

Wave energy absorption using an internal U-tank

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1 Introduction

We are interested in evaluating the wave energy absorption potential of ships as a means of both auxiliary power generation and ship motion reduction. Both passively- and actively-controlled anti-roll tanks are widely used to reduce motions for certain ship classes. The company GEPS Techno even offers an anti-roll tank with energy absorption from a water turbine [1]. They claim to be able to harvest up to 500 kW per device. In this abstract, we discuss some aspects of the hydrodynamic analysis of a ship-like body with an internal U-tank for energy absorption. The power take off (PTO) could be either a water turbine or an air turbine, modeled by an equivalent linearized damping of the U-tank motions.

2 Linear hydrodynamic analysis with an internal U-tank

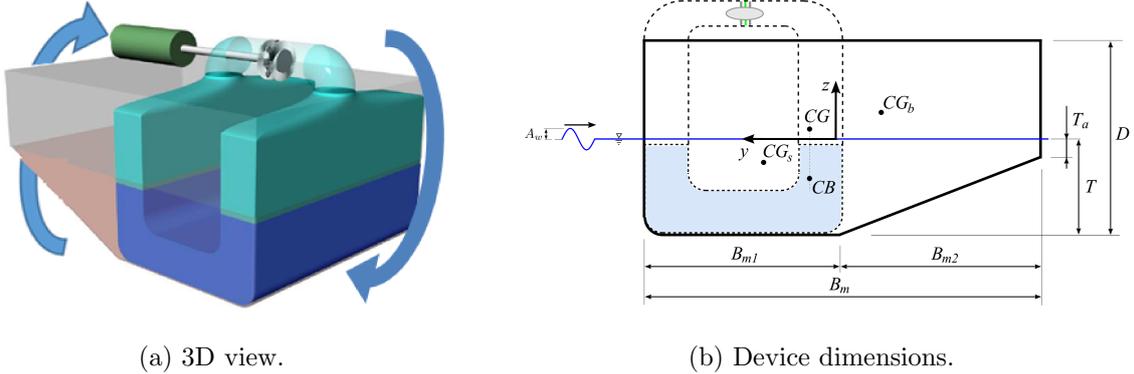


Figure 1: Schematic of the UGEN device (from [5]).

Consider a floating body with six rigid-body degrees of freedom ($j = 1, 2, \dots, 6$) plus the internal U-tank motion represented by generalized mode $j = 7$. Figure 1 shows a schematic of the UGEN device described in [4, 5], which will be used as a test case. The linear, potential flow, frequency-domain response, $x_j(t) = \Re\{\xi_j(\omega)e^{i\omega t}\}$, to a time-harmonic incident wave at frequency ω is given by the solution to the equations of motion:

$$\sum_{k=1}^7 Z_{jk} \xi_k = X_j, \quad j = 1, 2, \dots, 7. \quad (1)$$

Here,

$$Z_{jk} = R_{jk} + iI_{jk}, \quad R_{jk} = -\omega^2 \mathcal{M}_{jk} + C_{jk}, \quad I_{jk} = \omega \mathcal{B}_{jk}, \quad (2)$$

X_j is the wave exciting force coefficient, $\mathcal{M}_{jk} = M_{jk} + A_{jk} + M_{jk}^0$, with M_{jk} , A_{jk} , and M_{jk}^0 the inertia, added mass and possible external inertia matrices, $\mathcal{B}_{jk} = B_{jk} + B_{jk}^0$ the wave plus external damping matrices, and $C_{jk} = C_{jk} + C_{jk}^0$ the hydrostatic plus possible external restoring matrices. Invoking both the *internal tanks* and the *generalized modes* features of WAMIT [3],

all of the hydrodynamic coefficients can be computed. The generalized mode corresponding to the U-tank motion is defined by the generalized unit normal vector

$$n_7 = n_3(y - y_0) - n_2(z - z_0) = (y - y_0), \quad (3)$$

where y_0 is the position of the U-tank centerline, z_0 is the vertical position of the tank free surfaces and $n_3 = 1$, $n_2 = 0$ on these two horizontal planes. An equivalent linearized turbine damping coefficient, B_{77}^0 , can be estimated in various ways depending on the air (or water) turbine properties. We will present the damping in terms of the coefficient $\zeta_{77} = B_{77}^0/B_c$, where the critical damping, $B_c = 2\sqrt{C_{77}A_{77}}$.

2.1 The two-degree-of-freedom optimal damping coefficient

Restricting Eq. (1) to only $j = 4, 7$, corresponding to roll and the U-tank motion, the response is given by

$$\xi_4 = \frac{X_7 Z_{47} - X_4 Z_{77}}{Z_0}, \quad \xi_7 = \frac{X_4 Z_{74} - X_7 Z_{44}}{Z_0}, \quad Z_0 = Z_{47} Z_{74} - Z_{44} Z_{77}. \quad (4)$$

The extracted mean power per cycle is given by

$$\bar{P} = \frac{1}{2} B_{77}^0 \omega^2 |\xi_7|^2, \quad (5)$$

and since $X_7 = 0$, $B_{77} = B_{77}^0$ and all the coefficient matrices are symmetric, this can (after some algebra) be written

$$\bar{P} = \frac{\omega^2 B_{77}^0 N}{2[D_0 + B_{77}^0 D_1 + (B_{77}^0)^2 D_2]}, \quad (6)$$

where

$$N = (\omega^2 B_{47}^2 + R_{47}^2)[(X_4^R)^2 + (X_4^I)^2] \quad (7a)$$

$$D_0 = R_{47}^4 - 2R_{44}R_{47}^2R_{77} + R_{44}^2R_{77}^2 \quad (7b)$$

$$+ \omega^2(2B_{47}^2R_{47}^2 + 2B_{47}^2R_{44}R_{77} - 4B_{44}B_{47}R_{47}R_{77} + B_{44}^2R_{77}^2) + \omega^4B_{47}^4$$

$$D_1 = \omega^2(2B_{44}R_{47}^2 - 4B_{47}R_{44}R_{74}) - 2\omega^4B_{44}B_{47}^2 \quad (7c)$$

$$D_2 = \omega^2R_{44}^2 + \omega^4B_{44}^2, \quad (7d)$$

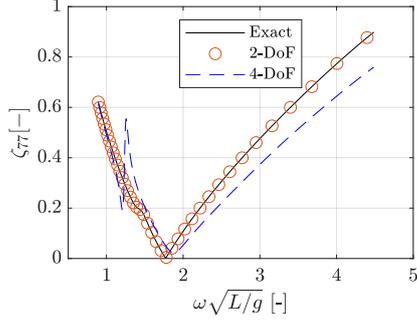
and the superscripts on X_4 indicate the real and imaginary parts. Taking the derivative of \bar{P} with respect to B_{77}^0 and setting it to zero gives the optimal damping as

$$B_{77}^{opt} = \sqrt{\frac{D_0}{D_2}}. \quad (8)$$

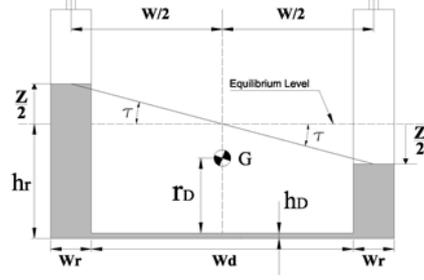
Fig. 2a shows ζ_{77} for the UGEN device comparing Eq. (8) with a numerical optimization using both 2 DoF (roll and U-tank) and 4 DoF (sway, heave, roll and U-tank). As expected, the numerical 2-DoF solution agrees essentially perfectly with the closed-form expression while some differences appear when all four degrees of freedom are included. The general trends are however similar.

2.2 An approximate solution for the U-tank response

In refs. [6, 4], the motions of the U-tank are estimated using an approximate method by Stigter [7] to determine the U-tank and coupling coefficients. Based on the parameters indicated in



(a) Optimal damping coefficients.



(b) U-tank parameters for the simplified model (from [6], where ξ_7 is denoted by τ).

Figure 2: Optimal damping coefficients and parameters of the simplified U-tank method.

Fig. 2b, the coefficients are assumed constant and given by

$$A_{77} = Q_t w_r \left(\frac{w}{2h_d} + \frac{h_r}{w_r} \right), \quad A_{27} = Q_t, \quad A_{47} = Q_t (r_d + h_r),$$

$$C_{77} = gQ_t, \quad C_{47} = gQ_t, \quad Q_t = \rho_t w_r w^2 \frac{L_t}{2}, \quad (9)$$

where ρ_t is the tank fluid density, g the gravitational acceleration, L_t the width of the tank and all coefficient matrices are symmetric. We note that the turbine damping coefficient, B_{77}^0 , is only applied to the U-tank motions and the power is computed from (5), in contrast to [6, 4] who apply the damping and compute the power based on the relative motion ($\xi_7 - \xi_4$).

Fig. 3 plots the predicted U-tank response (left) and absorbed power per meter of incident wave elevation squared (right) for the UGEN device with dimensions 15 by 20 m in the horizontal and a draft of 5 m, as described in [6]. Here a constant damping of $\zeta_{77} = 0.12$ is applied, 'WAMIT' refers to the internal tank plus generalized mode calculations while 'Simple' is the simplified method. The experimental measurements are from [5]. We can see from these plots that the two methods are in fairly good agreement, though the simplified method predicts a somewhat larger roll natural period (around $T = 7$ s) and U-tank resonant period (around $T = 5$ s), with somewhat more power absorption around these peaks. Both methods predict substantially higher response and power absorption than the experiments, indicating the need for further study to explain why. Additional comparisons and details can be found in [2].

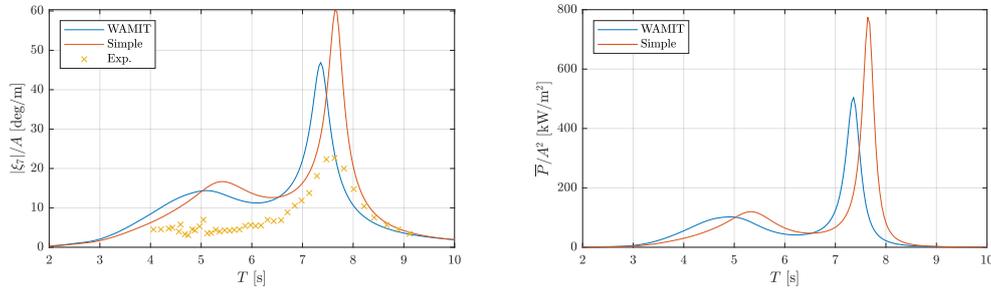


Figure 3: U-tank response (left) and power absorption per square meter of wave elevation (right) for the UGEN using a constant damping of $\zeta_{77} = 0.12$.

3 Preliminary calculations with a ship

Although the UGEN results presented above show some significant differences between the simplified and the full methods, we note that the U-tank mass in this case is nearly half of the total 'ship' mass, which is rather different from a typical ship where the U-tank mass would not likely be more than perhaps 5% of the total (see for example [6]). With that in mind, we use the simplified method to consider the energy harvesting potential of a U-tank installed on the KCS hull. Fig. 4 shows the absorbed power per wave amplitude squared for two cases. In

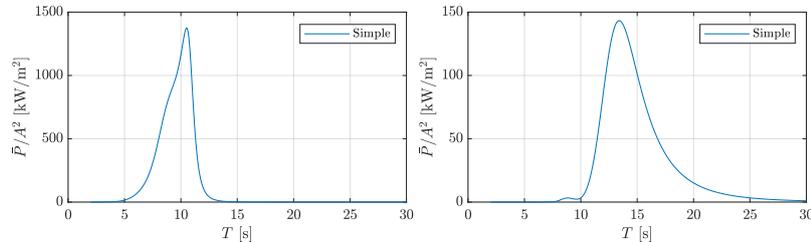


Figure 4: Power absorption in beam-seas using a roll tank (left) and in head-seas using a pitch tank (right). Constant damping of $\zeta_{77} = 0.20$ and zero forward speed.

the left-hand plot, the ship is in beam-seas with a roll tank installed across the midship section with dimensions $h_r = 3.69\text{m}$, $w_r = 4.49\text{ m}$, $h_d = 3.69\text{ m}$, $w = 25.51\text{ m}$, $L_t = 20\text{ m}$, $r_d = 3.55\text{ m}$, $w_d = 21.02\text{ m}$ giving a mass of 5% of the total ship mass. In the right-hand plot, the ship is in head seas with a pitch tank of dimensions $h_r = 7.50\text{ m}$, $w_r = 7.5\text{ m}$, $h_d = 7.5\text{ m}$, $w = 68.27\text{ m}$, $L_t = 3\text{ m}$, $r_d = 0.5\text{ m}$, $w_d = 24.76$ corresponding to 1.53% of the total ship mass. In both cases, a constant damping coefficient of $\zeta_{77} = 0.2$ is applied and the ship has zero forward speed. These preliminary examples indicate the potential for substantial energy harvesting even for large ships. More examples will be presented at the workshop.

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