Utilizing Localized Spectral Moments to Predict Conditions for Sway Instability in a Point Absorber

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HIGHLIGHTS

- A data-driven framework is derived to detect Mathieu-type sway instabilities in floating systems.
- This method accurately classifies and predicts instability regions in regular and irregular sea states.
- Results show stiffness-based dynamics alone are sufficient to infer sway stability.

1 INTRODUCTION

The accurate identification, prediction, and prevention of motion instabilities in floating systems remain major challenges in ocean engineering. Such instabilities can arise as a nonlinear coupling between dynamical degrees of freedom, and occur in both regular and irregular waves. While the resulting motions are often readily observed, the predictability and underlying causes remain poorly understood due to the complex, nonlinear nature of the marine environment.

This study presents a data-driven framework for the detection and prediction of Mathieu-type instabilities in coupled heave–sway motion of a point-absorbing wave energy converter (WEC). A supervised classification approach is employed, leveraging statistical and spectral features Fourier transformed from the vertical motion. The characteristic parameters of the Mathieu equation are formulated for irregular wave conditions, where the sway stiffness is expressed as a function of heave displacement, velocity, and acceleration, while the natural frequency is governed by the system's structural properties and added mass. A moving window is utilised to capture localised dynamics within time-series data obtained from tests conducted at the University of Plymouth COAST Laboratory whilst a comprehensive set of statistical and non-dimensional engineering features is extracted for supervised clustering and model training. Features capable of capturing the conditions for parametric resonance are identified.

2 THEORY

From the work of [1], consider a cylindrical floating buoy of radius R moving in three translational degrees of freedom, $(x(t), y(t), z(t))$ with incident waves propagating from the x-direction. The buoy is connected to a linear generator where the translator moves only vertically, $\zeta(t)$. The motion of the buoy and translator are coupled by the line force with magnitude $F_{\mathrm{l}}(t)$ as

$$m_b\ddot{x}(t) = F_{\mathrm{e},x}(t) + F_{\mathrm{r},x}(t) + F_{\mathrm{l},x}(t) \tag{1}$$

$$m_b\ddot{y}(t) = F_{\mathrm{r},y}(t) + F_{\mathrm{l},y}(t) \tag{2}$$

$$m_b\ddot{z}(t) = F_{\mathrm{e},z}(t) + F_{\mathrm{r},z}(t) + \rho g\pi R^2(d - z(t)) - m_b g + F_{\mathrm{l},z}(t) \tag{3}$$

$$m_t\ddot{\zeta}(t) = F_{\mathrm{l}}(t) + F_{\mathrm{PTO}}(t) - m_t g \tag{4}$$

where F_{e} and F_{r} are the excitation and radiation forces, $F_{\mathrm{PTO}} = -\gamma\dot{\zeta}$ is the generator damping force, d is the draft of the buoy at equilibrium, and m_b, m_t are the masses of the buoy and translator. Note that the sway motion is not affected by a hydrodynamic excitation force, and that the hydrostatic restoring force acts only in heave. Using the geometry in [1], the sway dynamics can be recast to fit the Mathieu type equation,

$$\ddot{y}(t) + \left[\omega_0^2 + q(t)\right]y(t) = 0, \tag{5}$$

Where $q(t)$ represents the system's sway time-varying stiffness coefficient, and ω_0 the damped natural frequency dependant on the translator length L, the mass of the buoy and added mass such that $m_x = m_b + m_{11}$,

$$\omega_0^2 = \frac{m_t g}{m_x L}, \qquad q(t) = \frac{1}{m_x L}\left(\gamma \dot{z}(t) + m_t \ddot{z}(t) - \frac{m_t g}{L} z(t)\right). \tag{6}$$

3 METHODOLOGY

An important property of the governing dynamics is that the stiffness parameter $q(t)$ depends solely on the heave displacement, velocity, and acceleration, and not explicitly on the sway response. This motivates a methodology that seeks to identify heave-based signatures indicative of sway instability.

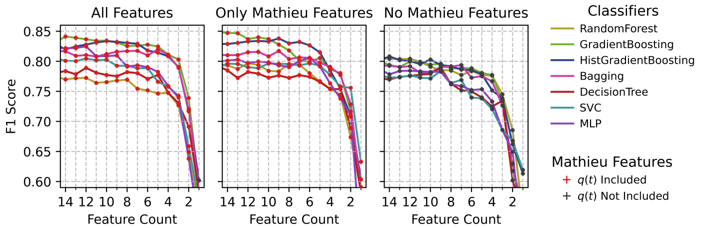
Short-term localized dynamics are captured using a sliding-window approach applied to experimental time-series data obtained from a point-absorber WEC. Each window yields localized time and frequency-domain statistics derived from the wave elevation $\eta(t)$, heave $z(t)$, stiffness $q(t)$, and their derivatives. Power spectral densities (PSDs) are estimated using Welch's method [2] leading to relative spectral moments of η, z, q and used to quantify localized frequency content across successive windows as

$$m_n^{\{\eta,z,q\}} = \int S_{\{\eta,z,q\}} \omega^n d\omega. \tag{7}$$

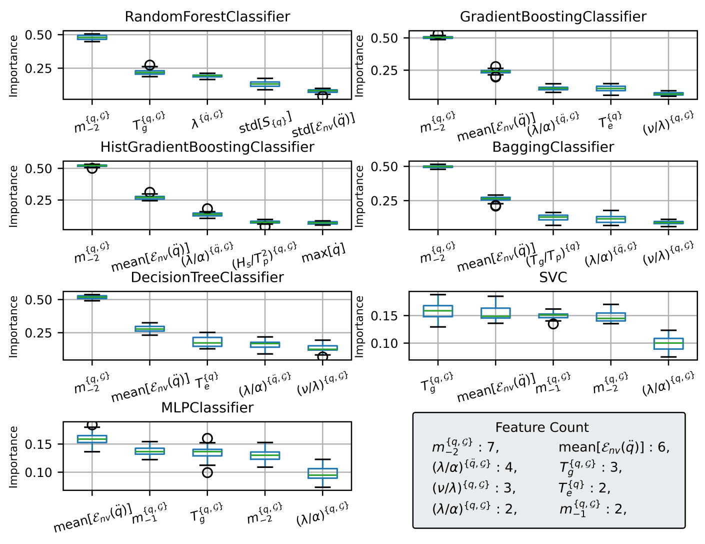
The selection of window width, step size, and Welch segmentation reflects a trade-off between temporal localization, spectral resolution, and estimator variance. The sampling frequency is 128 Hz, the step size $s = 512$ samples, the window width 2048 samples, segment length 1024 samples, and the frequency resolution 0.125 Hz. The natural frequency ω_0 of the Mathieu equation is estimated locally for each window by assuming a constant added mass over the window duration. This is found by obtaining the up-crossing period from the zeroth and second spectral moments, from which the added mass is inferred using a WAMIT transfer function.

Data labelling for supervised learning is performed using median significant amplitudes in heave and sway, $H_s^{\{z,y\}}$, computed from the zeroth spectral moment across all windows. Four stability regimes are defined based on combinations of high and low heave and sway amplitudes. While sway amplitude is used to define labels, features included in model training are not derived from sway, ensuring classifiers rely exclusively on features derived from heave. A comprehensive set of physics-informed features is extracted within each window. Time-domain features include statistical moments and Hilbert envelope metrics capturing localized growth or decay. Frequency-domain features are derived from spectral moments and their associated non-dimensional parameters, including bandwidth, irregularity, and narrowness measures. To emphasize dynamics associated with parametric resonance, PSDs are weighted using a Gaussian function centered at $2\omega_0$ [3], corresponding to the dominant Mathieu instability region. Prior to training, features are filtered using Pearson correlation with the significant sway amplitude, discarding the lowest 10% contributors. Redundant features exhibiting strong mutual correlation are further reduced using a bagging-based selection procedure. The remaining features are scaled and evaluated using repeated stratified K-fold cross-validation [4]. Model performance is quantified using the f_1-score, while feature relevance is assessed via permutation importance [5]. A combined cost metric is used to rank features based on both importance and consistency across folds, $I_f^{\text{cost}} = \overline{I}_f - \frac{1}{2}\sigma_{I_f}$.

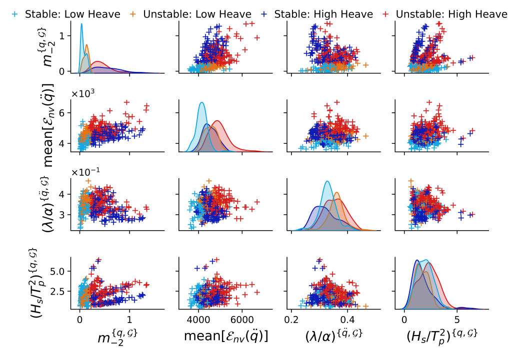
Feature reduction is performed iteratively across multiple classifiers, including ensemble methods such as Random Forests [5] and Gradient Boosting [6]. Feature sets are progressively reduced while tracking predictive performance, enabling identification of compact, high-performing subsets. To



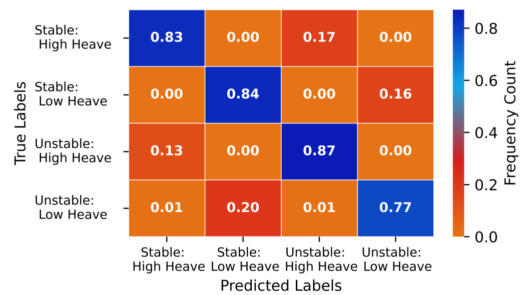
(a) Classifier F_1-score with decreasing feature-sets for the three feature regimes: full vertical-axis, Mathieu-only, and wave–heave.



(b) Permutation importance box plots of the most frequently selected Mathieu-based features across all classifiers.



(c) Pairwise correlations for the Gradient Boosting classifier trained on the top four Mathieu features.



(d) Confusion matrix for the Gradient Boosting classifier trained on the top ten Mathieu features.

Figure 1: Feature selection and prediction results.

assess the influence of Mathieu-related features, the analysis is repeated for two additional cases: one using only features from wave elevation and heave, and another using only stiffness-based features centered around $2\omega_0$.

4 RESULTS

Following correlation-based filtering and feature reduction, the majority of retained features were associated with the Mathieu stiffness parameter $q(t)$ and its Gaussian-weighted variants. Bagging based on mutual correlation found that Gaussian-weighted features consistently exhibited marginally stronger correlations with the significant sway amplitude $H_s^{\{y\}}$ than their unweighted counterparts. Progressive feature elimination enabled identification of a compact subset of features capable of maintaining stable predictive performance. As seen in Figure 1a, all classifiers achieved F_1-scores between 75% and 85% regardless of the features available. However, restricting the feature-set to Mathieu-related parameters did not degrade performance compared to the feature-set, but resulted in improvements of up to 5% relative to classifiers trained on wave elevation and heave alone, indicating that stiffness-based dynamics both yield improved performance and are sufficient on their own to infer sway stability.

Based on consistent performance, the Gradient Boosting classifier was selected for detailed analysis under the Mathieu-only feature regime. Figure 1d shows the normalized confusion matrix for this model trained on ten features. The classifier reliably distinguished between high and low heave labels, with most errors occurring associated with cases involving low heave with unstable sway, or high heave with stable sway, consistent with less common or transitional dynamical states.

The five strongest features across all classifiers are summarized in Figure 1b. The Gaussian-weighted spectral moment $m_{-2}^{\{q,\mathcal{G}\}}$ was consistently selected by all classifiers, while the mean Hilbert envelope of $\ddot{q}$, mean$[\mathcal{E}_{nv}(\ddot{q})]$, appeared in the majority. Dimensionless ratios derived from spectral moments, including bandwidth and irregularity measures were also repeatedly selected, together with features indicative of low-frequency Mathieu-type modulation, such as negative-order spectral moments and characteristic time periods (T_g and T_e), suggestive of excitation frequencies $\Omega = 2\omega_0/n$ for $n > 1$ [7]. Feature correlations for the four highest-ranked features of the Gradient Boosting classifier are shown in Figure 1c. The feature $m_{-2}^{\{q,\mathcal{G}\}}$ provides a clear separation between high and low heave regimes, explaining its high importance across classifiers. While this feature alone yields moderate classification performance, additional features related to the statistical behaviour of $\ddot{q}$ are required to distinguish stable and unstable sway. The combined contribution of these features increases the F_1-score from approximately 75% to 80% when using only four features, indicating applicability for reduced-order models and faster predictions.

Finally, the methodology was evaluated under regular wave conditions using nine discrete frequencies and amplitudes. Here, ensemble-based classifiers achieved F_1-scores of up to 95%–98%, with no significant performance difference observed between Mathieu-informed and non-Mathieu feature sets, confirming the validity of the framework across wave regimes.

5 CONCLUSIONS

This study presents a first application of machine learning methods in detecting Mathieu-type instabilities for offshore floating structures. Here, a data-driven framework is implemented to detect and predict Mathieu-type sway instabilities in a point-absorber wave energy converter using features solely derived from heave motion. By exploiting localized stiffness and frequency parameters governed by Mathieu-based vertical-axis dynamics, horizontal instability was inferred independently of sway measurements.

Using data recorded at the University of Plymouth's COAST Laboratory, This methodology was found to be reliable under both regular and irregular wave conditions. While individual features showed limited predictive capability, features indicative of low-frequency dynamics and those derived from the Mathieu stiffness parameter formed strong collective indicators of sway instability. Compact feature sets enabled classification accuracies of 75%–85%, demonstrating the framework's ability to capture dynamics linked with parametric resonance.

These findings establish the foundations for future work in real-time classification of parametric instabilities in floating offshore systems using a novel machine learning approach, with clear potential to identify and mitigate unwanted wave conditions associated with non-linear instabilities.

REFERENCES

[1] Giassi, M., Thomas, S., Tosdevin, T., Engström, J., Hann, M., Isberg, J., and Göteman, M. 2020. *Capturing the experimental behaviour of a point-absorber WEC by simplified numerical models. J. Fluids Struct. 99*, 103143.

[2] Welch, P. 1967. *The use of fast Fourier transform for the estimation of power spectra.* IEEE Trans. Audio Electroacoust. 15(2), 70–73.

[3] Brouwers, J. J. H. 2011. *Asymptotic solutions for Mathieu instability under random parametric excitation and nonlinear damping.* Physica D: Nonlinear Phenomena 240(12), 990–1000.

[4] Kohavi, R. A study of cross-validation and bootstrap for accuracy estimation and model selection. In *Proc. IJCAI* (1995), pp. 1137–1143.

[5] Breiman, L. 2001. *Random Forests.* Machine Learning 45(1), 5–32.

[6] Friedman, J. H. 2001. *Greedy function approximation: A gradient boosting machine.* Ann. Stat. 29(5), 1189–1232.

[7] Kovačić, I., Rand, R., and Sah, S. M. 2018. *Mathieu's Equation and Its Generalizations.* Springer, Cham.