Performance-enhanced HOS-NWT for generating and propagating short-crested seas

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1 INTRODUCTION

An accurate description of the ocean wave environment, together with its efficient numerical evaluation, is crucial in ocean engineering. These requirements have led to the widespread use of nonlinear potential flow models. Among these, High-Order Spectral (HOS) models offer an excellent compromise between computational efficiency and accuracy. In addition, HOS models are capable of representing a wide range of wave conditions, both in open-sea configurations (`HOS-Ocean` [1]) or in the numerical wave tanks simulations (`HOS-NWT` [2]).

The `HOS-NWT` model decomposes the overall Initial-Boundary Value Problem (I-BVP) into two subproblems: a wave generation problem, solved using a spectral formalism, and a wave propagation problem, solved with the HOS formulation [2, 3]. However, compared with `HOS-Ocean`, `HOS-NWT` exhibits significantly higher computational cost, especially in short-crested seas (*i.e.*, 3D), due to the presence of the wavemaker, which introduces additional quantities to be evaluated. The spectral formulation of the wavemaking problem expresses these additional quantities through modal expansions in the vertical and, in three dimensions, in the transverse direction. Their evaluation on the free surface depends on the instantaneous free-surface elevation, which breaks the separability between the transverse (y) and vertical (z) directions in the modal expansion. As a result, the use of the Fast Discrete Cosine Transform (FDCT) in the transverse direction is severely limited.

To address this limitation, the present work proposes applying a Taylor Expansion (TE) to the terms of the modal expansion involving the vertical coordinate z, thereby decoupling the dependence of z on y. This reformulation enables the efficient use of the FDCT in the transverse direction to compute the contributions of the additional wavemaking terms on the free surface, leading to a substantial reduction in computational cost for short-crested seas, particularly for large numbers of transverse modes (N_y).

2 FORMULATION

In the `HOS-NWT` model, the velocity potential $\phi(\mathbf{x}, z, t)$, solution of the problem, can be decomposed into two components in the whole domain: $\phi(\mathbf{x}, z, t) = \phi_{\text{HOS}}(\mathbf{x}, z, t) + \phi_{\text{add}}(\mathbf{x}, z, t)$, where ϕ_{HOS} is the velocity potential associated with wave propagation, and ϕ_{add} is the potential associated with wave generation, with $\mathbf{x} = (x, y)$. Theoretically, the total computational cost T_{total} is estimated as $T_{\text{total}} = T_{\text{HOS}} + T_{\text{add}}$ [2], with

$$T_{\text{HOS}} = \mathcal{O}\left(\left\{23 + \left[\frac{M^2}{2} + \frac{7M}{2} + 2E\left(\frac{M-1}{p}\right) + 31\right]\frac{p+1}{2}\right\} N_x N_y \log_2(N_x N_y)\right) \quad (1)$$

$$T_{\text{add}} = \mathcal{O}\big(\alpha N_y N_z \log_2(N_y N_z) + \beta N_x N_y \log_2(N_x N_y) + \gamma N_x N_y^2 N_z\big)\,. \quad (2)$$

T_{HOS} represents the (HOS) wave propagation CPU cost, T_{add} the wavemaking CPU cost, and $T_{\text{add3}} = \gamma N_x N_y^2 N_z$ is introduced for simplicity. N_x, N_y, and N_z are the numbers of grid points in the x, y, and z directions, and $E(\cdot)$ means taking integer; p is the de-aliasing order; M is the HOS order; and α, β, and γ are the numbers of products/FFTs needed in the additional (wavemaking) problem. For unidirectional waves (2D configuration), T_{HOS} and T_{add} are comparable, whereas T_{add} may dominate the CPU cost in 3D, since the term T_{add3} increases quadratically with N_y. For a second-order accurate wavemaking problem, as recommended practice in `HOS-NWT`, the parameters are as follows [2]: $\alpha = 9$, $\beta = 3$, $\gamma = 27$, $M = 5$, $p = 5$, $N_x = 128$, and $N_z = 32$. The relative contributions of T_{add}, T_{add3}, and T_{HOS} to the total computational cost of `HOS-NWT` with increasing N_y are presented on the left of Fig. 1, from which we can clearly observe the domination of the computational cost by T_{add3} as N_y increases.

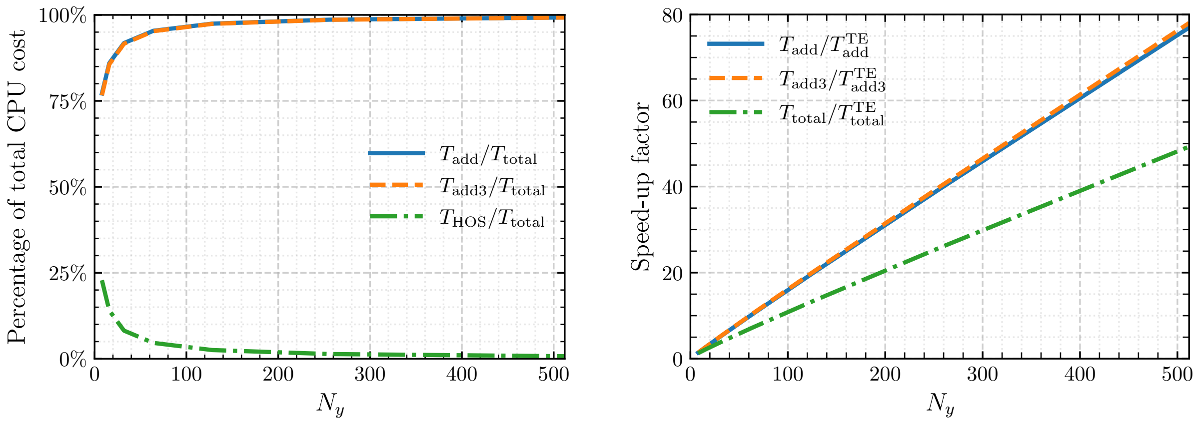

Figure 1: (Left) Percentage contributions of T_{add}, T_{add3}, and T_{HOS} to the total computational cost of `HOS-NWT`. (Right) Comparison of the computational cost between the original `HOS-NWT` and the TE-accelerated `HOS-NWT`.

In Eq. (2), T_{add3} represents the computational cost associated with the evaluation of the additional quantities on the instantaneous free-surface, such as $\phi_{\mathrm{add}}(\mathbf{x}, z = \eta(\mathbf{x}, t), t)$. The value at the grid point (x_p, y_q) on the free surface $\eta_{pq} = \eta(x_p, y_q)$ is given by

$$\phi_{\mathrm{add}}(x_p, y_q, \eta_{pq}, t) \quad = \quad \sum_{j=0}^{N_y} \sum_{l=0}^{N_z} B_{jl}^{\phi_{\mathrm{add}}}(t) \, \cos(k_j^y y_q) \, \cos[k_l^z(\eta_{pq} + h)] \, \frac{\cosh[k_{jl}(L_x - x_p)]}{\cosh(k_{jl} L_x)} \quad (3)$$

where L_x is the length of the wave tank, h is the water depth, $B_{jl}^{\phi_{\mathrm{add}}}(t)$ represents the modal amplitude, and k_j^y, k_l^z, and k_{jl} are wavenumber given by

$$k_j^y = j \, \frac{2\pi}{L_y}, \quad k_l^z = l \, \frac{2\pi}{h_{\mathrm{add}} + h}, \quad k_{jl} = \sqrt{(k_j^y)^2 + (k_l^z)^2}. \quad (4)$$

in which L_y is the wave tank width, and h_{add} the height of the extended region in `HOS-NWT` [2]. From Eq. (3), we recover that to evaluate the additional quantity on the whole free surface, the corresponding computational cost is $\mathcal{O}(N_x N_y^2 N_z)$, which is the dominant contribution in 3D configurations ($N_y > 1$).

To overcome this bottleneck, we propose a method that accelerates the computation of the additional quantities on the free surface by applying a Taylor expansion of their spectral expression about $z = 0$, thereby decoupling the dependence of z on y. This reformulation enables the efficient use of the FDCT in the transverse direction to evaluate the contributions of the additional terms on the free surface. Specifically, applying a TE to the term $\cos[k_l^z(\eta_{pq} + h)]$ in Eq. (3), and expanding about $\eta = 0$ up to order M^{TE} yields: $\cos\left[k_l^z(\eta_{pq} + h)\right] \approx \sum_{m=0}^{M^{\mathrm{TE}}} C_{lm}(\eta_{pq})^m$, where the coefficients C_{lm} are given by

$$C_{l,2m} = \frac{(-1)^m (k_l^z)^{2m}}{(2m)!} \cos(k_l^z h), \quad ; \quad C_{l,2m+1} = -\frac{(-1)^m (k_l^z)^{2m+1}}{(2m+1)!} \sin(k_l^z h). \tag{5}$$

Rewriting Eq. (3) with TE, we obtain

$$\phi_{\mathrm{add}}(x_p, y_q, \eta_{pq}, t) \approx \sum_{m=0}^{M^{\mathrm{TE}}} (\eta_{pq})^m \sum_{j=0}^{N_y} D_{pjm}(t) \cos(k_j^y y_q), \quad \text{with} \tag{6}$$

$$D_{pjm}(t) = \sum_{l=0}^{N_z} B_{jl}^{\phi_{\mathrm{add}}}(t)\, C_{lm} \frac{\cosh[k_{jl}(L_x - x_p)]}{\cosh(k_{jl} L_x)}. \tag{7}$$

The computational cost associated with evaluating a single additional term on the free surface using the decoupled formulation can therefore be decomposed into three contributions. Computing D_{pjm} requires $\mathcal{O}(N_x N_y M^{\mathrm{TE}} N_z)$ operations, the summation over p performed via FDCT costs $\mathcal{O}(N_x M^{\mathrm{TE}} N_y \log N_y)$, and multiplication by $(\eta_{pq})^m$, followed by the summation over m, requires $\mathcal{O}(N_x N_y M^{\mathrm{TE}})$ operations. Consequently, the total computational cost of the third term in Eq. (2) is replaced by $T_{\mathrm{add3}}^{\mathrm{TE}} = \gamma N_x N_y M^{\mathrm{TE}}(N_z + \log N_y + 1)$. The same procedure is applied to the other additional quantities. For quantities expressed using a sine basis, the corresponding Taylor expansion coefficients are given by

$$S_{l,2m} = \frac{(-1)^m (k_l^z)^{2m}}{(2m)!} \sin(k_l^z h) \quad ; \quad S_{l,2m+1} = \frac{(-1)^m (k_l^z)^{2m+1}}{(2m+1)!} \cos(k_l^z h). \tag{8}$$

As shown on the right-hand side of Fig. 1, the proposed TE-accelerated `HOS-NWT` provides a substantial speed-up compared to the classical formulation, particularly for large values of N_y. It should be noted, however, that the present approach may lose accuracy when the Taylor expansion order is insufficient.

3 RESULTS

To validate the proposed approach, a strongly nonlinear Directional Frequency Focusing Wave (DFFW), which is close to breaking, is generated using both the TE-accelerated `HOS-NWT` and the original `HOS-NWT`. The numerical results are compared with experiments, which were conducted in the Ocean Engineering tank of École Centrale de Nantes. This wave tank has dimensions $L_x = 46.2\,\mathrm{m}$ in length, $L_y = 29.7\,\mathrm{m}$ in width, and a water depth of $h = 5\,\mathrm{m}$. A 48 flap-type wavemaker is installed on one side, with an absorbing beach on the opposite side.

The DFFW is generated using a standard directional frequency focusing approach based on the superposition of multiple frequency and directional components. The wave phases are selected so as to focus wave energy at a prescribed location $(x_f, y_f) = (18.0\,\mathrm{m}, 15.0\,\mathrm{m})$ and at time $t_f = 45\,\mathrm{s}$, resulting in a highly nonlinear event. The free-surface elevation at the wavemaker, together with an appropriate transfer function, is used to prescribe the wavemaker motion. The frequency spectrum is defined by a constant steepness with a central frequency $f_c = 0.703\,\text{Hz}$, a bandwidth $\delta f = 0.781\,\text{Hz}$, and frequency bounds $f_{\min} = 0.3125\,\text{Hz}$ and $f_{\max} = 1.0935\,\text{Hz}$. In the present simulations, the target wave amplitude is set to $A = 0.3\,\text{m}$ with a directional spreading of 45°, and the setup is given by: $N_x = 128$, $N_y = 64$, $N_z = 32$, $M^{\text{TE}} = M = p = 5$. The acceleration achieved for this specific configuration is four.

The four left-hand panels of Fig. 2 show perfect agreement between the classical `HOS-NWT` simulation and the proposed TE-accelerated one, and excellent comparison with the experimental measurements at wave gauges BG12 (17.47 m, 14.87 m), BG09 (17.98 m, 14.87 m), BG02 (18.75 m, 15.23 m), and BG04 (18.78 m, 14.50 m). These results demonstrate that the proposed acceleration strategy preserves the accuracy of the original formulation, even for strongly nonlinear wave conditions (near-breaking event studied here).

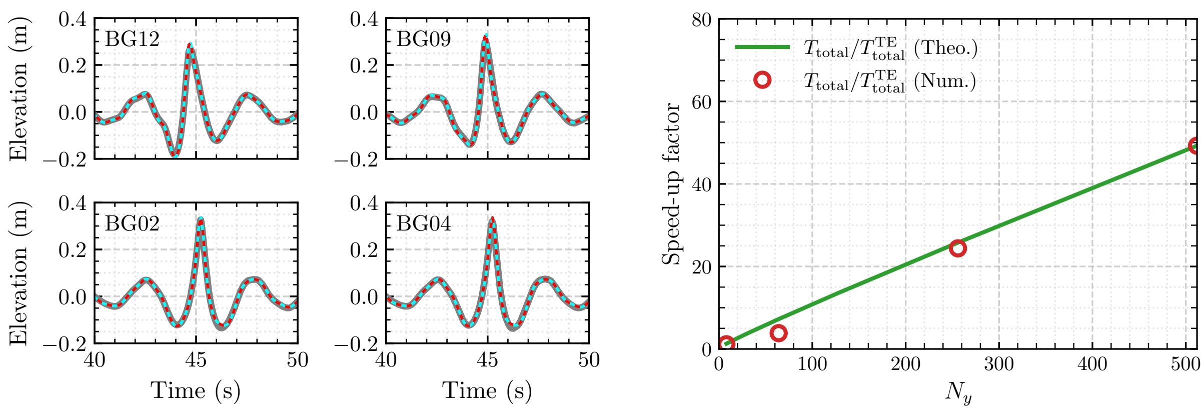

Figure 2: (Left) Comparison between the classical `HOS-NWT` (——), the TE-accelerated `HOS-NWT` (- - -), and the experiment (——). (Right) Comparison between the theoretical and the actual speed-up factor in the `HOS-NWT` simulation.

The right-hand panel of Fig. 2 compares the speed-up factor predicted by the theoretical cost analysis with that measured in the `HOS-NWT` simulations as a function of N_y. A very good agreement is observed, confirming the effectiveness of the proposed Taylor expansion-based acceleration, particularly for large transverse resolutions. Other test cases will be presented during the workshop, and the proposed acceleration enables the efficient simulation of large-scale short-crested seas in an NWT.

REFERENCES

[1] Ducrozet, G., Bonnefoy, F., Le Touzé, D., and Ferrant, P. 2016. *HOS-ocean: Open-source solver for nonlinear waves in open ocean based on High-Order Spectral method. Computer Physics Communications 203*, 245–254.

[2] Ducrozet, G., Bonnefoy, F., Le Touzé, D., and Ferrant, P. 2012. *A modified high-order spectral method for wavemaker modeling in a numerical wave tank. European Journal of Mechanics-B/Fluids 34*, 19–34.

[3] Bonnefoy, F., Ducrozet, G., Le Touzé, D., and Ferrant, P. Time domain simulation of nonlinear water waves using spectral methods. In *Advances in numerical simulation of nonlinear water waves*. World Scientific, 2010, pp. 129–164.