used:

$$\begin{aligned}(C_{(0,0)} + C_{(0,\pi)} - C_{(\pi,0)} - C_{(\pi,\pi)})/4 = \; & F_W + F^{-+}_{WWW} + F^{+-}_{MMW} + F^{-+}_{WMM} \\ & + F^{++}_{WWW} + F^{++}_{WMM} + \mathcal{O}(A^5).\end{aligned} \quad (3)$$

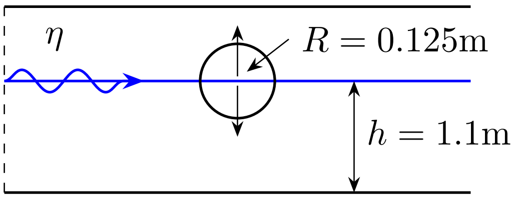
Here $C_{(p_w,p_m)}$ denotes the measured force time series resulting from a combined wave-motion input with wave phase p_w and motion phase p_m. Note, superscript–subscript ordering indicates frequency contribution: $F^{-+}_{WMM} \neq F^{+-}_{MMW}$. It is clear that phase-only combinations are unable to fully isolate the linear first-harmonic, with third-order first harmonic in wave terms (F^{-+}_{WWW} and F^{-+}_{WMM}) being indistinguishable by phase-manipulation. Only by combining amplitude scaled runs may higher-order first-harmonic terms be separated. However, using four phase shifts for each input and the Hilbert transform (H), third-order wave-motion difference cross terms may be isolated, e.g.

$$\begin{aligned}\frac{1}{16}\Big[& H\Big(C_{(\frac{3\pi}{2},0)} + C_{(\frac{3\pi}{2},\pi)} - C_{(\frac{\pi}{2},0)} - C_{(\frac{\pi}{2},\pi)} + C_{(\frac{\pi}{2},\frac{\pi}{2})} + C_{(\frac{\pi}{2},\frac{3\pi}{2})} - C_{(\frac{3\pi}{2},\frac{\pi}{2})} - C_{(\frac{3\pi}{2},\frac{3\pi}{2})}\Big) \\ & + C_{(0,0)} + C_{(0,\pi)} - C_{(\pi,0)} - C_{(\pi,\pi)} - C_{(0,\frac{\pi}{2})} - C_{(0,\frac{3\pi}{2})} + C_{(\pi,\frac{\pi}{2})} + C_{(\pi,\frac{3\pi}{2})}\Big] = F^{+-}_{MMW} + \mathcal{O}(A^5)\end{aligned} \quad (4)$$

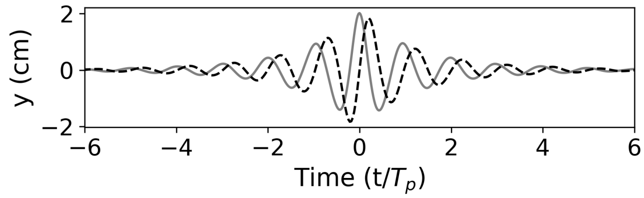
Equivalent expressions may be achieved for the first harmonic in motion and F^{+-}_{WWM} by swapping the motion and wave phases. Likewise, free surface measurements combine to produce equivalent results. These examples will be expanded on at the workshop.

3 NUMERICAL MODEL

The OpenFOAM solver interFOAM was used to solve the Navier-Stokes equations under the assumption of laminar flow and incompressibility, using the Volume of Fluid method to track the air-water interface. The utility waves2Foam [3] was used to couple the OpenFOAM domain to the potential flow solver OceanWave3D [4], through boundary conditions on the inlet. This allowed fully developed nonlinear waves to be input into the domain, mitigating the generation of error waves at higher order. A focused wave group derived from a JONSWAP spectrum with $k_pR = 0.32$ was used. The numerical wave tank was validated for the undisturbed wave against the same wave in OceanWave3D. As this work aimed to investigate third-order effects, the mesh was substantially refined to agree with experimental results for the linear signal and the second and third harmonics from OceanWave3D. The resulting undisturbed free surface measured at the focus location is shown in Figure 1b. A horizontal cylinder of radius $R = 0.125$m centred on the free surface was constructed using the snappyHexMesh utility. The domain was restricted to 2D by applying empty boundary conditions to the front and back of the domain, greatly expediting the simulation. The imposed cylinder motion was solved using the displacementLaplacian and specified with the codedFixedValue pointDisplacement boundary condition. Diffraction tests were completed with four phases using a $A = 0.16R$ focused wave. Radiation tests used heave motions scaled from the linear wave input to maximum displacements $Y = A$, $2A$, and $3A$. These input signals are seen in Figure 1b. Both radiation and diffraction tests were compared against linear theory [5] for the smallest amplitudes, showing excellent agreement, see Figure 2. Combined tests were then completed for the 16 total combinations of four phase-shifted waves and motions. The combined tests were only run up to $Y = 3A$ to avoid severe nonlinearity. Forces are only presented in heave and with the hydrostatic contribution relative to the still water level removed.



(a) Simulation set up.



(b) Wave elevation (solid line) and body displacement (dashed line) relative to mean free surface ($y = 0$) with $p_m/p_w = 0$.

Figure 1: Problem setup and input conditions.

4 RESULTS AND DISCUSSION

The first and third harmonics, isolated using 16 phase combinations and a bandpass filter respec-

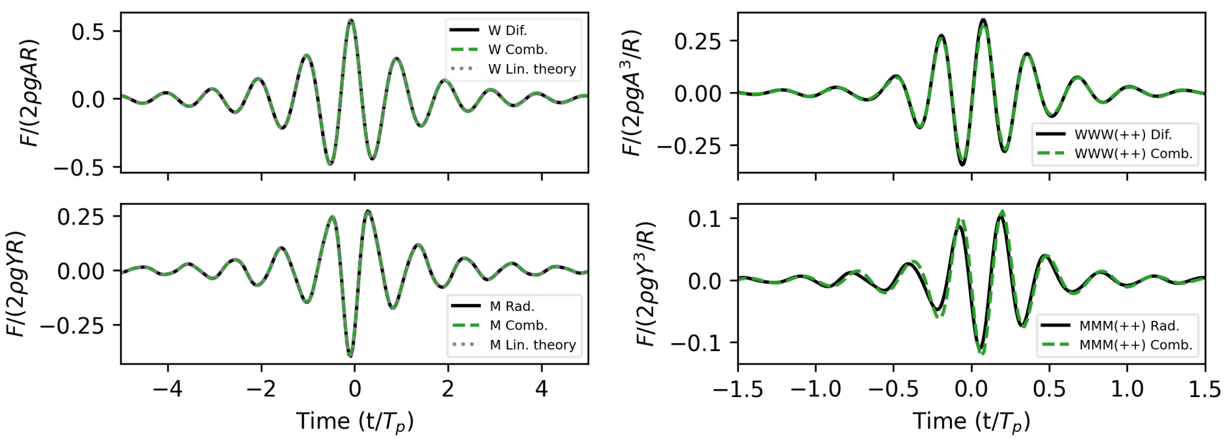


Figure 2: Extracted first and third harmonics from one-input phase combinations for radiation (rad.) and diffraction (dif.) tests, and from two-input phase combinations for combined (comb.) tests. The first harmonic is compared with linear theory (lin. theory).

tively at $0.6f_p$ to $2.5f_p$ and $2.4f_p$ to $5f_p$, from the combined test results at $Y = A$ are compared against the corresponding radiation and diffraction results in Figure 2. The results agree very closely even at higher order and demonstrate the high level of controllability of applying phase combinations numerically. The isolated first harmonic in wave (W) closely aligns for all three motion amplitudes, as seen in Figure 3a. However, the peaks are clearly diminishing with increasing body motion amplitude. This may be attributed to the first-harmonic cross term WMM($-+$),

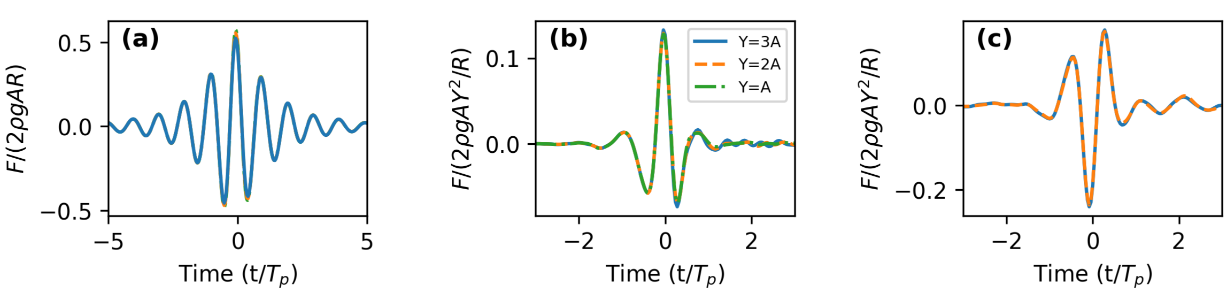


Figure 3: Total first-harmonic force in waves (a), MMW($+-$) cross term from phase combinations (b), and WMM($-+$) cross term from amplitude scaling (c).

which is second-order in motion and first-order in wave. To verify this, the smallest motion case was subtracted from the larger two test cases, Y $= 2A$ and $3A$. The remaining signal should collapse when normalised by $(Y_{large}^2 - Y_{small}^2)$, which accounts for both the large and small motions' cross terms. The MMW($+-$) cross term can also be extracted directly using (4). Both the extracted and subtracted signals show matching structure and collapse when normalised, see Figure 3. For the largest motion tested the isolated cross terms are respectively 5% and 10% of the total wave dependent first-harmonic force.

We may similarly compare the isolated first harmonics in motion. The cross term WWM($+-$) can similarly be extracted using phase combinations with the normalised results in Figure 4b displaying exceptionally good agreement. The linearly scaled smallest motion was then subtracted from the largest motion signals. The remaining signal collapses when normalised by $(Y_{large}^3 - Y_{small}^3)$, as shown in Figure 4c, supporting the conclusion that it is third-order in motion. For $Y = 3A$ the MMM($--$) term is approximately 5% of the total motion dependent first-harmonic force.

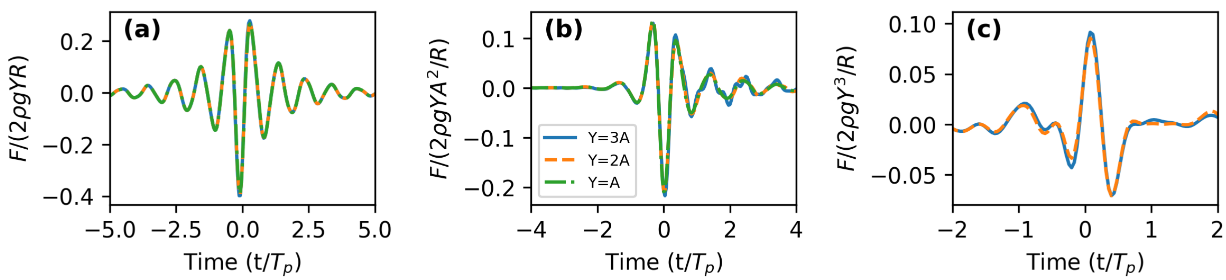


Figure 4: Total first-harmonic force in motion (a), WWM($+-$) cross term from phase combinations (b), and the motion third-order first harmonic MMM($--$) isolated from amplitude scaling (c).

This work demonstrated the effectiveness of applying multiple-input phase combinations computationally. The isolated higher-order contributions to the first-harmonic forces would directly affect WEC (e.g. [6]) operation. Further analysis, including the behaviour of added mass and damping, will be presented at the workshop. Future work will make the extension to 3D geometries.

ACKNOWLEDGEMENTS

MZ is supported by the Aus. Gov. RTP Fees Offset and Stipend at UWA. This work was supported by resources provided by the Pawsey Supercomputing Research Centre with funding from the Australian Government and the Government of Western Australia. This work forms part of DP250104899, funded by the Australian Research Council.

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