Response of a Porous Viscoelastic Ice Plate to External Pressure under Wind and Current Effects

Linhua Sun[a], **Baoyu Ni**[a], **Alexander Korobkin**[b], **Hang Xiong**[a], **Tatyana Khabakhpasheva**[b]

[a]College of Shipbuilding Engineering, Harbin Engineering University, Harbin, China
[b]School of Mathematics, University of East Anglia, UK

Email:nibaoyu@hrbeu.edu.cn

1 INTRODUCTION

In recent years, analytical models of wave propagation in ice-covered oceans have evolved from classical elastic plate formulations to viscoelastic and porous viscoelastic descriptions. Early elastic plate models successfully captured flexural gravity wave dispersion but lacked intrinsic damping and therefore underestimated wave attenuation observed in the field [1]. To remedy this, viscoelastic constitutive laws, typically of Kelvin-Voigt type, were introduced, yielding complex dispersion relations and frequency-dependent attenuation [2, 3]. More recently, porous viscoelastic models have been developed, treating ice as a fluid-saturated porous medium and introducing additional dissipation due to fluid-solid interaction, which enables more complex attenuation behavior [4, 5]. Further refinements of hydroelastic dispersion relations have also been reported [6]. Despite these advances, most analytical studies assume quiescent conditions; although current effects beneath ice have been considered in a few works [7], and wind-induced hydroelastic wave phenomena have been analyzed separately [8], analytical frameworks incorporating coupled wind, current, and ice effects remain relatively limited.

2 FORMULATION OF THE PROBLEM

The propagation of flexural–gravity waves generated by a periodic external load is investigated in the presence of wind above the ice cover and a current beneath it. The ice cover is modeled as a thin porous viscoelastic plate. The flow is described in the Cartesian coordinate system Oxy, where $y = 0$ corresponds to the interface between ice and water at equilibrium. The plate is of constant thickness h. The air layer is of thickness H_1 and the water is of depth H_2. The air and water are of densities ρ_1 and ρ_2, respectively. The ambient velocities of the air and water flows are U_1 and U_2, respectively. Both the air and water are inviscid and incompressible, with their flows being two–dimensional and potential. The velocity potential in each fluid layer is expressed as

$$\Phi_j^U(x, y, t) = U_j x + \Phi_j(x, y, t) \tag{1}$$

where for the air $j = 1$ and $j = 2$ for the water. Here $\Phi_j(x, y, t)$ are the wave-induced perturbation potentials. Within the linear wave theory, Φ_j satisfies the Laplace equation in each respective fluid domain,

$$\nabla^2\Phi_j = 0, \quad -\infty < x < +\infty \quad \text{and} \quad \begin{cases} 0 < y < +H_1 & \text{for } j = 1, \\ -H_2 < y < 0 & \text{for } j = 2, \end{cases} \tag{2}$$

the top and the bottom boundary condition,

$$\frac{\partial \Phi_1}{\partial y} = 0 \quad y = H_1, \qquad \frac{\partial \Phi_2}{\partial y} = 0 \quad y = -H_2, \tag{3}$$

the linearised kinematic boundary condition on the air/plate and the plate/water interface,

$$\frac{\partial \Phi_1}{\partial y} = \left(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x} \right) W, \quad y = 0^+ \tag{4}$$

$$\frac{\partial \Phi_2}{\partial y} = \left(\frac{\partial}{\partial t} + U_2 \frac{\partial}{\partial x} \right) W + \alpha (P_2 - P_1), \quad y = 0^- \tag{5}$$

where $W(x,t)$ is the plate deflection positive upwards and the parameter α describes the porosity of the ice plate. In a more general scenario where the plate is immersed in fluid, the normal velocity is expressed as $v_n = \alpha(p_- - p_+)$, where p_- is the hydrodynamic pressure on the lower surface of the plate, and p_+ is the hydrodynamic pressure above the fluid that has permeated into the porous plate. The dynamic boundary condition on the viscoelastic plate subjected to external pressure reads

$$\left[EI \left(1 + \eta \frac{\partial}{\partial t} \right) \frac{\partial^4}{\partial x^4} + M \frac{\partial^2}{\partial t^2} \right] W + P_{\text{ext}}(x,t) = P_2(x,t) - P_1(x,t), \quad y = 0 \tag{6}$$

where η is the viscosity coefficient of the ice plate, $M = \rho_i h$ is the mass of the ice per unit area, ρ_i is the ice density. $P_{\text{ext}}(x,t) = p_{\text{ext}}(x) \cos(\omega t)$ is the function describing the distribution of the given external load along the plate, $P_{\text{ext}}(x) \to 0$ as $|x| \to \infty$, and ω is the frequency of the load, $EI = Eh^3 / \big(12(1-\nu^2)\big)$ is the flexural rigidity of the plate, where E is Young's modulus and ν is Poisson's ratio. The hydrodynamic pressure $P_j(x,t)$ ($j = 1$ for air and $j = 2$ for water) is described by the linearised Bernoulli equation

$$P_j(x,t) = -\rho_j \left[\left(\frac{\partial}{\partial t} + U_j \frac{\partial}{\partial x} \right) \Phi_j(x,0,t) + gW \right] \tag{7}$$

where ρ_j is the fluid density, and g is the gravitational acceleration. Note that in equilibrium without any external load, we have $\Phi_j = 0$ and $W = 0$, which implies that the reference ambient pressure is selected in such a way that $P_1 = P_2 = 0$ on $y = \pm 0$ correspondingly. Due to the porosity of the viscoelastic plate, there is a damping in air/plate/water system. Therefore the following conditions at infinity far away from the load should be satisfied,

$$W \to 0, \quad \Phi_j \to 0 \quad (|x| \to \infty). \tag{8}$$

3 DISPERATION RELATION

For linear propagating waves with $P_{ext} = 0$, we seek the functions $W(x,t)$ and $\Phi_j(x,y,t)$ in the form

$$\begin{cases} W(x,t) = \text{Re}\left\{ w \cdot e^{i(kx - \omega t)} \right\} \\ \Phi_j(x,y,t) = \text{Re}\left\{ \phi_j(y) \cdot e^{i(kx - \omega t)} \right\} \end{cases} \tag{9}$$

where w is the amplitude of the plate deflection, k and ω are the wavenumber and frequency of the plane wave, respectively. $\phi_j(y)$ is the spatial velocity potential. Equations (2)-(7) are transformed into

$$\phi_j'' - k^2\phi_j = 0, \qquad \begin{cases} 0 < y < H_1, & j = 1, \\ -H_2 < y < 0, & j = 2, \end{cases} \tag{10}$$

where potentials $\phi_j(y)$ satisfy equations (10) and the boundary conditions

$$\phi_1' = 0 \quad (y = +H_1), \qquad \phi_2' = 0 \quad (y = -H_2), \tag{11}$$

$$\phi_1' = (-i\omega + ikU_1)\, w \quad (y = +0), \tag{12}$$

$$\phi_2' = \big(-i\omega + ikU_2\big) w + \alpha[\rho_1(-i\omega + ikU_1)\phi_1 - \rho_2(-i\omega + ikU_2)\phi_2 + (\rho_1 - \rho_2)gw] \;\; (y = -0), \tag{13}$$

$$\big[EI(1 - i\eta\omega)k^4 - M\omega^2\big] w = \rho_1(-i\omega + ikU_1)\phi_1 - \rho_2(-i\omega + ikU_2)\phi_2 + (\rho_1 - \rho_2)g \;\; w(y = 0). \tag{14}$$

A general solutions of (10) which satisfy conditions (11) read

$$\phi_1(y) = A_1 \cosh[k(H_1 - y)], \quad \phi_2(y) = A_2 \cosh[k(y + H_2)]. \tag{15}$$

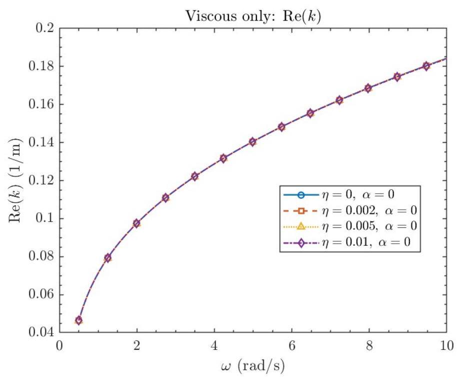
Substituting (15) in (12)-(14), we arrive at a homogeneous system of three algebraic equations with respect to A_1, A_2 and w. The system has a nontrivial solution if its determinant is equal to zero, which leads to the following dispersion relation,

$$\begin{aligned} EI(1 - i\eta\omega)\, k^4 - M\omega^2 - (\rho_1 - \rho_2)g = {} & \frac{\rho_1(\omega - kU_1)^2}{k\tanh(kH_1)} + \frac{\rho_2(\omega - kU_2)^2}{k\tanh(kH_2) - i\alpha\rho_2(\omega - kU_2)} \\ & + \frac{i\alpha\rho_2(\omega - kU_2)\left[\dfrac{\rho_1(\omega - kU_1)^2}{k\tanh(kH_1)} + (\rho_1 - \rho_2)g\right]}{k\tanh(kH_2) - i\alpha\rho_2(\omega - kU_2)} \end{aligned} \tag{16}$$

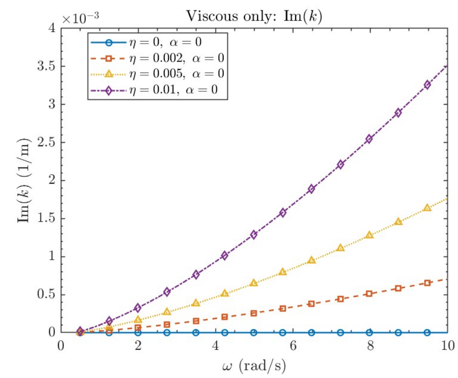
On the right-hand side of the dispersion relation, the first term comes from the air layer, capturing the dynamic pressure of the airflow. The second term reflects the water layer's response, modified by the porosity parameter α. The last term represents the interaction between air and water through the ice cover.

4 DISPERSION UNDER DISTINCT DISSIPATION MECHANISMS

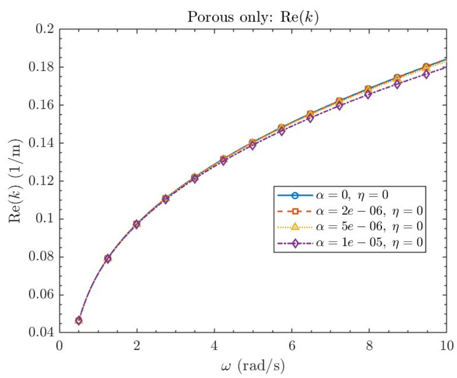
we distinguish two distinct attenuation mechanisms embedded in the dispersion relation. The viscoelastic parameter η enters the plate constitutive relation through the complex bending rigidity $EI(1 - i\eta\omega)$, representing intrinsic structural dissipation of the ice cover, whereas the porosity parameter α appears in the hydrodynamic terms of the water layer and introduces a complex correction associated with $(\omega - kU_2)$, accounting for seepage-induced energy loss and phase lag due to fluid exchange through the porous ice. As illustrated in Fig. 1, viscoelastic dissipation mainly affects the attenuation of hydroelastic waves, leading to a frequency-dependent increase in the imaginary part of the wavenumber while leaving the real part almost unchanged. In contrast, porous dissipation modifies both the real and imaginary parts of the wavenumber, indicating that porosity influences not only wave attenuation but also the dispersion characteristics through hydrodynamic coupling.



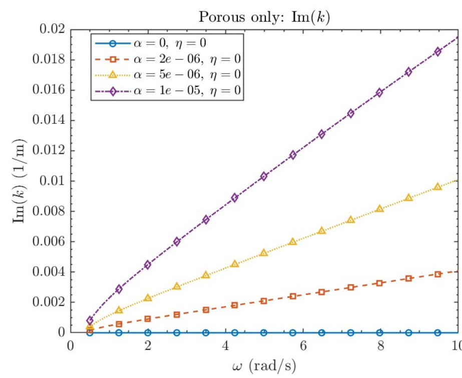
(a) Real part of the wavenumber accounting for viscoelastic dissipation



(b) Imaginary part of the wavenumber accounting for viscoelastic dissipation



(c) Real part of the wavenumber accounting for porous dissipation



(d) Imaginary part of the wavenumber accounting for porous dissipation

Figure 1: Dispersion characteristics of hydroelastic waves under combined wind and current. Top row: viscoelastic dissipation ($\alpha = 0$). Bottom row: porous-flow-induced dissipation ($\eta = 0$). Left and right columns show the real and imaginary parts of the wavenumber, respectively. The parameters are $U_1 = 8$ m/s, $U_2 = 0.2$ m/s, $H_1 = 1$ m, $H_2 = 10$ m, $h = 0.05$ m, and $EI = 5 \times 10^8$ N/m^2.

REFERENCES

[1] Fox, C., and Squire, V. A. 1994. *On the oblique reflection and transmission of ocean waves at shore fast sea ice*. Philosophical Transactions of the Royal Society of London. Series A: Physical and Engineering Sciences 347(1682), 185–218.

[2] Wang, R., and Shen, H. H. 2010. *Gravity waves propagating into an ice-covered ocean: A viscoelastic model*. Journal of Geophysical Research: Oceans 115(C6), C06024.

[3] Mosig, J. E. M., Montiel, F., and Squire, V. A. 2015. *Comparison of viscoelastic-type models for ocean wave attenuation in ice-covered seas*. Journal of Geophysical Research: Oceans 120(9), 6072–6090.

[4] Chen, H., Gilbert, R. P., and Guyenne, P. 2019. *Dispersion and attenuation in a porous viscoelastic model for gravity waves on an ice-covered ocean. European Journal of Mechanics - B/Fluids 78*, 88–105.

[5] Xu, B., and Guyenne, P. 2022. *Assessment of a porous viscoelastic model for wave attenuation in ice-covered seas. Applied Ocean Research 122*, 103122.

[6] Korobkin, A., and Khabakhpasheva, T. 2025. *Consistent Models of Flexural-Gravity Waves in Floating Ice*. Journal of Marine Science and Engineering 13(6), 1191.

[7] Bhattacharjee, J., and Sahoo, T. 2007. *Interaction of current and flexural gravity waves. Ocean Engineering 34*, 1505–1515.

[8] Boral, S., Sahoo, T., and Stepanyants, Y. 2021. *Modulation Instability of Hydro-Elastic Waves Blown by a Wind with a Uniform Vertical Profile*. Fluids 6(12), 458.