Occurrence of Parametric roll in Bichromatic Incident Waves

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HIGHLIGHTS

- Parametric roll has been studied in the present of bichromatic waves, particularly focusing on the sum-frequency effects.
- A quasi-periodic Mathieu equation was derived from the second-order equation of ship motion.
- Numerical calculations were performed to verify the validity of the theory.
- The possibility of parametric roll occurrence due to sum frequency of bichromatic waves is obviously shown.

1 INTRODUCTION

The rolling motion of a ship is a major concern for ship engineers and operators due to its highly nonlinear behavior and associated risks. Despite advances in ship design and operational technologies, accidents related to ship stability have persisted for decades (Peter and Belenky, 2022). To mitigate operational risks associated with ship stability and to enhance the safety of ships at sea, the International Maritime Organization (IMO) recently established the second-generation *Standards for the Maintenance of Ship Stability* (MSC.1/Circ.1627). Nevertheless, parametric roll phenomena remain a challenging research topic due to their highly nonlinear and non-ergodic characteristics.

Parametrically excited roll motion, which arises from periodic variations in transverse stability, has been extensively studied over several decades. Early experimental studies by Kerwin (1955) and Paulling (1961) demonstrated that large roll motions can occur in longitudinal waves without direct transverse excitation. To explain this phenomenon, a Mathieu second-order differential equation based on a simplified one-degree-of-freedom (1-DoF) roll motion was introduced. Furthermore, several studies proposed nonlinear dynamic models for the roll restoring moment to better represent the observed behavior (Francescutto, 2001; Bulian et al., 2004).

Kim and Kim (2011) introduced a new mechanism for the occurrence of parametric roll. Traditionally, parametric roll has been explained by the presence of a single wave component, where the wave encounter frequency is approximately twice the natural roll frequency. However, Kim and Kim numerically and experimentally demonstrated that very large roll motions can also occur in the presence of bichromatic waves. They explained this phenomenon through the relationship between the second-order sum or difference frequencies of the waves and the natural roll frequency. Their study, however, focused on oblique sea conditions, in which sufficient external rolling moments exist to trigger roll motion, thereby indicating the need for a more systematic validation study.

In the present study, roll motion is simulated under bichromatic wave conditions in head seas in order to investigate the pure nonlinear characteristics of roll motion. The simulations focus on sum-frequency cases, and the results clearly demonstrate the occurrence of parametric roll when the sum frequency is approximately twice the natural roll frequency.

2 PARAMETRIC EXCITATION DUE TO BICHROMATIC WAVERS

2.1 Roll motion in bichromatic waves

Consider a free-floating ship moving with a steady speed $\vec{U}$ under single harmonic roll motion. If there is a change of metacetric height $\overline{GM}$ due to the ship motion and if it can be approximated up to the first harmonic fluctuation s.t.

$$\overline{GM} = \overline{GM_0}(1 + \delta\cos(\omega_e t + \alpha)) \tag{1}$$

where $\delta = \overline{\Delta GM}/2\overline{GM_0}$, $\overline{\Delta GM}$ is the fluctuation amount of $\overline{GM}$, and $\overline{GM_0}$ is the mean metacenter height. In addition, ω_e and α are the encounter frequency and phase of wave excitation. Then the 1-DOF roll equation of motion can be written as Eq.2.

$$\ddot{\phi} + 2\nu\omega_n\dot{\phi} + \omega_n^2\{1 + \delta\cos(\omega_e t + \alpha)\}\phi = M_{roll}(t) \tag{2}$$

where ν indicates the damping coefficients and $M_{roll}(t)$ means the roll moment normalized by the mass and added mass moment of inertia.

It is assume that roll angle ϕ can be written into a series form as follows:

$$\phi = \phi^{(1)} + \varepsilon\phi^{(2)} + O(\varepsilon^2) \tag{3}$$

Then the normalized linear and second-order equation of roll motion can be written as

$$\ddot{\phi}^{(1)} + 2\nu\omega_n\dot{\phi}^{(1)} + \omega_n^2\phi^{(1)} = M_{roll}(t) = M_0\cos(\omega_e t + \beta) \tag{4}$$

$$\ddot{\phi}^{(2)} + 2\nu\omega_n\dot{\phi}^{(2)} + \omega_n^2\phi^{(2)} = -\omega_n^2\delta\cos(\omega_e t + \alpha)\,\phi^{(1)} \tag{5}$$

β are the phase differences of the roll exciation moment.

If the linear roll motion can be expressed to $\phi^{(1)} = \phi_L\cos(\omega_e t + \gamma)$ where ϕ_L means the amplitude of first-order motion and γ is the phase difference. Then, the second-order problem takes the form as below:

$$\begin{aligned}\ddot{\phi}^{(2)} + 2\nu\omega_n\dot{\phi}^{(2)} + \omega_n^2\phi^{(2)} &= -\omega_n^2\delta\phi_L\cos(\omega_e t + \alpha)\cos(\omega_e t + \gamma) \\ &= -\frac{\omega_n^2\delta\phi_L}{2}\cos(2\omega_e t + \alpha + \gamma) - \frac{\omega_n^2\delta\phi_L}{2}\cos(\alpha - \gamma)\end{aligned} \tag{6}$$

The last non-homogeneous term means the heeling due to the second-order effect, and the harmonic term includes the resonance of the second-order roll motion.

When the ship encounters bichromatic waves. the equation of motion can be written similary

$$\begin{aligned}&\ddot{\phi} + 2\nu\omega_n\dot{\phi} + \omega_n^2\{1 + \delta_1\cos(\omega_{e,1}t + \alpha_1) + \delta_2\cos(\omega_{e,2}t + \alpha_2)\}\phi \\ &\qquad = M_{0,1}\cos(\omega_{e,1}t + \beta_1) + M_{0,2}\cos(\omega_{e,2}t + \beta_2)\end{aligned} \tag{7}$$

Subscripts 1 and 2 represent two frquency components.

Like the single wave case, a perturbation solution can be assumed and the linear and second-order problems can be written as follows:

$$\phi^{(1)} = \phi_{L,1}\cos(\omega_{e,1}t + \gamma_1) + \phi_{L,2}\cos(\omega_{e,2}t + \gamma_2) \tag{8}$$

$$\begin{aligned}&\ddot{\phi}^{(2)} + 2\nu\omega_n\dot{\phi}^{(2)} + \omega_n^2\phi^{(2)} = \\ &\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\delta_i\phi_{L,j}}{2}\{\cos((\omega_{e,i} + E_{ij}\omega_{e,j})t + \alpha_i + \gamma_j) + \cos(\alpha_i - \gamma_j)\}\end{aligned} \tag{9}$$

where $E_{ij} = -1$= if $i = 2$ and $j = 1$, otherwise E_{ij}=1. Then, there are four frequencies which may cause parametric roll, i.e. $2\omega_{e,1}$, $2\omega_{e,2}$, $|\omega_{e,1} + \omega_{e,2}|$, or $|\omega_{e,1} - \omega_{e,2}|$ are twice of the roll natural frequency.

2.2 Stability of quasi-periodic Mathieu equation

Eq. (7) is the form of quasi-periodic Mathieu equation. For example, bichromtaic components of Eq. (7) without damping term can be expressed as follows, taking the form of Rand(2003):

$$\frac{d^2\phi}{d\tau^2} + [\sigma + \varepsilon\cos(\tau) + \varepsilon\mu\cos(1+\Lambda)\,\tau]\phi = 0 \tag{10}$$

where $\tau = \omega_{e,1}t$ and σ, μ, Λ are all functions of natural and encounter frequencies, e.g. $\sigma = \left(\frac{\omega_n}{\omega_{e,1}}\right)^2$, $\varepsilon = \left(\frac{\omega_n}{\omega_{e,1}}\right)^2 \frac{\overline{\Delta GM_1}}{\overline{GM_0}}$, $\mu = \frac{\overline{\Delta GM_2}}{\overline{\Delta GM_1}}$, and $\Lambda = \frac{\omega_{e,2}-\omega_{e,1}}{\omega_{e,1}}$. The stability of Eq.(10) without damping term can be obtained by a few different approaches such as numerical integration, Lyapunov exponents, or perturbation. The stability diagram is much more complicated that single harmonic Mathieu equation. For example, Figure 1 is examples of the *σ-Λ* stability chart for different ε and μ.

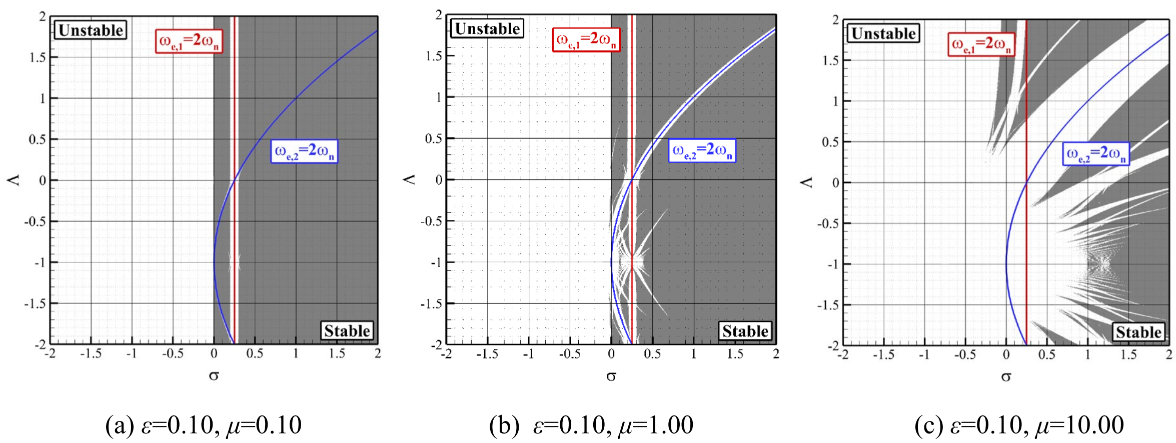

Figure 1. Examples of *σ-Λ* stability chart of quasi-periodic Mathieu equation

3 NUMERICAL SIMULATION & OBSERVATION

3.1 Ship model and simulation method

The ship model used for numerical validation is the KCS model, a widely used containership model with a length of 230 m. Its metacentric height in still water is assumed to be 1.27 m, and the natural roll frequency is approximately 0.308 rad/s. As a containership, the KCS model is well suited for parametric roll simulations. The simulations are based on an impulse response function approach combined with nonlinear Froude–Krylov forces and nonlinear restoring moments resulting from instantaneous variations in the wetted surface. Although this approach does not fully capture all nonlinear effects, it is well known as a computationally efficient method capable of predicting the occurrence of parametric roll. In the present study, head-sea conditions and zero forward speed are applied to various combinations of bichromatic waves.

3.2 Stability of quasi-periodic Mathieu equation

Figure 2 shows the roll amplitude distribution under head-sea conditions. This case corresponds to bichromatic waves with individual wave amplitudes of 2.3 m. The maximum roll amplitudes are obtained over a simulation duration of 2000 s. When each wave encounter frequency is at or

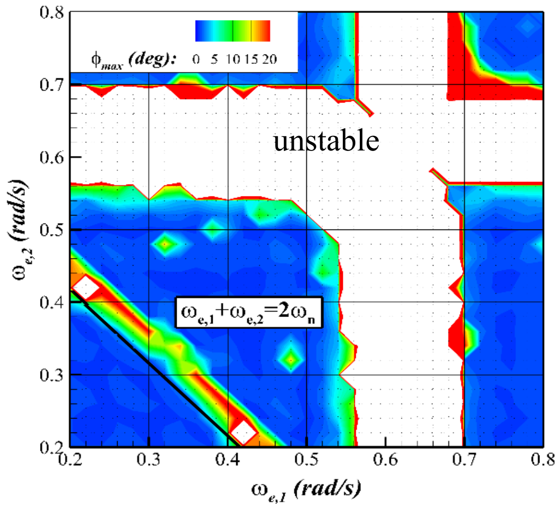

Fig.2 Maximum roll amplitude distributions in bichromatic wave condition: $A_1 = A_2$=2.3m

near twice the natural roll frequency, the roll motion becomes unstable due to parametric roll induced by a single wave component, i.e. $\omega_{e,1} \approx 2\omega_n$ and $\omega_{e,2} \approx 2\omega_n$ (white zone in Fig. 2). Furthermore, it should be noted that a region of significant roll amplitudes exisits when $\omega_{e,1} + \omega_{e,2} \approx 2\omega_n$, which corresponds to sum-frequency-induced parametric roll. Figure 3 presents the time history of roll motion in bichromatic waves with $\omega_1 = 0.240$ rad/s and $\omega_2 = 0.420$rad/s. After the onset of parametric roll, a clear harmonic roll motion is observed in head sea.

According to our computational observations, the occurrence of sum- or difference-frequency-induced parametric roll requires a stronger triggering mechanism to develop over time. Nevertheless, this phenomenon should be considered as one of the possible mechanisms for the occurrence of parametric roll.

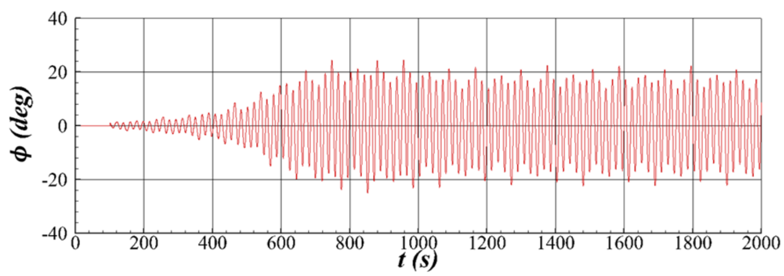

Fig.3 Roll motion signal in bichromatic waves: ω_1=0.240 rad/sec, ω_2=0.420 rad/sec, $A_1 = A_2$=2.3m

ACKNOWLEDGEMENT

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