Submitted to the 41st International Workshop on Water Waves and Floating Bodies, Plitvice, Croatia, 26-29 April 2026

Analysis of ship seakeeping in following seas

X.B. Chen[1], **L. Diebold**[1], **H. Iwashita**[2] & **P-Y Wuillaume**[1]

[1]Research Department, Bureau Veritas Marine & Offshore, Paris, France
[2]Dept. Transportation and Environment Systems, Hiroshima University, Hiroshima, Japan
xiao-bo.chen@bureauveritas.com

Some important aspects of ship seakeeping in following seas including stern quartering seas are analyzed here based on the linear potential flow theory. Unlike in head or bow quartering seas, the encounter frequencies of ship motions could be small and even negative when the ship speed is larger than wave phase velocity. At the zero encounter frequency, it is important to consider the radiation forces due to ship displacements as the stiffness and added to the restoring forces due to forward speed which has been ignored in previous studies. At the negative encounter frequency, it is shown that the radiation coefficients, unlike the diffraction loading, respect the symmetry properties. Some preliminary numerical results on Wigley IV and RIOS ship are included.

1 Introduction

In the coordinate system translating with the ship at a constant speed U, the flow around the ship described in [1] is decomposed into the sum of a base flow called ship-shaped stream $\mathbf{w}(\mathbf{r})$ with the space coordinates $\mathbf{r} = (x, y, z)$, the wavy steady flow represented by its velocity potential $\phi(\mathbf{r})$ and the unsteady flow by $\psi(\mathbf{r}, t)$ which is assumed to be time-harmonic and can be written by

$$\psi(\mathbf{r}, t) = \Re\mathrm{e}\{\varphi(\mathbf{r})\mathrm{e}^{-\mathrm{i}\omega t}\} = \Re\mathrm{e}\{\varphi^*(\mathbf{r})\mathrm{e}^{+\mathrm{i}\omega t}\} \tag{1}$$

with $\varphi(\mathbf{r})$ the complex function in space and its complex conjugate $\varphi^*(\mathbf{r})$, and the encounter frequency ω in the time-harmonic factor. Furthermore, the complex unsteady potential $\varphi(\mathbf{r})$ is written as the sum of 14 components

$$\varphi(\mathbf{r}) = \overbrace{\xi_0\left[\varphi_0(\mathbf{r}) + \varphi_d(\mathbf{r})\right]}^{\varphi^D(\mathbf{r})} + \left\{\sum_{j=1}^{6}(-\mathrm{i}\omega\xi_j)\varphi_j^n(\mathbf{r}) + F_\mathrm{r}\sum_{j=1}^{6}\xi_j\varphi_j^m(\mathbf{r})\right\}^{\varphi^R(\mathbf{r})} \tag{2}$$

in which $\varphi_0(\mathbf{r})$ and $\varphi_d(\mathbf{r})$ represent incoming and diffraction waves, respectively, and their sum denoted by $\varphi^D(\mathbf{r})$ are associated with the incoming wave amplitude ξ_0. The radiation potential $\varphi^R(\mathbf{r})$ is decomposed into $\varphi_j^n(\mathbf{r})$ for $j = 1, 2, \cdots, 6$ which are due to the six elementary *velocities* $(-\mathrm{i}\omega\xi_j)$ of the ship and those $\varphi_j^m(\mathbf{r})$ due to the six elementary *displacements* ξ_j for $j = 1, 2, \cdots, 6$. The ship translations and rotations are denoted by (ξ_1, ξ_2, ξ_3) and (ξ_4, ξ_5, ξ_6), respectively. The Froude number F_r in (2) is defined by $F_\mathrm{r} = U/\sqrt{gL}$ with the ship speed U, the acceleration of gravity g and the ship length L. All potentials $(\varphi, \varphi_0, \varphi_d, \varphi_j^n, \varphi_j^m)$ for $j = 1, 2, \cdots, 6$ in (2) are scaled by $L\sqrt{gL}$, the encounter frequency ω by $\sqrt{g/L}$, the time t by $\sqrt{L/g}$ and (ξ_0, ξ_j) same as $\mathbf{r}$ by L, accordingly.

The velocity potential of incoming waves propagating in the direction with an angle β with respect to the positive $x-$axis is given by

$$\varphi_0(\mathbf{r}) = -\omega_0^{-1}\mathrm{e}^{k_0 z + \mathrm{i}k_0(x\cos\beta + y\sin\beta)} \tag{3}$$

in which (ω_0, k_0) are wave frequency and wavenumber, respectively. They are scaled by $(\sqrt{g/L}, L^{-1})$ and satisfy the dispersion relation $\omega_0^2 = k_0$ in the deepwater case considered here.

2 Negative encounter frequencies

The relationship between the wave frequency $\omega_0 > 0$ and the encounter frequency ω is written equivalently by

$$\omega = \omega_0 - F_\mathrm{r}k_0\cos\beta \quad \Rightarrow \quad \tilde{\omega} = \begin{cases} \tilde{\omega}_0(1 - \tilde{\omega}_0) & \cos\beta > 0 \ \text{(following or stern quartering seas)} \\ \tilde{\omega}_0(1 + \tilde{\omega}_0) & \cos\beta < 0 \ \text{(head or bow quartering seas)} \end{cases} \tag{4}$$

by using the dispersion relation $k_0 = \omega_0^2$ and re-scaling (ω_0, ω) with $F_\mathrm{r}|\cos\beta|$, i.e., $(\tilde{\omega}_0, \tilde{\omega}) = (\omega_0, \omega)F_\mathrm{r}|\cos\beta|$ with β denoting the wave heading. In head or bow quartering seas, we have $\cos\beta < 0$ and $\omega/\omega_0 = \tilde{\omega}/\tilde{\omega}_0 > 1$. In beam seas $\cos\beta = 0$, the wave frequency and encounter frequency are equal $(\omega/\omega_0 = 1)$. In following or stern quartering seas $\cos\beta > 0$, the encounter frequency $\tilde{\omega}$ has a maximum equal to 1/4 at $\tilde{\omega}_0 = 1/2$. In this case of stern quartering and following seas $(\beta < \pi/2)$ as illustrated on the left of Fig.1, the encounter frequency ω is small and can be zero or negative for $\tilde{\omega}_0 \geq 1$ depicted in the middle and on the right of Fig.1. In fact, the value $F_\mathrm{r}\cos\beta$ is equal to the projection of forward speed in the direction of wave propagation and $1/\omega_0$ the phase velocity of incoming waves. The

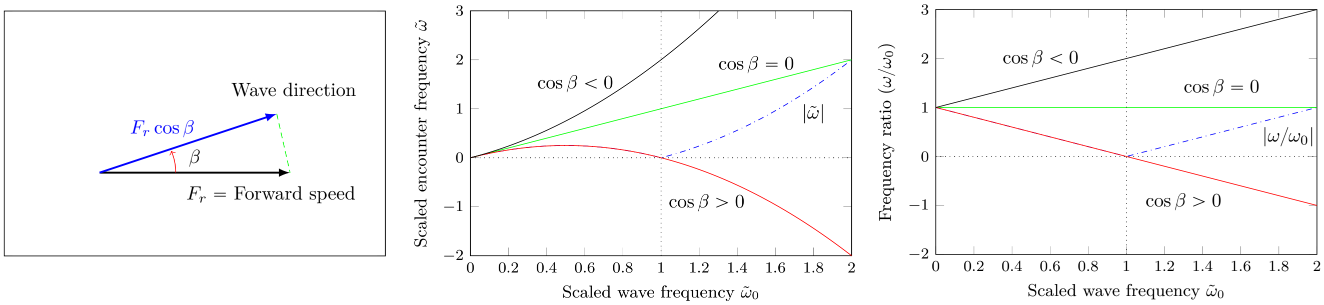

Figure 1: Wave heading (left) and Wave frequency *vs* encounter frequency (middle + right)

ratio of both is then $F_{\rm r}\cos\beta/(1/\omega_0) = \tilde{\omega}_0$ the re-scaled wave frequency. The encounter frequency $\tilde{\omega} = 0$ when and only when the speed projection is equal to the wave phase velocity, i.e. $\tilde{\omega}_0 = 1$. When $\tilde{\omega}_0 > 1$, the ship is overtaking waves and the encounter frequency becomes negative ($\tilde{\omega} < 0$). In this case, the usual way to take the absolute value of encounter frequency and make computations is examined in the following.

3 Radiation and diffraction solutions

All elementary potentials $(\varphi_0, \varphi_d, \varphi_j^n, \varphi_j^m)$ satisfy the Laplacian equation in the fluid domain and a set of boundary conditions. Beside the incoming wave potential $\varphi_0(\mathbf{r})$ which is analytically given, all others disappear (radiation condition) for $|\mathbf{r}| \to \infty$, the boundary condition on the mean free surface F given in [1] and written in general

$$\mathcal{L}(\varphi,\omega) = 0 = \partial_z\varphi - \omega^2\varphi - 2\mathrm{i}\tau\mathbf{w}\cdot\nabla\varphi + F_{\rm r}^2\mathbf{w}\cdot\nabla(\mathbf{w}\cdot\nabla\varphi) + F_{\rm r}^2\nabla\varphi\cdot(\mathbf{w}\cdot\nabla)\mathbf{w} + \bar{\phi}_{zz}(\mathrm{i}\tau\varphi - F_{\rm r}^2\mathbf{w}\cdot\nabla\varphi) \tag{5}$$

with φ denoting $(\varphi_0+\varphi_d, \varphi_j^n, \varphi_j^m)$ for $j = 1, 2, \cdots, 6$. In (5), the parameter $\tau = \omega F_{\rm r}$ and the base flow is denoted by the real function $\mathbf{w}(\mathbf{r}) = (\bar{\phi}_x - 1, \bar{\phi}_y, \bar{\phi}_z)$. The boundary conditions on the hull H are written by

$$\begin{aligned} \partial_n\varphi_j^n &= n_j \\ \partial_n\varphi_j^m &= m_j \\ \partial_n\varphi_d &= -\partial_n\varphi_0 \end{aligned} \tag{6}$$

in which $(n_1, n_2, \cdots, n_6) = (\mathbf{n}, \mathbf{r}\wedge\mathbf{n}) = \mathbf{N}$ are components of the generalized normal vector and $(m_1, m_2, \cdots, m_6) = -[(\mathbf{n}\cdot\nabla)\mathbf{w}, (\mathbf{n}\cdot\nabla)(\mathbf{r}\wedge\mathbf{w})]$ are the well-known m-terms representing one of interaction effects with the base flow.

Since the radiation solutions $(\varphi_j^n, \varphi_j^m)$ are dependent only on the encounter frequency and the operators

$$\mathcal{L}(\varphi_j, -\omega) = \mathcal{L}^*(\varphi_j, +\omega)\ ; \quad \partial_n\varphi_j = \partial_n\varphi_j^* \quad \text{with} \quad \varphi_j = \varphi_j^n \text{ or } \varphi_j^m \tag{7}$$

on F and H, respectively, it is easy to show that

$$\varphi_j(-\omega) \equiv \varphi_j^*(+\omega) \tag{8}$$

an interesting symmetry property which can be used to obtain radiation solutions by changing the sign of encounter frequency. On the other side, since the potential of incoming waves $\varphi_0(\mathbf{r})$ is defined by the wave frequency ω_0 and heading β, i.e. for the same encounter frequency ω defined by (4), we have a series of combination (ω_0, β) and

$$\varphi_0(-\omega) \neq \varphi_0^*(+\omega) \quad \Rightarrow \quad \varphi_d(-\omega) \neq \varphi_d^*(+\omega) \tag{9}$$

Fortunately, the Green function formulated in [3] and its integration on ship hull and over the free surface presented in [4] involved in the boundary integral equations (BIEs) established in [1] satisfies the same symmetry identity (8), so that all BIEs for radiation and diffraction potentials at a negative encounter frequency can be solved exactly in the same way but using the complex conjugate Green function at the absolute value of encounter frequency.

The unsteady hydrodynamic pressure $P(\mathbf{r},t)$ scaled by (ρgL) can be expressed in the same form as (1) with a complex function $p(\mathbf{r}) = p^D(\mathbf{r}) + p^R(\mathbf{r})$ and the time factor. Associated with the radiation potentials $\varphi^R(\mathbf{r})$, we have given $p^R(\mathbf{r})$ by the formula (eq.10) in [2]. In addition, there are 3 components of steady pressure: hydrostatic one $p^H(\mathbf{r}) = -z$, that $p^S(\mathbf{r})$ due to the base flow $\mathbf{w}(\mathbf{r})$ and that $p^W(\mathbf{r})$ due to the wavy steady flow $\phi(\mathbf{r})$. Associated with ship motions, the variation of $p^H(\mathbf{r})$ and the gravity force gives the classical hydrostatic stiffness H_{ij} (eq.15) in [2] and (eq.59) in [1]. The restoring forces S_{ij} due to $p^S(\mathbf{r})$ have been explicitly given for the first time by (eq.16) in [2] and by an equivalent but simpler form for the horizontal components

$$\{S_{11}, S_{22}, S_{66}\} = \iint_H \left\{\partial_x p^S n_1, \partial_y p^S n_2, (x\partial_y p^S - y\partial_x p^S)(xn_2 - yn_1)\right\} \mathrm{d}s \quad \text{with} \quad p^S(\mathbf{r}) = -(\mathbf{w}\cdot\mathbf{w} - 1)/2 \tag{10}$$

In addition, there exist restoring forces S_{ij}^W due to the wavy steady pressure $p^W(\mathbf{r})$ associated with the wavy steady potential $\phi(\mathbf{r})$.

Concerning the radiation forces due to the time-harmonic pressure $p^R(\mathbf{r})$ defined by (eq.10) in [2] and the free-surface elevation $\eta^R(x,y) = p^R(\mathbf{r})|_{z=0}$ along the waterline, we decompose them

$$\mathbf{F}^R = \{F_i^R\} = -\iint_H p^R(\mathbf{r})\mathbf{N}\,\mathrm{d}s - F_\mathrm{r}^2 \oint_\Gamma \eta^R(x,y)p^S\mathbf{N}\,\mathrm{d}l = -\sum_{j=1}^{6}\Big(-\omega^2 R_{ij}^\omega - \tau R_{ij}^\tau + F_\mathrm{r}^2 R_{ij}^F\Big)\xi_j \tag{11}$$

for $i = 1, 2, \cdots, 6$, with the components

$$\begin{aligned}
R_{ij}^\omega &= -\iint_H \varphi_j^n n_i\,\mathrm{d}s - F_\mathrm{r}^2 \oint_\Gamma \varphi_j^n p^S n_i\,\mathrm{d}l \\
R_{ij}^\tau &= -\iint_H \mathrm{i}\big(\mathbf{w}\cdot\nabla\varphi_j^n + \varphi_j^m\big) n_i\,\mathrm{d}s - F_\mathrm{r}^2 \oint_\Gamma \mathrm{i}\big(\mathbf{w}\cdot\nabla\varphi_j^n + \varphi_j^m\big) p^S n_i\,\mathrm{d}l \\
R_{ij}^F &= -\iint_H \big(\mathbf{w}\cdot\nabla\varphi_j^m\big) n_i\,\mathrm{d}s - F_\mathrm{r}^2 \oint_\Gamma \big(\mathbf{w}\cdot\nabla\varphi_j^m\big) p^S n_i\,\mathrm{d}l
\end{aligned} \tag{12}$$

and the coefficients (A_{ij}, B_{ij}, C_{ij}) are given by

$$\begin{aligned}
A_{ij} &= \Re\mathrm{e}\Big\{R_{ij}^\omega + (F_\mathrm{r}/\omega)R_{ij}^\tau - (F_\mathrm{r}^2/\omega^2)\big(R_{ij}^F - R_{ij}^S\big)\Big\} \\
B_{ij} &= \Im\mathrm{m}\Big\{\omega R_{ij}^\omega + F_\mathrm{r} R_{ij}^\tau - (F_\mathrm{r}^2/\omega) R_{ij}^F\Big\} \\
C_{ij} &= H_{ij} + F_\mathrm{r}^2\,\big(S_{ij} + R_{ij}^S\big)
\end{aligned} \tag{13}$$

The terms R_{ij}^S in above (13) are defined as the limit at zero frequency of the component R_{ij}^F given in (12), i.e.

$$R_{ij}^S = R_{ij}^F(\omega \to 0) \tag{14}$$

which are purely real. A preliminary analysis on R_{ij}^S and S_{ij} in [2] shows that $S_{jj} + R_{jj}^S = 0$ for $j = 1, 2, 6$ in the horizontal translations (surge, sway and yaw), as expected to be consistent with physics.

At a negative frequency of encounter, we have

$$\begin{aligned}
R_{ij}^\omega(-\omega) &= +R_{ij}^{\omega*}(+\omega) \\
R_{ij}^\tau(-\omega) &= -R_{ij}^{\tau*}(+\omega) \\
R_{ij}^F(-\omega) &= +R_{ij}^{F*}(+\omega)
\end{aligned} \tag{15}$$

according to (8), and the radiation coefficients

$$\begin{aligned}
A_{ij}(-\omega) &= +A_{ij}(+\omega) \\
B_{ij}(-\omega) &= +B_{ij}(+\omega) \\
C_{ij}(-\omega) &= +C_{ij}(+\omega)
\end{aligned} \tag{16}$$

obtained by introducing (15) to (13). The symmetric feature of radiation damping coefficients is not surprise and consistent with the definition of radiation coefficients (eq.14) in [2].

On the other side, the diffraction wave loads $\mathbf{E} = (E_1, E_2, \cdots, E_6)$ are defined by the integration of time-harmonic pressure due to wave diffraction and of steady pressure over time-harmonic varying surface around the waterline

$$\mathbf{E} = \{E_i\} = -\iint_H p^D(\mathbf{r})\mathbf{N}\,\mathrm{d}s - F_\mathrm{r}^2 \oint_\Gamma \eta^D(x,y)p^S\mathbf{N}\,\mathrm{d}l \tag{17}$$

with $p^D(\mathbf{x})$ by

$$p^D(\mathbf{r}) = -(-\mathrm{i}\omega\varphi^D + F_\mathrm{r}\mathbf{w}\cdot\nabla\varphi^D) = -\big[-\mathrm{i}\omega(\varphi_0+\varphi_d) + F_\mathrm{r}(\mathbf{w}\cdot\nabla)(\varphi_0+\varphi_d)\big]\xi_0 \tag{18}$$

and $\eta^D(x,y) = p^D(\mathbf{r})|_{z=0}$. At a negative frequency of encounter, the associated wave frequency $\omega_0(-\omega)$ is different from that $\omega_0(+\omega)$, we have inequality relation

$$E_i(-\omega) \neq E_i^*(+\omega) \tag{19}$$

for $i = 1, 2, \cdots, 6$, consistent with previous discussions concerning the diffraction potentials (9). The same inequality (19) holds for ship motions.

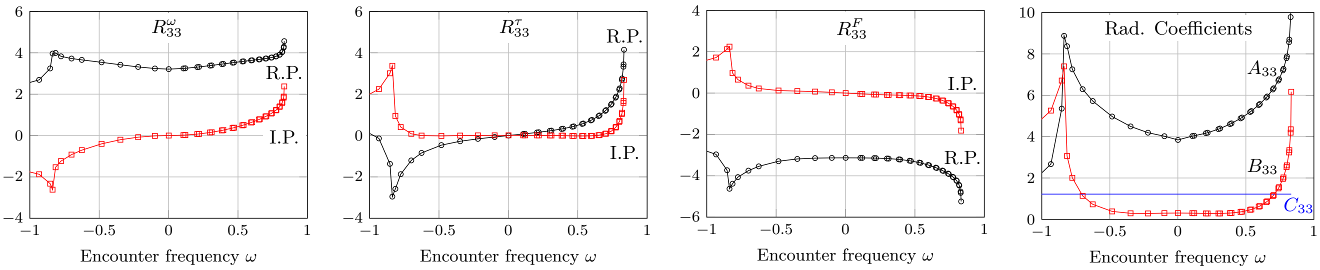

Figure 2: Radiation forces in heave (left 3 pictures) and added-mass/damping coefficients (right)

4 Numerical results and discussions

First, we examine the 3 components of radiation forces in heave defined by (12) for the hull of Wigley IV at the speed of $F_{\rm r} = 0.3$ within a rang of encounter frequencies $\omega \in (-1, 0.8)$ depicted on the left 3 pictures of Fig.2. The real part (R.P.) of R^{ω}_{33} and R^{F}_{33}, and the imaginary part (I.P.) of R^{τ}_{33} are symmetrical with respect to the sign of encounter frequency. On the other side, the imaginary part of R^{ω}_{33} and R^{F}_{33}, and the real part of R^{τ}_{33} are anti-symmetrical. Both added-mass and damping coefficients (A_{33}, B_{33}), defined by (13) and illustrated on the right of Fig.2, are symmetrical with respect to the encounter frequency consistent with (16). The added-mass coefficients A_{33} are of finite value at $\omega = 0$ since the value of the real part of $R^{F}_{ij}(\omega \to 0) \approx O(\omega^2)$ is subtracted and added to the stiffness coefficients. The value of B_{33} is not zero at $\omega = 0$ since the imaginary part of $R^{F}_{ij}(\omega) \approx O(\omega)$. The sharp variation in the vicinity of $\omega = -0.833$ is associated with the critical parameter $\tau = -1/4$, same as $\tau = 1/4$, where one system (ring waves) of ship generated waves are blocked in front of the ship bow, as explained in [3]. Other coefficients like (A_{55}, B_{55}), (A_{35}, B_{35}), ... etc have the same properties.

Second example concerns the bulk carrier RIOS (Research Initiative on Oceangoing Ships) for which experimental results have been presented in [5] in both the head $(\beta = \pi)$ and following $(\beta = 0)$ seas. Only amplitudes of RAOs in surge, heave and pitch are illustrated on the left, middle and right pictures of Fig.3, respectively. Excellent agreement has been observed in both head and following seas. Unlike in head seas, the surge motion in following seas is amplified by a factor about 3 which has been explained by less inertial and damping forces due to small encounter frequencies while the diffraction loads are of same order as in head seas. Since the resonance frequency in heave and pitch is out off the range of encounter frequencies, smaller heave and pitch motions are expected in following seas.

At small encounter frequencies $(|\omega| \to 0)$ in following seas, ships at moderate or large speeds could experience large motions like surf-riding on waves during which ships are directionally unstable. The present study provides some elements to improve understanding of this "broaching-to" phenomenon. In this business, the well-known Munk moment could be considered as a negative restoring torque in yaw. The restoring moment S^{W}_{66} due to the wavy steady flow $p^{W}(\mathbf{r})$ in addition to S_{66} defined by (10) are to be analyzed further.

References

[1] X.B. Chen, M.Q. Nguyen, I. Ten, C. Ouled Housseine, Y.M. Choi, L. Diebold, S. Malenica, G. De-Hautecloque & Q. Derbanne (2022) New seakeeping computations based on potentials linearised over the ship-shaped stream. *Proc. 15th Intl PRADS'2022, Dubrovnik, Croatia.*

[2] X.B. Chen, L. Diebold & Y.M. Choi (2022) Speed-effect restoring forces on ship motions. *Proc. 37th IWWWFB, Giardini Naxos, Italy*, pp37-41.

[3] X.B. Chen (2023) Fundamental solutions to ship-motion problems with viscous effects. *Physics of Fluids* (35).

[4] X.B. Chen, Y.M. Choi, M.Q. Nguyen & P-Y Wuillaume (2025) Surface integration of ship-motion Green function *Proc. 40th IWWWFB, Shanghai, China.*

[5] H. Iwashita, M. Kashiwagi, Y. Ito & Y. Seki (2016) Calculations of ship seakeeping in low-speed/low-frequency range by frequency-domain Rankine panel methods. *J Japn Soc Naval Archit Ocean Eng*, (24) 129–46.

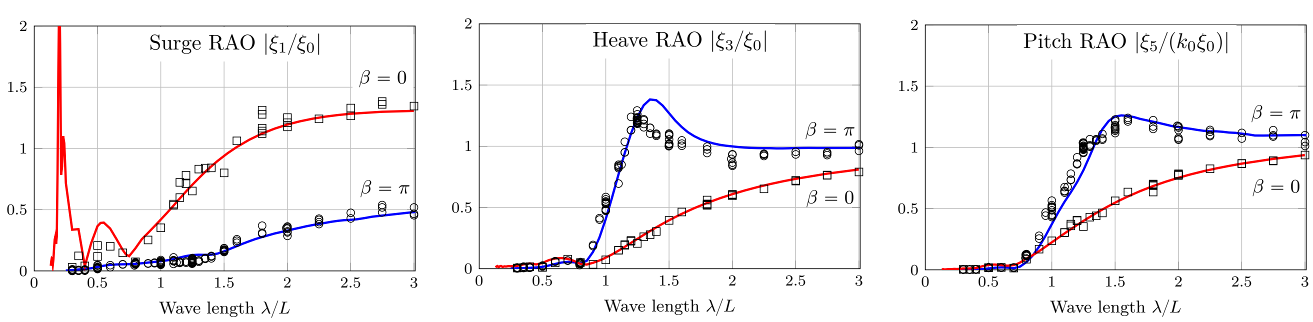

Figure 3: RAOs of surge (left), heave (middle) and pitch (right) in head seas $(\beta = \pi)$ and following seas $(\beta = 0)$