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Wave Added Resistance Using Double-Body Linearization and Finite-Difference Method

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1 Introduction

During the past decade, we have developed a linear potential-flow solver for calculation of the seakeeping response of ships and marine structures. Many features of this solver have already been demonstrated in several previous workshops. This year our objective is to present some preliminary results for wave added resistance based on the *near-field* formulation and a *double-body* linearization. First, we briefly describe the underlying principles of our numerical model, afterwards we present the formulation and the results for computation of the wave added resistance.

2 The Numerical Method

The seakeeping model, OceanWave3D-Seakeeping [1], is based on the classical radiation diffraction decomposition where the hydrodynamic problems are solved in time domain. The computational space is comprised of a volume enclosed by the free surface S_f, the body surface S_b, the sea bed S_d and the far-field surface S_∞, where the numerical domain is truncated at a convenient horizontal location. The *entire* computational space is discretized using a 4th order *finite-difference* method on *overset* structured grids via the Overture library [2]. See Figure 1, where we show the part of the overset grid covering the body and free surface for a modified Wigley hull. The Laplace equation is solved in the entire domain subject to boundary conditions on the above-mentioned surfaces. The body boundary condition is specified by a pseudo-impulsive Gaussian function, either as the body motion (radiation) or the incident wave

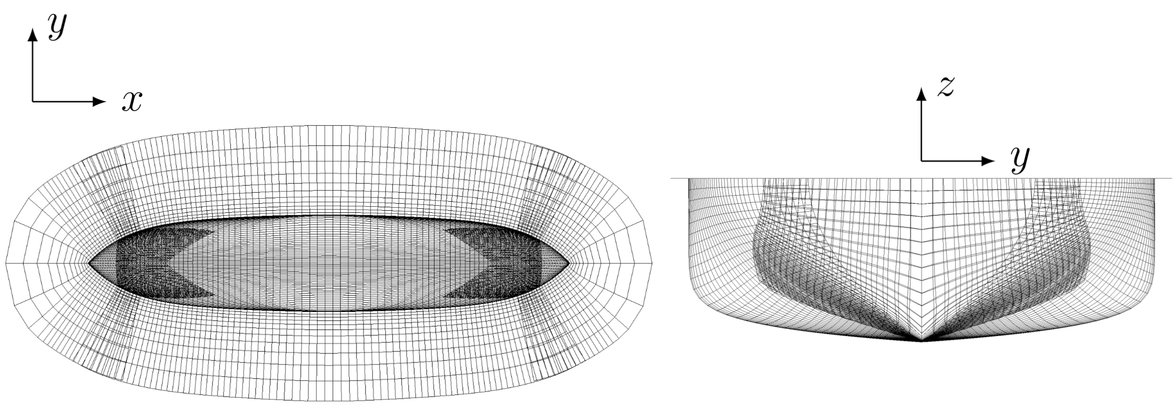

Figure 1: The overset grid. Note the coordinate systems are shown schematically.

elevation (diffraction). The conventional *m-terms* for the double-body linearization are computed, and the velocity potential ϕ and the surface elevation ζ are integrated in time according to

$$\frac{\partial \phi}{\partial t} = -g\zeta + U\frac{\partial \phi}{\partial x} - \nabla\phi_b \cdot \nabla\phi \qquad \text{at } z = 0, \tag{2.1}$$

$$\frac{\partial \zeta}{\partial t} = \frac{\partial \phi}{\partial z} + U\frac{\partial \zeta}{\partial x} - \nabla\phi_b \cdot \nabla\zeta + \zeta\frac{\partial^2 \phi_b}{\partial z^2} \qquad \text{at } z = 0. \tag{2.2}$$

Here ϕ_b is the double-body velocity potential, U is the forward speed and g is the gravitational acceleration. All first-order solutions obtained in the time domain are then Fourier transformed to the frequency domain. The details can be found in [1].

3 Wave Added Resistance

There are two well-known and classical methods for computing the second-order mean wave drift forces. The far-field method, where conservation of momentum is applied to a control surface comprised of S_b, S_f, S_d and S_∞ and the mean wave drift force is finally expressed in terms of an integral over S_∞. Traditionally this *infinity* integral has been dealt by two approaches. In the first approach, a far-field surface is selected to represent S_∞, where the flow properties can be computed accurately and conveniently. This is known as the *middle-field* method in the community. The second approach is to employ the Kochin function, by which the S_∞ integral can be converted to an integral over the body surface S_b. Based on this approach, Maruo in the 1960s developed a formulation for wave added resistance. As we have presented in the 37th workshop [3], there are still some *unsolved* issues related to this formulation, and it cannot be reliably applied in 3D potential-flow models for wave added resistance. At the 38th workshop [4], we showed that there exists in fact another approach for converting the S_∞ integral to a body surface integral. Here, we also presented results for some analytical geometries. For this new approach to computing wave added resistance for ship hulls, we are working to prove numerical convergence and will present our results in future workshops. Therefore our focus in this abstract is on the second classical method based on the *near-field* formulation, which is expressed by a body and a waterline integral as

$$\boldsymbol{F} = \overline{\iint_{S_b} (\mathbf{I} + \mathbf{II} + \mathbf{III} + \mathbf{IV} + \mathbf{V} + \mathbf{VI})\, ds} + \frac{1}{2}\rho g \overline{\oint_{C_b} \eta^2 \boldsymbol{n} dl} - \rho \overline{\oint_{C_b} (\mathbf{VII} + \mathbf{VIII})\, dl}.$$

$$\text{Where} \quad \mathbf{I} = -\rho\frac{1}{2}(\nabla\phi)^2 \boldsymbol{n}, \quad \mathbf{II} = -\rho\left(\frac{\partial \phi}{\partial t} - U\frac{\partial \phi}{\partial x} + \nabla\phi_b \cdot \nabla\phi\right)(\boldsymbol{\alpha} \times \boldsymbol{n}),$$

$$\mathbf{III} = -\rho(\boldsymbol{\xi} + \boldsymbol{\alpha} \times \boldsymbol{r}) \cdot \nabla\left(\frac{\partial \phi}{\partial t} - U\frac{\partial \phi}{\partial x} + \nabla\phi_b \cdot \nabla\phi\right)\boldsymbol{n},$$

$$\mathbf{IV} = -\rho(\boldsymbol{\xi} + \boldsymbol{\alpha} \times \boldsymbol{r}) \cdot \nabla\left(-U\frac{\partial \phi_b}{\partial x} + \frac{1}{2}(\nabla\phi_b)^2 + gz\right)(\boldsymbol{\alpha} \times \boldsymbol{n}),$$

$$\mathbf{V} = -\rho\left(-U\frac{\partial \phi_b}{\partial x} + \frac{1}{2}(\nabla\phi_b)^2 + gz\right)H\boldsymbol{n},$$

$$\mathbf{VI} = -\rho H\boldsymbol{r} \cdot \nabla\left(-U\frac{\partial \phi_b}{\partial x} + \frac{1}{2}(\nabla\phi_b)^2 + gz\right)\boldsymbol{n},$$

$$\mathbf{VII} = (\boldsymbol{\xi} + \boldsymbol{\alpha} \times \boldsymbol{r}) \cdot \nabla\left(-U\frac{\partial \phi_b}{\partial x} + \frac{1}{2}(\nabla\phi_b)^2\right)\eta\boldsymbol{n},$$

$$\mathbf{VIII} = \left(-U\frac{\partial \phi_b}{\partial x} + \frac{1}{2}(\nabla\phi_b)^2\right)\eta(\boldsymbol{\alpha} \times \boldsymbol{n}).$$

Here $\boldsymbol{r}$ is the position vector, and $\eta = \zeta - (\xi_3 + y\alpha_1 - x\alpha_2)$ is the relative surface elevation. The translational and the rotational body motions are denoted by $\boldsymbol{\xi}$ and $\boldsymbol{\alpha}$, and H is the second-order transformation matrix. The wave added resistance R_w is the x- component of the total mean drift force $\boldsymbol{F}$. As can be seen, the formulation contains *second derivatives* of the unsteady and steady velocity potentials. Accurate computation of these derivatives is fundamental, and becomes challenging specially over the complex geometries like ship hull. According to [5], we have employed an efficient method to compute these second derivatives. The main principle of this method is to employ identities from differential geometry, and express the derivatives in terms of their tangential and normal components over the body surface. The the accuracy of the computation is remarkably improved, since the normal component of the derivative is already known in closed-form.

4 Results

We present results for the classical Wigley hull (I), and the modified Wigley hull with the geometry according to [6], see Figure 2. For both cases the results are for head seas and with $Fr = 0.2$. In the plots, L is the ship length, B is the breadth, λ is wave length and A is wave amplitude. In the top two rows, the heave ξ_3 and pitch ξ_5 motion amplitudes are compared with measurements and other numerical solutions based on the Rankine panel method from [7, 8, 9, 10] all based on the double-body linearization.

5 Acknowledgment

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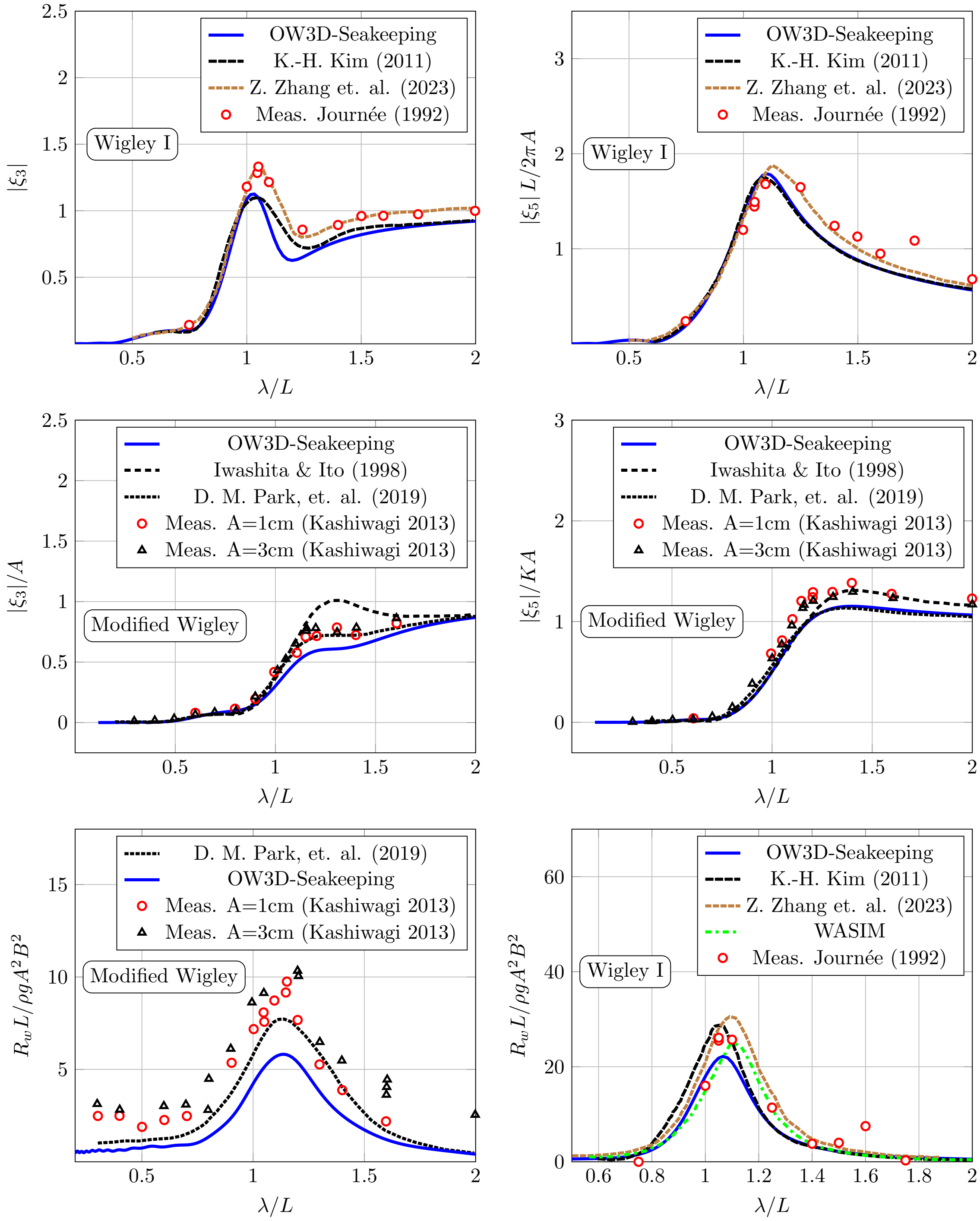

Figure 2: Results for the motion amplitude and wave added resistance for the Wigley hulls. Only heave and pitch modes are considered, and all results are for $Fr = 0.2$.