Water wave scattering by a floating or submerged rectangular anisotropic elastic plate

Ben Wilks[a,b], Michael H. Meylan[b], Zachary J. Wegert[c], Vivien J. Challis[c] & Ngamta Thamwattana[b]
[a] School of Mathematical Sciences , Adelaide University, Mawson Lakes, Australia
[b] School of Computer and Information Sciences, University of Newcastle, Callaghan, Australia
[c] School of Mathematical Sciences, Queensland University of Technology, Brisbane, Australia
Email: b.wilks@adelaide.edu.au

1 HIGHLIGHTS

- Water wave scattering by a rectangular anisotropic elastic plate is considered.
- The plate is either submerged or surface mounted, with either free or clamped edges.
- The problem is obtained as an expansion over the dry modes of the elastic plate, which are computed using a Rayleigh–Ritz method.
- The component diffraction and radiation problems are solved by formulating a boundary integral equation (BIE) and solving numerically using a constant panel method. The BIE is hypersingular in the case of the submerged plate.

2 INTRODUCTION

Hydroelasticity, which is concerned with the interaction of fluids with elastic bodies, is an important topic with numerous applications, including those in the cryosphere (ocean wave scattering by sea ice and ice shelves) and those related to the engineering of very large floating structures (VLFS). The large body of research into hydroelastic wave scattering has focussed almost entirely on isotropic plates [1]. Recently, hydroelastic modelling was applied to study piezoelectric wave energy converters (PWECs), which a device that extracts energy from ocean waves using the piezoelectric effect, in which materials become electrically polarised in response to elastic deformation [2]. In order to model PWECs in three dimensions, there is a need to develop numerical methods for anisotropic hydroelasticity, since common piezoelectric materials (e.g., polyvinylidene fluoride) are anisotropic. This abstract, which extends prior work on one-dimensional plates [3, 4] encompasses previous developments for surface mounted plates [5] and includes a more recent extension to submerged plates.

3 PROBLEM OUTLINE

The problem under consideration is described in Figure 1. Time harmonic linear water wave theory and Kirchhoff–Love thin plate theory (with time dependence $e^{-i\omega t}$) are assumed to govern the fluid and plate, respectively. This leads to the following boundary value problem for the complex velocity potential ϕ:

$$\triangle\phi = 0 \qquad (x,y,z)\in\Omega, \tag{1a}$$
$$\partial_z\phi = 0 \qquad z=-H, \quad (x,y)\in\mathbb{R}^2, \tag{1b}$$
$$\partial_z\phi = \frac{\omega^2}{g}\phi \qquad z=0, \quad (x,y)\in\mathbb{R}^2\setminus\Gamma, \tag{1c}$$
$$\partial_z\phi = -i\omega \mathrm{w} \qquad z=0, \quad (x,y)\in\Gamma, \tag{1d}$$
$$\sqrt{r}(\partial_r - ik)(\phi-\phi^{\mathrm{inc}}) \to 0 \qquad \text{as } r\to\infty, \tag{1e}$$

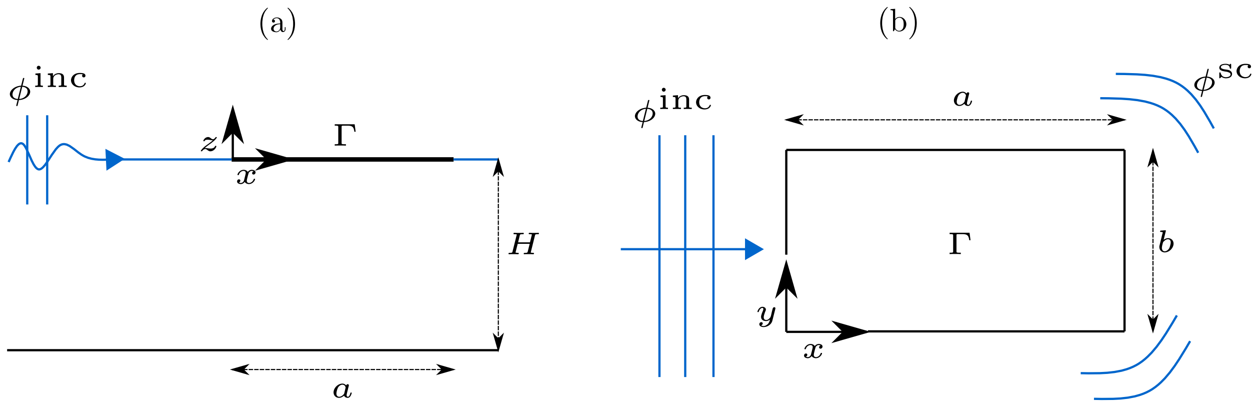

Figure 1: (a) Side and (b) plan views of the surface mounted plate scattering problem. The rectangular plate, labelled Γ, has side lengths a and b and the fluid is of depth H. The incident wave ϕ^{inc} excites the plate into motion, generating scattered waves ϕ^{sc}. In the case of the submerged plate, the plate is horizontal and submerged to a depth d.

where Ω is the fluid domain and g is acceleration due to gravity, which must be solved in tandem with the dynamic equation of the plate. For a surface mounted plate, this is

$$\mathcal{D}\mathrm{w} - \omega^2 \rho h \mathrm{w} + \rho_w g \mathrm{w} = \mathrm{i}\omega\rho_w \phi\Big|_{z=0}, \tag{2}$$

where ρ and ρ_w are the densities of the plate and water, h is the plate thickness $\mathcal{D}$ is the following fourth order differential operator [6]

$$\mathcal{D} = D_{11}\partial_x^4 + 4D_{16}\partial_x^3\partial_y + 2(D_{12} + 2D_{66})\partial_x^2\partial_y^2 + 4D_{26}\partial_x\partial_y^3 + D_{22}\partial_y^4.$$

If the plate is submerged, (2) becomes

$$\mathcal{P}\mathrm{w} - \omega^2\rho h\mathrm{w} = -\mathrm{i}\omega\rho_w[\phi], \tag{3}$$

where $[\phi(\mathbf{x})] = \lim_{z\to -d^+}\phi - \lim_{z\to -d^-}\phi$ is the jump in ϕ across the plate. Lastly, we prescribe boundary conditions at the plate edges.

4 PLATE MODES

The modes w_j and frequencies ω_j of vibration are computed using a Rayleigh–Ritz method [7, 6] and validated for both isotropic and anisotropic plates [8, 9].

5 DIFFRACTION AND RADIATION

For the submerged plate, integral equations for the diffraction potential ϕ^{di} and radiation potentials ϕ_j^{ra} (associated with radiation from mode of vibration w_j) are obtained as

$$\phi^{\text{di}}(\mathbf{x}) = \alpha \iint_{\Gamma\times\{0\}} G(\mathbf{x}', \mathbf{x})(\phi^{\text{inc}}(\mathbf{x}') + \phi^{\text{di}}(\mathbf{x}'))\mathrm{d}S_{\mathbf{x}'}, \tag{4a}$$

$$\phi_j^{\text{ra}}(\mathbf{x}) = \iint_{\Gamma\times\{0\}} G(\mathbf{x}', \mathbf{x})(\alpha\phi_j^{\text{ra}}(\mathbf{x}') + \mathrm{i}\omega\mathrm{w}_j(\mathbf{x}'))\mathrm{d}S_{\mathbf{x}'}, \tag{4b}$$

for all $\mathbf{x} \in \Gamma \times \{0\}$. When the plate is submerged, we obtain hypersingular integral equations

$$\partial_z \phi^{\mathrm{inc}}(\mathbf{x}) = \mathcal{H} \iint_{\Gamma \times \{0\}} [\phi^{\mathrm{di}}(\mathbf{x}')] \partial_z \partial_{z'} G(\mathbf{x}', \mathbf{x}) \mathrm{d}S_{\mathbf{x}'}, \tag{5a}$$

$$\mathrm{i}\omega \mathrm{w}_j(\mathbf{x}) = \mathcal{H} \iint_{\Gamma \times \{0\}} [\phi_j^{\mathrm{ra}}(\mathbf{x}')] \partial_z \partial_{z'} G(\mathbf{x}', \mathbf{x}) \mathrm{d}S_{\mathbf{x}'}. \tag{5b}$$

where $\mathcal{H} \iint$ denotes the Hadamard finite part integral [10]. The integral equations are solved using a constant panel method.

6 HYDRODYNAMIC COUPLING

The velocity potential and plate displacement are expanded over the dry modes as

$$\phi = \phi^{\mathrm{inc}} + \phi^{\mathrm{di}} + \sum_{j=1}^{\infty} c_j \phi_j^{\mathrm{ra}} \quad \text{and} \quad \mathrm{w} = \sum_{j=1}^{\infty} c_j \mathrm{w}_j, \tag{6}$$

where the coefficients c_j are unknown. These are then solved by substituting the above into the dynamic equation of the plate (either (2) or (3) for surface mounted or submerged plates, respectively). The method is based on that of Newman [11].

7 RESULTS FOR A SURFACE MOUNTED PLATE

An example result for a surface mounted anisotropic plate is given in Figure 2. The rigidity coefficients D_{ij} are those considered by An et al. [9].

REFERENCES

[1] Squire, V. A. Jan. 2020. *Ocean wave interactions with sea ice: a reappraisal.* Annual Review of Fluid Mechanics 52(1), 37–60.

[2] Renzi, E. Nov. 2016. *Hydroelectromechanical modelling of a piezoelectric wave energy converter.* Proceedings of the Royal Society A 472(2195), 20160715.

[3] Meylan, M. H., Challis, V. J., Thamwattana, N., Wegert, Z. J., and Wilks, B. 2025. *Wave power absorption by floating plates with application to piezoelectric or other bending-based absorption. Ocean Engineering 328*, 120931.

[4] Wegert, Z. J., Wilks, B., Thamwattana, N., Challis, V. J., Koley, S., and Meylan, M. H. 2025. *Wave energy conversion by floating and submerged piezoelectric bimorph plates. arXiv preprint arXiv:2512.17965*.

[5] Wilks, B., Meylan, M. H., Wegert, Z. J., Challis, V. J., and Thamwattana, N. 2025. *Water wave scattering by a surface-mounted rectangular anisotropic elastic plate. arXiv preprint arXiv:2510.17872*.

[6] Reddy, J. N. 2006. *Theory and Analysis of Elastic Plates and Shells*, 2nd edition ed. CRC Press.

[7] Young, D. 12 1950. *Vibration of Rectangular Plates by the Ritz Method.* Journal of Applied Mechanics 17(4), 448–453.

[8] Leissa, A. 1973. *The free vibration of rectangular plates.* Journal of Sound and Vibration 31(3), 257–293.

[9] An, D., Ni, Z., Xu, D., and Li, R. Dec. 2021. *New Straightforward Benchmark Solutions for Bending and Free Vibration of Clamped Anisotropic Rectangular Thin Plates.* Journal of Vibration and Acoustics 144(3), 031011.

[10] Martin, P. A., and Farina, L. 1997. *Radiation of water waves by a heaving submerged horizontal disc. Journal of Fluid Mechanics 337*, 365–379.

[11] Newman, J. N. Jan. 1994. *Wave effects on deformable bodies.* Applied Ocean Research 16(1), 47–59.

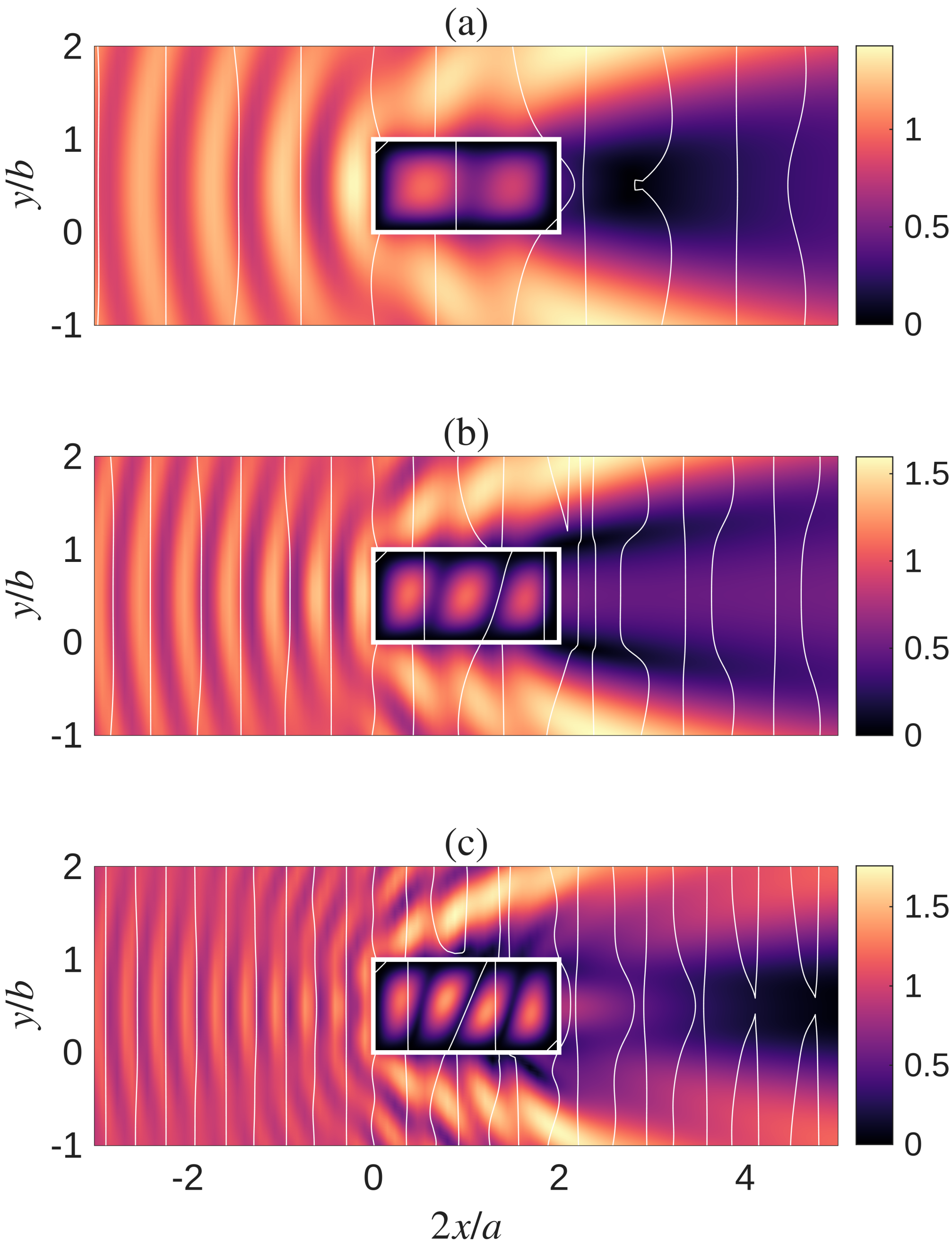

Figure 2: Surface elevation of the excited rectangular ($a = 2\,\mathrm{m}$ and $b = 1\,\mathrm{m}$) anisotropic plate with clamped edges for $H = 20\,\mathrm{m}$ and $\rho h = 1\,\mathrm{kg\,m^{-1}}$ at the resonant frequencies (a) $\omega = 6.42\,\mathrm{s}^{-1}$, (b) $\omega = 8.09\,\mathrm{s}^{-1}$ and (c) $\omega = 9.82\,\mathrm{s}^{-1}$.