

## Wave Cancellation by A Four-Hull Vessel in A Diamond Configuration

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### 1 INTRODUCTION

Ships moving on or close to the free-water surface at a constant speed inherently create a unique steady wave pattern, known as Kelvin wake. To maintain such wave pattern, ships need to consume energy to overcome a resistance, which is called wave-making resistance. For high-speed vessels, such as container ships or passenger ferries, such wave resistance could contribute more than 50% to the total resistance. For over a century, naval architects have tried to understand and minimize such resistance component to reduce fuel consumption. Measures taken so far include hull form optimization, bulbous bow design, catamarans or multi-hull design. These measures share a common principle: wave cancellation. Optimizing of a monohull would result in a destructive wave interference, where the waves generated by different parts of the hull cancel each other out. A typical example is to design a bulbous bow to generate a wave that interferes destructively with the wave pattern generated by the ship hull. Multi-hull vessels, such as catamarans and trimarans, can achieve a very small wave resistance when the waves generated by each hull are cancelled with each other [1].

Our interest in this study is simple: can we design a waveless ship? If so, what should this waveless ship look like? If not, what are the minimum waves a ship can generate?

To answer the above questions, we need to have a fundamental understanding of the steady wave pattern generated by a moving body close to free-water surface. Let's start with moving source points.

### 2 WAVE INTERFERENCE AMONG SOURCE POINTS

Wave patterns by a moving pressure disturbance have been well established. According to Peters [2], the wave elevation by a moving source point on free water surface can be written as

$$\begin{aligned} \zeta(x, y) = & -\frac{\varepsilon F_n^2 \operatorname{sgn}(x)}{\pi^2} \int_0^{\pi/2} \cos \theta \int_0^{\infty} \frac{k e^{-k|x|} \cos(ky \sin \theta) g(k, \theta)}{F_n^4 k^2 + \cos^2 \theta} dk d\theta \\ & + \frac{\varepsilon H(x)}{\pi} \int_{-\infty}^{\infty} \xi e^{-F_n^2 \xi^2} \cos(x\xi) \cos(y\xi) d\xi \end{aligned} \quad (1)$$

where  $g(k, \theta) = F_n^2 k \sin(k \cos \theta) + \cos \theta \cos(k \cos \theta)$ , and  $\xi(\lambda) = \sqrt{\lambda^2 + 1} / F_n$ . Here  $F_n$  is the Froude number,  $k$  is the wave number,  $\operatorname{sgn}(x)$  is the sign function, and  $H(x)$  is the Heaviside function.

The wave patterns generated by a moving single source point is shown in *Figure 1 (a)*. Two distinguish wave systems can be observed: transverse and diverging waves. If we want to

eliminate waves, both of these two wave systems need to be vanished. This can only be achieved by adding one or more source points travelling at the same constant speed. A source doesn't have geometrical size; therefore, linear superposition principle can be applied to find the wave patterns by multiple source points. To quantify wave cancellation effect, the wave energy produced by a single source point (S1) is normalized as  $E$ . The best configuration of two sources to achieve wave cancellation is "two-in-tandem", as shown in *Figure 1 (b)*. This simply corresponds to placing S2 at about one half-wavelength ( $0.5\lambda$ ) behind S1, with their transverse waves out of phase. In such case, the wave energy is only  $0.27E$ . One can anticipate that similar destructive interference can also be observed when S2 is located at  $(n+0.5)\lambda$  behind S1. However, in such layout, diverging waves could not be perfectly cancelled. If the distance between these two sources gets larger, there will be no interference between diverging wave systems of S1 and S2 in a tandem configuration. Diverging waves can only be cancelled when there is a transverse stagger. Here, transverse waves are defined to be those propagating within  $35.3^\circ$  wedge angle behind the source; diverging waves propagating at angle greater than  $35.3^\circ$  [3]. *Figure 1 (c)* shows the minimum waves generated by three sources in a V-shape configuration. It can be observed the portside diverging waves of S1 is cancelled by the portside diverging waves of S2, while the starboard diverging waves of S1 is cancelled by the starboard diverging waves of S3. Although the diverging waves are almost vanished, the starboard diverging waves of S2 and the portside diverging waves of S3 still persist, forming a new and complete diverging wave system. As a result, wave cancellation effects of such V-shape configuration are not apparent, only achieving a wave energy of  $0.75E$ . However, it brings a perfect scenario to place S4 to achieve destructive wave interference. In *Figure 1 (d)*, both diverging and transverse waves behind the triangle (S1+S2+S3) are 180 out of phase with S4's waves, leading to a dramatic wave cancellation. The wave energy produced by such diamond configuration is only  $0.21E$ , and most of them are concentrated within the diamond area. The waves energy escaped from the diamond area is negligible, and they are damped very quickly when propagating to the far field. It inspires us to investigate a 3-dimensional waveless ship design by adopting such diamond configuration.

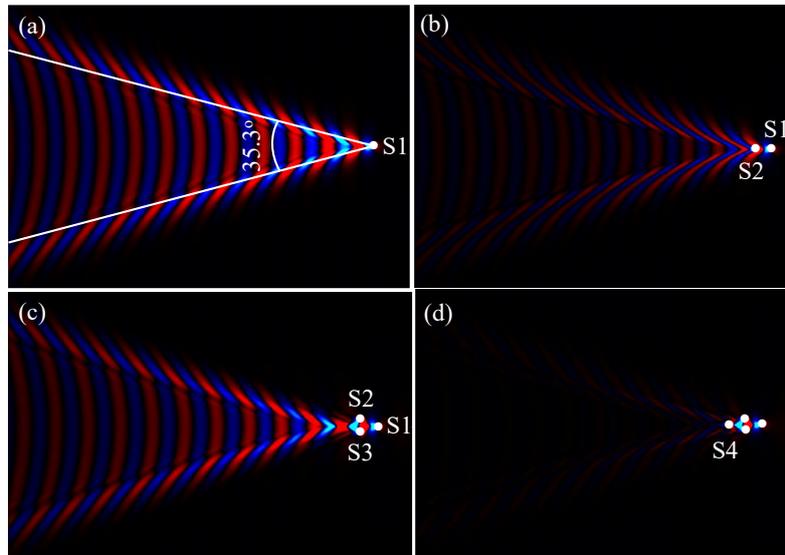


Figure 1. (a) The wave pattern generated by a single source point submerged at a depth of  $h=0.3u^2/g$ , where  $u$  is the travel speed, and  $g$  is gravitational acceleration. The wave energy produced by this single source is  $E$ . (b) Destructive wave pattern generated by two source points in a tandem configuration. Wave energy is  $0.27E$ . (c) Destructive wave pattern generated by three source points in a V-shape configuration. Wave energy is  $0.75E$ . (d) Destructive wave pattern generated by four source points in a diamond configuration. Wave energy is  $0.21E$ .

### 3 WAVE AND WAVE RESISTANCE OF MULTI-HULL VESSELS

Being different from source points, ship hulls have 3D geometry. Wave interference does not only occur among multiple hulls, but also among different parts of a monohull. When multiple hulls are arranged very close to each other, near-field wave interference cannot be ignored, and wave reflection is inevitable. The superposition principle applied to source points may not be valid. A more sophisticated method needs to be adopted. A linear potential flow solver, MHydro, based on Rankine source panel method, is used in this study to solve the boundary value problem [4]. The free-surface is discretized into panels, and a linearized boundary condition is applied. An ellipsoid ( $a=10\text{m}$ ,  $b=c=2\text{m}$ ) is used to represent the ship hull. To simplify the numerical modelling of the free-surface, the ellipsoid is submerged at a depth of 0.2m. Similar to source points, we calculated the waves generated by a single ellipsoid, two-in-tandem, V-shape, and diamond configurations. To find the minimum waves generated by multiple hulls, we varied the transverse ( $X$ ) and longitudinal ( $Y$ ) staggers of the multi-hull vessels. When minimum total wave resistance is achieved, the corresponding configuration corresponds to one with minimum wave energy.

The wave patterns are shown in *Figure 2*. In general, the results are consistent with the wave patterns by source points. Of course, more near-field interactions are observed among 3D ellipsoids. In a “two-in-tandem” configuration, as shown in *Figure 2 (b)*, E2 positions its fore part in a wave trough and aft part in a crest behind E1. This is so-called “wave-riding” position, where the ducklings are observed in the nature to following their mother [4]. In such configuration, only small amount of the diverging waves is cancelled. The interference mainly occurs in the transverse wave region, and the transverse wave pattern almost vanished behind E2. However, the diverging waves still persist, propagating to the far field, making it not to be an ideal “waveless” ship. From the resistance point of view, the total resistance of such a dihull is about 69% of a monohull moving at the same speed. It should be noted the total displacement of the dihull is twice as that of the monohull. Decomposing the total wave resistance to each individual hull, it is found that  $R_{E2}/R_m = -0.39$ , indicating E2 utilizes E1’s waves as propulsion. Surprisingly, it is also found that  $R_{E1}/R_m = -1.07$ , showing the presence of E2 slightly increases E1’s resistance.

The waves generated by a trihull in a V-shape configuration (as shown in *Figure 2 (c)*) is similar to those by 3 source points. Although E1’s portside and starboard diverging waves are cancelled by E2 and E3 respectively, it leaves a complete Kelvin wake behind E2 and E3. The position of E4 can be easily determined: it should be placed close to the point where E2’s starboard diverging waves intersect with E3’s portside diverging waves. Longitudinally, its position can be tuned by aligning E4’s fore part to a wave trough. This is how a waveless four-hulled vessel looks like, as shown in *Figure 2 (d)*. The total wave resistance of this four-hulled vessel is almost the same as the monohull vessel, while its displacement is four times larger. Compared to the V-shape configuration, E4 recycles the wave energy in the trihull vessel’s wake. As a result, its wave resistance turns to be a propulsion force of 2.1 kN. Compared to the “two-in-tandem” configuration, the total wave resistance of such quadhull in a diamond configuration is larger. It should be noted the four-hulled vessel possesses twice larger displacement than the dihull vessel in our example, and more importantly, about **88%** of the total wave energy produced by this four-hulled vessel is concentrated within the diamond area shown in *Figure 2 (d)*. The remaining waves are damped quickly as they propagate to the far field, making this diamond configuration as the best candidate for waveless ship design. In a scenario where natural river banks are eroded by boat-generated waves, such four-hulled vessel in a diamond configuration could be a solution.

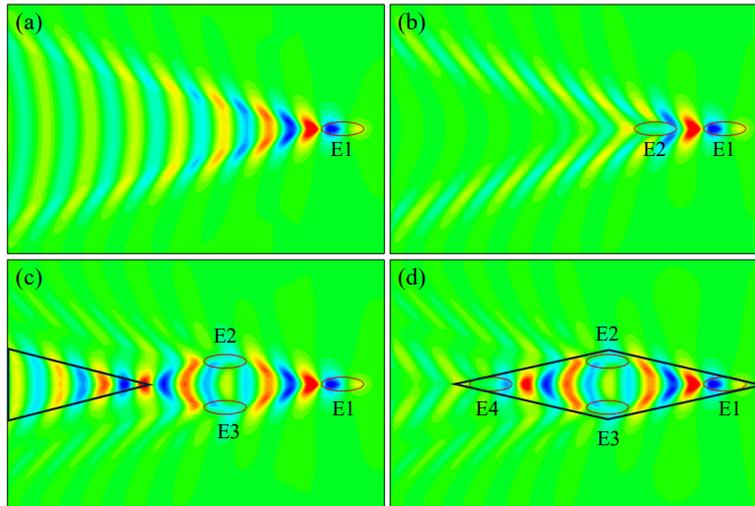


Figure 2. (a) Waves generated by a submerged ellipsoid moving at  $u=3.96$  m/s. Its wave resistance  $R_m=10.8$ kN. (b) Wave pattern generated by a di-hull in a tandem configuration. Its total wave resistance  $R_{di}/R_m=0.69$ . (c) Wave pattern generated by a tri-hull in a V-shape configuration. Its total wave resistance  $R_{tri}/R_m=1.25$ . (d) Wave pattern generated by a four-hulled vessel in a diamond configuration. Its total wave resistance  $R_{quad}/R_m=1.02$ .

So in the end, can we design a waveless ship, and what does the waveless ship look like? For a monohull ship, the answer is clearly no. While achieving a completely "waveless" ship is physically impossible, the waves can be effectively eliminated by multi-hull configurations. "Two-in-tandem" configuration could achieve a minimum total wave resistance, hence generating minimum waves. However, these waves are visibly propagating to the far-field in the form of a diverging pattern. Therefore, it does not match "waveless" concept. The best waveless solution is through a diamond configuration. It does not represent minimum wave-making resistance, but the most visible waves are "cloaked" within a diamond area. In the far field, the waves are nearly vanished.

#### 4 REFERENCES

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