Simulation of hydroelastic slamming by simplified models

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The two-dimensional water-entry problem is formulated in Cartesian coordinates (x, y). At the initial moment $t = 0$, the liquid is at rest and occupies the lower half-plane $y < 0$; the line $y = 0$ represents the equilibrium position of the free surface. A body touches the free surface at a single point, which is chosen as the origin of the coordinate system, $x = 0$. The body's shape is described by the equation $y = f_0(x)$ for $x_L < x < x_R$, where $f_0(0) = 0$ and $f_0(x) > 0$ for $x \neq 0$. At $t = 0$, the body begins to move vertically, with its vertical displacement $h(t)$, where $h(0) = 0$, $h'(0) = V$, and $h'(t) > 0$ for all $t > 0$. The liquid is assumed to be inviscid and incompressible. The motion of the body causes the free surface to rise, increasing the wetted portion of the body surface. The contact region between the liquid and the body $\Gamma_w(t)$ is unknown in advance and must be determined as part of the solution.

The position at time t of the elastic body is described by

$$y = f(x, t) = f_0(x) + w(x, t) - h(t),$$

where $w(x, t)$ represents the elastic deflection of the body surface in the vertical y-direction. The function $w(x, t)$ will be obtained from a structural model specified later. We focus on the initial (impact) stage of the body's penetration into the liquid. Deformation of the entering body is caused by the hydrodynamic pressure acting on the body surface along the contact region $\Gamma_w(t)$. To determine this pressure, we first need to determine the flow field generated by the entering body. We assume that the resulting flow is potential, with a velocity potential $\varphi(x, y, t)$ defined in the flow region $\Omega(t)$. This region is bounded above by the free surface $\Gamma_f(t)$ and by the contact region $\Gamma_w(t)$. The hydrodynamic pressure $p(x, y, t)$ in the flow region is given by the Bernoulli equation.

We intend to show how this coupled problem of hydroelastic slamming can be solved accurately within the fully nonlinear potential model and some simplified models.

General approach

The nonlinear problem with respect to the velocity potential can be solved numerically for prescribed functions $f_0(x)$, $w(x, t)$, and $h(t)$. Once the solution is obtained, the pressure distribution along the wetted part of the elastic body surface can be evaluated as

$$P(x, t) = p(x,\ f(x, t),\ t)\,.$$

It is convenient to introduce the velocity potential along the contact region,

$$\phi(x, t) = \varphi(x,\ f(x, t),\ t)$$

and to use the body boundary condition to present the function $P(x, t)$ in the form, see [2-4],

$$P(x, t) = -\rho \left[\frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial x} \frac{f_t f_x}{1 + f_x^2} + \frac{1}{2} \frac{\phi_x^2 - f_t^2}{1 + f_x^2} + g f(x, t) \right]. \qquad (1)$$

The formula (1) is exact within the theory of potential flows with free surfaces.

In this study, we focus on an iterative method for numerical solving coupled problems of hydroelastic slamming using a structural solver and a hydrodynamic solver. To demonstrate the method, we consider a simple elastic body – a symmetric wedge with simply supported elastic plates – similar to the configuration studied in [1]. Then $P(x,t)$ is an even function of x. Note the structural model is not simplified, rather, the most tractable configuration of the structure is considered.

The normal deflection $W(\xi,t)$ of the wedge side plating is governed by the equation

$$m\frac{\partial^2 W}{\partial t^2}+D\frac{\partial^4 W}{\partial \xi^4}=P(\xi\cos\beta,t),\qquad (-L<\xi<L,\ t>0). \tag{2}$$

The plates are simply supported and initially undeformed. The forcing function $P(\xi\cos\beta,t)$ in (2) is given by (1) along the wetted part of the plating $|\xi|<b(t)$, and is equal to zero along the dry part of the plate. We do not specify how the wetting part of the elastic plating is determined/defined in numerical calculations. Within each model considered in this study, exact definition of wetted part will be explained as part of the model.

The normal deflection is an even function, $W(-\xi,t)=W(\xi,t)$. Symmetric modes of the dry simply-supported plating are used to represent a symmetric wedge plating deflection,

$$W(\xi,t)=\sum_{n=1}^{\infty}a_n(t)\psi_n(\xi),\quad \psi_n(\xi)=\frac{1}{\sqrt{L}}\sin(\lambda_n|\xi|),\qquad \lambda_n=\frac{\pi n}{L}. \tag{3}$$

Substituting the series (3) in the plate equation (2), multiplying both sides of the equation by $\psi_m(\xi)$, and integrating in ξ, we obtain the so-called structural model of the problem,

$$\frac{\mathrm{d}^2\vec{a}}{\mathrm{d}t^2}+\omega_1^2\mathbb{D}\vec{a}=\vec{P}\Big(\vec{a},\frac{\mathrm{d}\vec{a}}{\mathrm{d}t},\frac{\mathrm{d}^2\vec{a}}{\mathrm{d}t^2},h(t),\frac{\mathrm{d}h}{\mathrm{d}t},\frac{\mathrm{d}^2h}{\mathrm{d}t^2},t\Big), \tag{4}$$

where $\mathbb{D}=\mathrm{diag}(1^2,2^2,3^2,...)$ is a diagonal matrix, and $\vec{P}=(P_1,P_2,...,P_n,...)$,

$$P_n=\frac{1}{m}\int_{-b(t)}^{b(t)}P(\xi\cos\beta,t)\psi_n(\xi)\mathrm{d}\xi.$$

The system (4) is integrated in time subject to zero initial conditions. The integration of the system is challenging because its right-hand side must be evaluated at each time step by solving the hydrodynamic problem. Moreover, this right-hand side depends strongly on the elastic acceleration.

The right-hans side of the system can be presented in the vector form

$$\vec{P}=-\frac{\mathrm{d}\vec{q}}{\mathrm{d}t}+\vec{P}_R,\quad \vec{q}=(q_1,q_2,...),\qquad q_n=\frac{\rho}{m}\int_{-b(t)}^{b(t)}\phi(\xi\cos\beta,t)\psi_n(\xi)\mathrm{d}\xi. \tag{5}$$

where the vectors $\vec{q}$ and $\vec{P}_R$ depend on $\vec{a}$, $\frac{\mathrm{d}\vec{a}}{\mathrm{d}t}$, $h(t)$, $\frac{\mathrm{d}h}{\mathrm{d}t}$, t and require only the distribution of the potential over the contact region. Then the plate equation (2) leads to the following system

$$\frac{\mathrm{d}\vec{a}}{\mathrm{d}t}+\vec{q}\left(\vec{a},\frac{\mathrm{d}\vec{a}}{\mathrm{d}t},h(t),\frac{\mathrm{d}h}{\mathrm{d}t},t\right)=\vec{u},\qquad \frac{\mathrm{d}\vec{u}}{\mathrm{d}t}=\vec{Q}\left(\vec{a},\frac{\mathrm{d}\vec{a}}{\mathrm{d}t},h(t),\frac{\mathrm{d}h}{\mathrm{d}t},t\right), \tag{6}$$

where $\vec{Q}=\vec{P}_R-\omega_1^2\mathbb{D}\vec{a}$. To simplify the analysis while preserving the physical essence of the problem, the rigid-body displacement $h(t)$ is assumed to be prescribed.

The velocity potential in the contact region can be decomposed as

$$\phi(x,t_k)=\phi_f[f(x,t_k),\eta(x,t_k),\varphi(x,\eta(x,t_k),t_k)](x,t_k)+$$
$$\sum_{\alpha=1}^{\infty}a'_\alpha(t_k)\phi_\alpha[f(x,t_k),\eta(x,t_k)](x,t_k)-h'(t)\phi_w[f(x,t_k),\eta(x,t_k),t_k)](x,t_k). \quad (7)$$

where $\phi_f(x,t_k)$ is the potential in the contact region at $t=t_k$ caused only by the given distribution of the potential along the free surface, $\phi_w(x,t_k)$ and $\phi_\alpha(x,t_k)$ are the potentials as the solutions of the formulated problem with zero potential on the free surface and rigid/elastic motions of the body surface correspondingly.

The decomposition (7) provides

$$\vec{q}(t_k)=\mathbb{A}(\vec{a}(t_k),t_k)\frac{\mathrm{d}\vec{a}}{\mathrm{d}t}(t_k)+\vec{q}_R(\vec{a}(t_k),t_k), \quad (8)$$

where $\mathbb{A}(\vec{a}(t_k),t_k)$ is called the added-mass matrix of the flow region $\Omega(t_k)$, and $\vec{q}_R(\vec{a}(t_k),t_k)$ does not depend explicitly on the time derivative of the elastic deflection of the body surface. Then (6) and (8) give the formula for the velocity of elastic deflection,

$$\frac{\mathrm{d}\vec{a}}{\mathrm{d}t}(t_k)=\left(\mathbb{I}+\mathbb{A}(\vec{a}(t_k),t_k)\right)^{-1}(\vec{u}(t_k)-\vec{q}_R(\vec{a}(t_k),t_k)),$$

which depends on both inertia of the structure and the fluid.

Determining elements of the decomposition (8) could be time-consuming if many natural modes of the structure are retained. Another approach, which could be faster within the same accuracy, is based on the following decomposition

$$\vec{q}(t_k)=\tilde{\mathbb{A}}(t_k)\frac{\mathrm{d}\vec{a}}{\mathrm{d}t}(t_k)+\vec{q}_{ER}\left(\vec{a}(t_k),\frac{\mathrm{d}\vec{a}}{\mathrm{d}t}(t_k),t_k\right),$$

where $\tilde{\mathbb{A}}(t_k)$ approximates the added-mass matrix $\mathbb{A}(\vec{a}(t_k),t_k)$ and $\vec{q}_{ER}$ "weakly" depends on the velocity of elastic deflection. Then system (6) yields the following equation for the velocity of the entering body surface at $t=t_k$,

$$\frac{\mathrm{d}\vec{a}}{\mathrm{d}t}(t_k)=\left(\mathbb{I}+\tilde{\mathbb{A}}(t_k)\right)^{-1}\left[\vec{u}(t_k)-\vec{q}(t_k)-\tilde{\mathbb{A}}(t_k)\frac{\mathrm{d}\vec{a}}{\mathrm{d}t}(t_k)\right],$$

which is solved by iterations. The rest of the algorithm is the same as in the first fully coupled approach.

It is shown below how the presented approaches work for simplified models of hydroelastic slamming. Simplifications are concerned only with the hydrodynamic part of the coupled problem.

Within the **Wagner Model** equations of hydrodynamics are linearised. The boundary conditions are imposed at the initial undisturbed level of the liquid surface, $y=0$. The solution of the linearised problem yields, see [4],

$$\frac{\partial\varphi^{(w)}}{\partial x}(x,0,t)=\frac{2x}{\pi\sqrt{c^2(t)-x^2}}PV\int_0^{c(t)}\frac{(w_t(\tau,t)-h'(t))\sqrt{c^2(t)-\tau^2}}{\tau^2-x^2}\mathrm{d}\tau,$$

where the radius of the contact region $c(t)$ is a solution of the Wagner equation

$$\int_0^{\pi/2} f[c(t)\sin\theta, t]\mathrm{d}\theta = 0.$$

The added-mass was calculated in [1]. The system (6) takes the form

$$\frac{\mathrm{d}\vec{a}}{\mathrm{d}t} = (\mathbb{I} + \mathbb{S}(c))^{-1}(\vec{u} - h'(t)\vec{g}_(c)), \quad \frac{\mathrm{d}\vec{u}}{\mathrm{d}t} = -\omega_1^2\mathbb{D}\vec{a},$$

which is similar to the system derived in [1].

Within **Modified Logvinovich Model (MLM)**, the following approximation is used

$$\phi(x,t) \approx \varphi^{(w)}(x,0,t) + f_t(x,t)f(x,t).$$

The contact region is defined in such a way that the pressure in this region, $P(x,t)$ is positive. Equation (1) shows that the pressure depends on the acceleration of elastic deflection, which makes calculations of $b(t)$ complicated. Note that accelerations are not needed in the Wagner model. Acceleration and the dimension of the contact region are calculated using the formulae from [4] and iterations. In a simplified approach, the function $b(t)$ is taken from the previous time step. Having an initial guess of $b(t_k)$, we solve the system (6) to find the deflection velocity and acceleration at t_k. Then the value $b(t_k)$ is corrected and all calculations are repeated until the equation $P(b(t_k), t_k) = 0$ is satisfied with a required accuracy.

The **Generalised Wagner Model (GWM)** is the simplified model, which is closest to the original nonlinear potential model. Hydroelastic slamming problems are solved in GWM using a conformal mapping of the flow region onto a half plane, see [5], and theory of analytic functions to solve the resulting mixed boundary value problems at each time instant. In this model, $\phi_f(x,t) = 0$. However, calculations of the dimension $b(t)$ of the contact region is still a problem, which requires close attention.

Numerical results and their discussions will be presented at the workshop.

References

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