On the Resonant Responses of Twin Inter-Connected Floating Bodies under Water Wave Actions

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HIGHLIGHTS

A coupled dynamic model was proposed for predicting the resonant frequency of twin floating bodies with inter linear elastic connection subjected to regular water waves. Different resonant modes were identified based on viscous numerical simulations. Attentions were further paid on the dynamic responses of floating bodies connected with inter-cables and fenders. The results are compared with those obtained with simplified spring connection.

ABSTRACT

The resonance behavior between twin inter-connected rectangular floating bodies with sway motions in side-by-side arrangement under regular water wave actions was numerically investigated based on a two-dimensional viscous fluid model and a potential flow model. Two categories of the inter-connections are considered: (1) a simplified linear spring, and (2) an assembly of cables and fenders, where no compression can be applied on the former and no tension on the latter. A coupled analytical model was proposed based on the potential flow theory, which is able to produce accurate predictions on the resonant frequencies. Three resonant modes were identified in the study, including one in-phase mode and two out-of-phase modes in terms of the motions of the twin floating bodies and the oscillation of the fluid bulk between the bodies. Numerical results show that the intrusion of the interconnecting spring may lead to significant changes of the two out-of-phase resonance frequencies. But it has a rather limited effect on the in-phase mode. As the spring connection is replaced with the real cables and fenders, which are modelled by different stiffness parameters and displacement limiters, the dynamic responses of the coupling system exhibit rich nonlinearities, including the harmonic responses with half and one-and-a-half-wave frequency, and net shift from the initial equilibrium position. The present study shows evidence that widely used spring approximation may lead to unexpected uncertainties, especially the missing of critical nonlinearity.

Key words: inter-connection; coupled analytical model; resonance modes

As illustrated in Fig.1(a), a side-by-side twin-body system composed of two identical floating boxes are placed in the middle of the numerical wave flume and subjected to regular water waves. The twin floating bodies are restricted to sway motion only in this study. Referring to Fig.1(b), each floating body is connected to a horizontal spring with an identical stiffness $k_{11}^{AA} = k_{11}^{BB} = k_{11}$. And an inter-connected spring is set between the two bodies with a different stiffness k_{11}^{AB}. Such that, the coupled dynamic system comprises two floating bodies with sway motions and the quasi-vertical oscillating fluid mass in the narrow gap between the twin bodies.

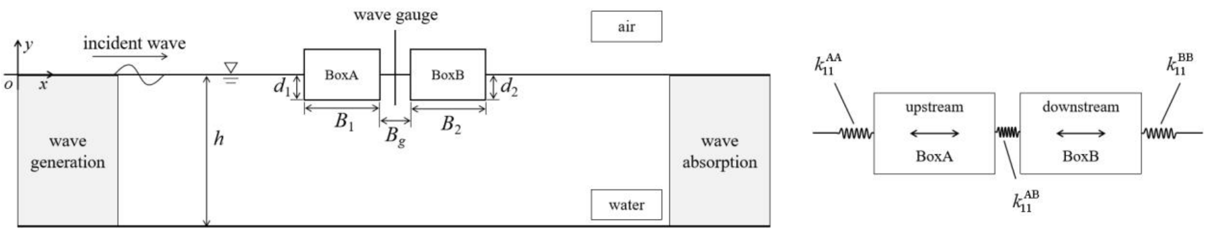


Figure 1: Schematic diagram of two-dimensional viscous flow model for studying the wave-induced coupling resonance behavior of two closely spaced floating bodes in a side-by-side arrangement.

Based on the linear potential flow theory, the motion equations of the two bodies can be expressed as follows in the frequency-domain,

$$\begin{cases}\left[-\omega^2 M a_{11}^{\mathrm{AA}}(\omega)-\mathrm{i}\omega b_{11}^{\mathrm{AA}}(\omega)+k_{11}^{\mathrm{AA}}+k_{11}^{\mathrm{AB}}\right]\zeta_1^{\mathrm{A}}+\left[-\omega^2 a_{11}^{\mathrm{AB}}(\omega)-\mathrm{i}\omega b_{11}^{\mathrm{AB}}(\omega)-k_{11}^{\mathrm{AB}}\right]\zeta_1^{\mathrm{B}}=f_{ex1}^{\mathrm{A}}\\ \left[-\omega^2 a_{11}^{\mathrm{BA}}(\omega)-\mathrm{i}\omega b_{11}^{\mathrm{BA}}(\omega)-k_{11}^{\mathrm{BA}}\right]\zeta_1^{\mathrm{A}}+\left[-\omega^2 M a_{11}^{\mathrm{BB}}(\omega)-\mathrm{i}\omega b_{11}^{\mathrm{BB}}(\omega)+k_{11}^{\mathrm{BB}}+k_{11}^{\mathrm{BA}}\right]\zeta_1^{\mathrm{B}}=f_{ex1}^{\mathrm{B}}\end{cases} \tag{1}$$

For conditions without considering any dissipative effects, the natural frequencies of the motions can be obtained through Eq. (2) as below,

$$\omega_{n1}=\sqrt{\frac{k_{11}+2k_{11}^{\mathrm{AB}}}{m+a_{11}^{\mathrm{AA}}(\omega_{n1})-a_{11}^{\mathrm{AB}}(\omega_{n1})}},\quad \omega_{n2}=\sqrt{\frac{k_{11}}{m+a_{11}^{\mathrm{AA}}(\omega_{n2})+a_{11}^{\mathrm{AB}}(\omega_{n2})}} \tag{2}$$

The radiation damping ratios associated with the sway motions can be written as follows,

$$\mu_1(\omega)=\frac{b_{11}^{\mathrm{AA}}(\omega)-b_{11}^{\mathrm{AB}}(\omega)}{2\sqrt{\left(k_{11}+2k_{11}^{\mathrm{AB}}\right)\left(m+a_{11}^{\mathrm{AA}}(\omega)-a_{11}^{\mathrm{AB}}(\omega)\right)}},\quad \mu_2(\omega)=\frac{b_{11}^{\mathrm{AA}}(\omega)+b_{11}^{\mathrm{AB}}(\omega)}{2\sqrt{k_{11}\left(m+a_{11}^{\mathrm{AA}}(\omega)+a_{11}^{\mathrm{AB}}(\omega)\right)}} \tag{3}$$

By substituting Eq. (2) and (3) into Eq. (4), the resonant frequencies are ready to be obtained:

$$\omega_{dni}=\omega_{ni}\sqrt{1-\left(\mu_i(\omega_{dni})\right)^2},\quad (i=1,\ 2) \tag{4}$$

The resonant frequencies are identified in Fig. 2, where the mooring stiffness of the bodes holds k_{11} = 0.2c_{22}, and the inter-connection stiffness is k_{11}^{AB} = 0.5c_{22} with c_{22} the vertical hydrostatic restoring stiffness of the floating bodies. Fig. 2 shows three resonant frequencies that correspond to the intersections between the curves of ω^2 and ω_{dn}^2. Numerical results show one has an in-phase mode of the sway motion of the twin bodies, which is confirmed to be essentially dependent on the mooring stiffness k_{11}. The other two hold an out-of-phase mode of the sway motions, further distinguished by the large amplitude fluid oscillation in the narrow gap, referring to as the gap resonance problem[1-3]. The latter two out-of-phase modes are controlled by the collaboration of the mooring stiffness k_{11} and the inter-spring stiffness k_{11}^{AB}. Fig. 3 illustrates the resonant modes associated with the motions of the floating bodies as well as the fluid in the gap.

Figure 4 depicts the amplitude-frequency responses for the cases with and without the interconnecting spring based on both potential and viscous flow models. Numerical solutions suggest significant resonant peaks. The peak frequencies agree well with the analytical solutions, as shown in Fig. 2. Much similar sway motion responses can be found for the cases with and without inter-connection around the first resonance frequency, which indicates the rather limited influence of the inter-connection on the in-phase mode. After introducing the interconnecting spring, the second and third resonant frequencies shift to higher values, demonstrating the effect of the inter-connection stiffness on the out-of-phase motion modes of the twin floating bodies. Two significant relative motion amplitude peaks can be found in the figure, corresponding to the above two out-of-phase resonance frequencies.

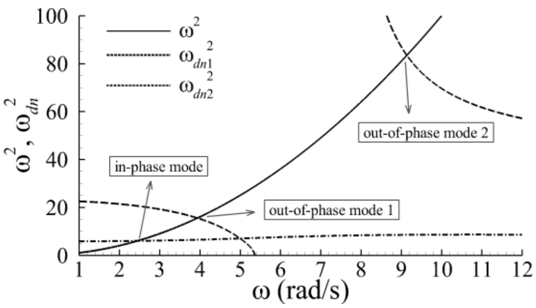


Figure 2: Identification of the system resonant frequencies.

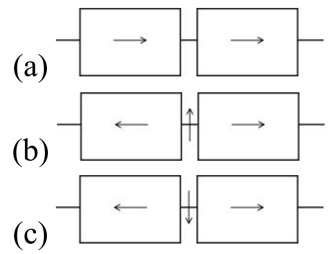


Figure 3: Illustration of the resonance modes. The sequence of the resonance modes corresponds to the frequency order from low to high under the present parameters, (a) in-phase mode, (b) out-of-phase mode 1, (c) out-of-phase mode 2.

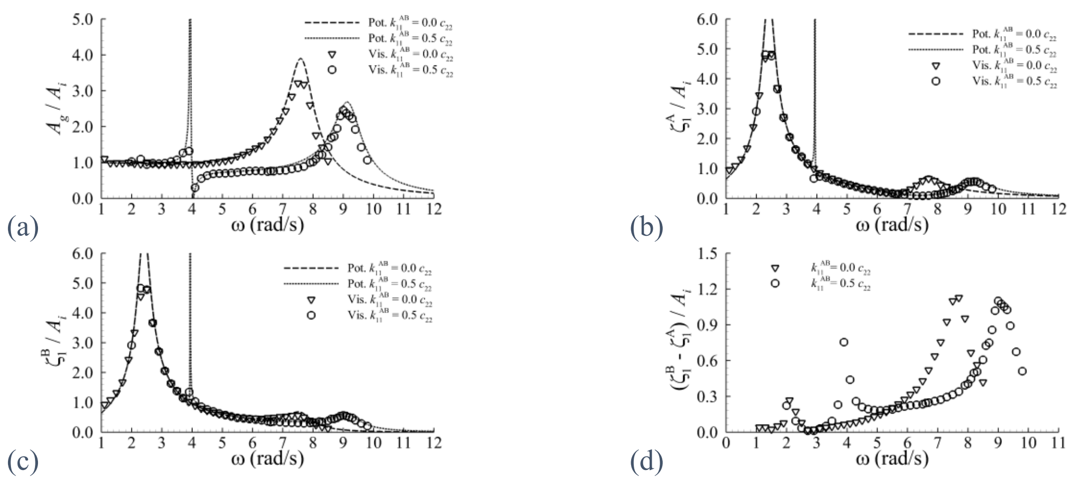
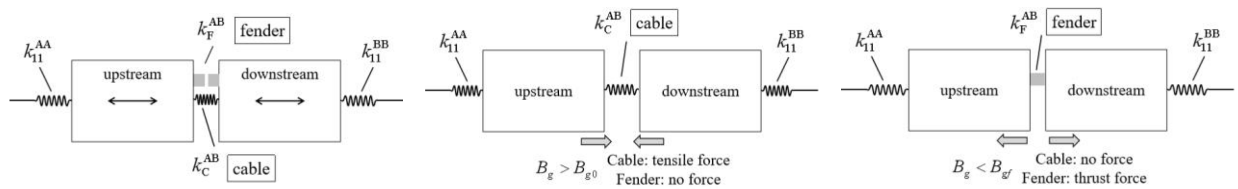


Figure 4: Comparison of the amplitude-frequency responses for the cases with and without inter-connection, (a) gap fluid, (b) upstream box A, (c) downstream box B, (d) relative motion between the two bodies.

Further investigations are conducted to understand the different effects between the inter-connections with cables and fenders and the above simplified linear spring on the coupling dynamic resonances. The stiffness of cable is $k_C^{AB} = 0.5\ c_{22}$ and the fender stiffness is $k_F^{AB} = 0.1\ c_{22}$. The numerical set-up is shown in Fig. 5(a). As indicated by Fig. 5(b) and (c), the cable only provides tensile force when the gap distance B_g exceeds the initial gap width $B_{g0} = 0.10$ m due to the sway motions of the two boxes, while the fender only provides thrust force when the gap distance B_g is less than the double thickness of the fender $B_{gf} = 0.09$ m.

Figure 6 shows the motion responses of the system after considering the effects of the cables and fenders. The incident wave frequency $\omega_i = 5.1$rad/s is selected in the vicinity of the second resonant mode, where the two floating bodies have significant relative displacement with the aforementioned out-of-phase resonant mode. The viscous numerical results in Fig. 6(a) show notable nonlinearity with the appearances of the response components of zero-frequency, half-wave frequency, wave frequency, and one-and-a-half-wave frequency. The dynamic responses exhibit pronounced differences from those identified in the previously examined simplified linear-spring interconnection. It is seen that the mean position of the oscillatory fluid in gap rises, indicating a nonlinear harden stiffness effect. As shown in Fig. 6(b), a low-frequency response with a period equal to twice the wave period appears in the relative displacement between the two floating bodies ($\zeta^B - \zeta^A$). In addition, the relative displacement curve exhibits asymmetric behavior. The mean position of the relative displacement has shifted away from zero to a non-dimensional value –0.4. Physically speaking, the nonlinear responses are resulted from the asymmetric restoring stiffness, and even involving discontinuous stiffness, which is far beyond the validity of a simplified linear spring[4]. More details on the explanations of numerical studies will be presented in the workshop.



(a) (b) (c)

Figure 5: Schematic diagram of the twin-body with inter-mooring cable and fender.

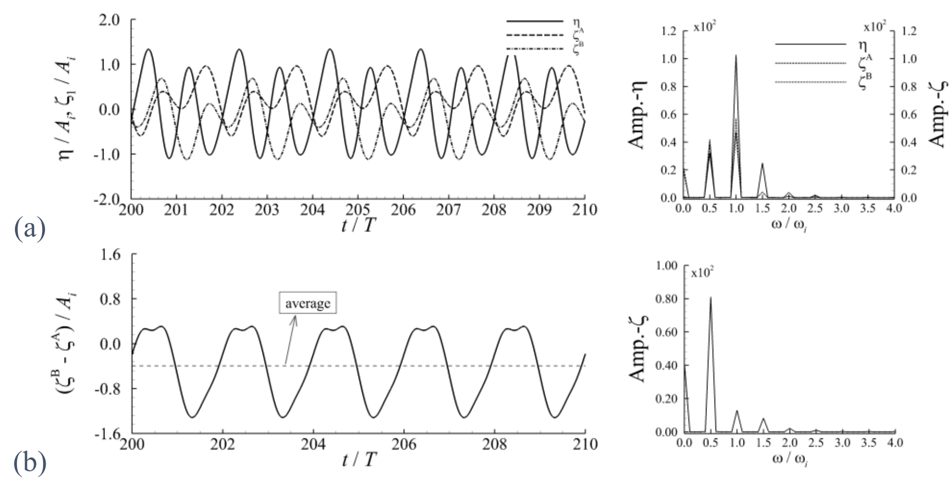


Figure 6: Time series and FFT results of motion responses under water wave with an angular frequency ω_i = 5.10 rad/s. (a) gap fluid motion and bodies motions, (b) relative displacement between two bodies.

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