Layout and device parameter optimisation of a wave energy park in a broadband sea

Ben Wilks[a*], **Michael H. Meylan**[a], **Fabien Montiel**[b], **Dasun Shalila Balasooriya**[a], **Tahir Jauhar**[a], **Alan Blair**[c], **Craig Wheeler**[a], **Stephan Chalup**[a]

a. School of Information and Physical Sciences, University of Newcastle, Callaghan NSW 2308, Australia

b. Department of Mathematics and Statistics, University of Otago, Dunedin 9016, New Zealand

c. School of Computer Science and Engineering, University of New South Wales, Sydney NSW 2052, Australia

Highlights

- Linear water wave scattering by an array of partially-submerged cylindrical wave energy converters (WECs) is considered.
- A genetic algorithm is used to find the array layout and device parameters which maximise the power takeoff of the array in an irregular sea.
- When constrained to a rectangular region, the optimally-configured array consists of two parallel graded line arrays.

1 Introduction

The development of technologies for water wave energy conversion is an important avenue of research for the renewable transition. The optimisation of wave energy parks (namely, arrays of WECs) by analytic, numerical or experimental methods is an active area of research [3]. Here, we consider the problem of simultaneously optimising both the layout and device parameters of an array impacted by a broadband incident spectrum. This problem is computationally challenging due to the high cost of individual simulations (which involve multiple scattering calculations at multiple frequencies) and the large number of variables to be optimised. Presumably due to these reasons, we are not aware of any previous studies of this nature. The layouts of wave energy parks in irregular seas were optimised by [7, 9], although the device parameters were fixed throughout these optimisations. We consider a model of a wave energy park consisting of an array of partially submerged truncated cylinders coupled to a spring and damper—the layout of an array of such devices was previously optimised in [2] for a regular sea. By simultaneously optimising the device parameters and the park layout, we find an optimal array consisting of parallel graded line arrays. Although graded arrays of WECs are known to have impressive energy absorption capabilities [13, 12], a graded array has not previously arisen from a joint layout/parameter optimisation without imposing grading as a constraint.

2 Problem formulation and solution

We initially consider a single WEC consisting of a truncated, partially-submerged cylinder, which is constrained to move in heave and coupled to a spring-damper power takeoff system. The scattering problem is posed using time-harmonic linear water wave theory, which assumes that the fluid is incompressible and inviscid and undergoing irrotational, time-harmonic motion with time dependence $\mathrm{e}^{-\mathrm{i}\omega t}$. In cylindrical coordinates, the problem can be reduced to finding

*Email for correspondence: ben.wilks@newcastle.edu.au

the complex potential ϕ, which satisfies the boundary value problem

$$\triangle\phi = 0 \qquad (r,\theta,z) \in \Omega \tag{1a}$$
$$\partial_z\phi = -\mathrm{i}\omega s \qquad r < a, z = -d \tag{1b}$$
$$\partial_z\phi = \frac{\omega^2}{g}\phi \qquad r > a, z = 0 \tag{1c}$$
$$\partial_z\phi = 0 \qquad z = -H \tag{1d}$$
$$\partial_r\phi = 0 \qquad r = a, z > -d, \tag{1e}$$

where Ω is the fluid domain, g is acceleration due to gravity, a and d are the radius and equilibrium submergence of the cylinder and s is its heave amplitude, which we assume to be small. Equations (1) are solved in conjunction with a prescribed incident wave, a Sommerfeld radiation condition, a weak singularity condition at the submerged edge of the cylinder, and an equation of motion of the form

$$-\omega^2\rho d\pi a^2 s = \mathrm{i}\omega\mu s - k_s s - \rho g\pi a^2 s + \mathrm{i}\omega\rho\int_{-\pi}^{\pi}\int_0^a \phi(r,\theta,-d)r\mathrm{d}r\mathrm{d}\theta, \tag{2}$$

where the terms on the right hand side correspond to the damping, spring, hydrostatic and hydrodynamic forces, respectively, with ρ, μ and k_s being the fluid density, damping coefficient and spring coefficient, respectively. By rotational symmetry, the solution to (1) can be expressed as

$$\phi(r,\theta,z) = \sum_{n=-\infty}^{\infty}\phi_n(r,z)\mathrm{e}^{\mathrm{i}n\theta}, \quad \text{where} \quad \phi_n(r,z) = \begin{cases}\varphi_n(r,z) & r > a, \quad z \in (-H,0) \\ \chi_n(r,z) & r < a, \quad z \in (-H,-d).\end{cases} \tag{3}$$

Expressions for φ_n and χ_n, which we do not state here for brevity, are obtained using the method of separation of variables. Equation (3) gives rise to a matching problem between the interior region $r < a$ and the exterior region $r > a$, as ϕ_n must be continuously differentiable at $r = a$ for all n. The matching problem is solved using an integral equation/Galerkin method, in which the normal derivative $\partial_r\phi_n(a,z)$ is expanded in a basis of weighted Gegenbauer polynomials [4].

Next, we consider the multiple scattering problem of N WECs centred at (x_j, y_j) for $1 \leq j \leq N$, which we solve using the self consistent theory of multiple scattering and Graf's addition theorem [6]. We invoke the wide spacing approximation to simplify the computation, which assumes that evanescent modes do not contribute to the interaction between cylinders. Wiscombe's formula is used as a heuristic for the minimum number of angular modes required for accuracy [14]. A generalised optical theorem, which relates the amplitudes of the pistons to the far field function, is used to validate conservation of energy by the array [8].

We optimise the rate of energy absorption by the array in an irregular sea, given by

$$P(X) = \int_0^\infty P(X,\omega)\sqrt{2S(\omega)\mathrm{d}\omega}, \tag{4}$$

where X is a vector containing the coordinates of the WECs and their parameters (here comprising their spring and damping coefficients) and $P(X,\omega)$ is the average rate of energy absorption by the array when acted on by a monochromatic plane wave of unit amplitude with angular frequency ω. The spectral density function S is taken to be the Pierson-Moskowitz spectrum [11] with significant wave height $H_s = 2\,\mathrm{m}$. The reader is directed to [10] for the technical details of the integral in (4).

3 Results and conclusion

The objective function (4) is optimised for an array of $N = 25$ WECs using MATLAB's built-in genetic algorithm `ga`. The WECs are constrained to the rectangular region $0 \leq x_j \leq D_x = 200\,\mathrm{m}$

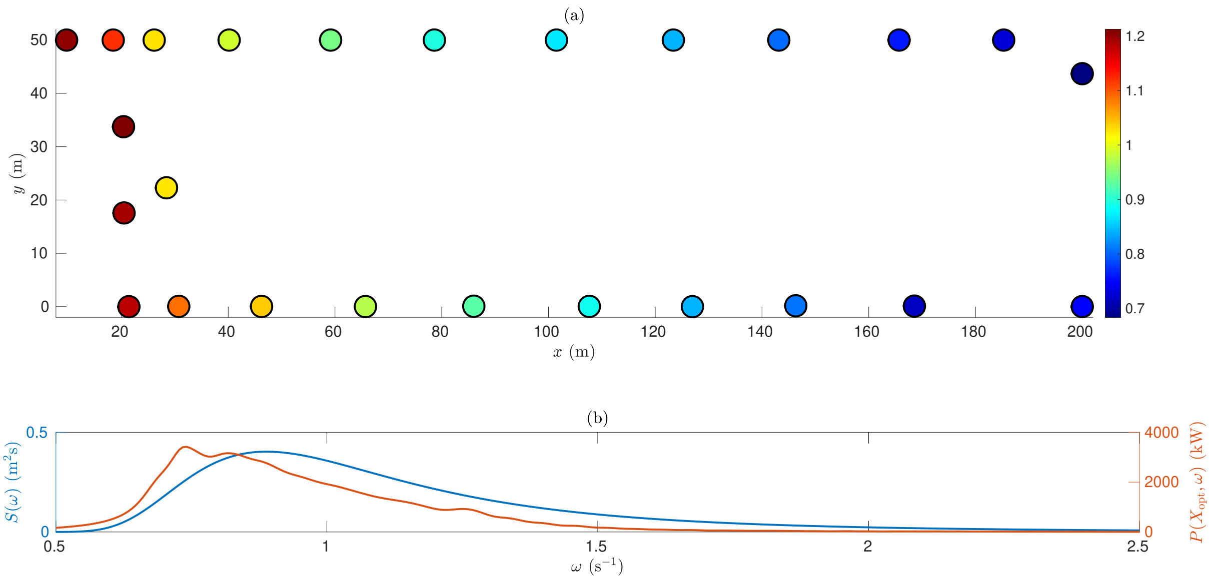

Figure 1: (a) Layout of the optimal array, with WECs marked as circles not drawn to scale. Colour indicates the frequency at which the WEC, if isolated, would draw maximum power from monochromatic plane waves. Note the direction of wave propagation is from left to right. (b) Pierson-Moskowitz spectrum used in the optimisation problem (blue line) and power extraction of the array from unit amplitude monochromatic waves as a function of frequency (red line).

and $0 \le y_j \le D_y = 50\,\mathrm{m}$ for all $1 \le j \le N$. The radius and submergence of the WECs are fixed to be $a = d = 1\,\mathrm{m}$. Non-restrictive constraints are imposed on the spring and damping coefficients—notably, negative spring coefficients are permitted (as in [12]). The output of one run of the genetic algorithm, which does not necessarily coincide with the global optimum, is given in Fig 1.

Of note, two parallel graded line arrays emerge as elements in the optimally configured array, which suggests that rainbow absorption is occurring. This is a phenomenon exhibited by graded arrays, in which waves slow down and amplify at different locations depending on frequency, where their energy can be subsequently absorbed. Previous studies of rainbow absorption in water waves [13, 12] imposed grading as a constraint in the optimisation process. The fact that grading arises naturally in our problem suggests that rainbow absorption is of fundamental importance in wave energy parks optimally configured for broadband spectra.

Future work will consider this problem as a multi-condition optimisation problem (MCOP), in which the effect of certain parameters on the optimal solution are sought. In our context, the significant wave height H_s and the aspect ratio of the bounding rectangle of the array D_x/D_y would both be interesting to consider as conditions. MCOPs are of the form

$$\min_{\mathbf{x}\in X} f(\mathbf{x}; c) \quad \text{for all } c \in \Phi, \tag{5}$$

where f is the objective function, X is the space of decision vectors and Φ is the condition space. Although traditional methods (e.g. genetic algorithms) can only solve MCOPs by discretising them into a set of standard optimisation problems, algorithms based on deep reinforcement learning efficiently solve MCOPs through a seamless incorporation of the condition space [5, 1].

Acknowledgements

BW, DSB, TJ, CW and SC are supported by the Australian Research Council's Linkage Projects funding scheme (LP190100378) and Australian Government Research Training Program Scholarships to DSB and TJ. BW and MHM are supported by the Australian Research Council's

Discovery Projects funding scheme (DP240102104). FM is supported by the Royal Society Te Apārangi (Marsden Fund projects 20-UOO-173) and New Zealand's Antarctic Science Platform (project ANTA1801).

References

[1] D. S. Balsooriya, A. Blair, C. Wheeler, and S. Chalup, *Multi-condition multi-objective airfoil shape optimisation using deep reinforcement learning compared to genetic algorithms*, in Optimization, Learning Algorithms and Applications. Proceedings of the OL2A 2024 Conference, Springer Nature Switzerland, in press. Accepted 02.06.2024.

[2] B. Child and V. Venugopal, *Optimal configurations of wave energy device arrays*, Ocean Engineering, 37 (2010), pp. 1402–1417.

[3] M. Göteman, M. Giassi, J. Engström, and J. Isberg, *Advances and challenges in wave energy park optimization—a review*, Frontiers in Energy Research, 8 (2020), p. 26.

[4] M. Kanoria, D. Dolai, and B. Mandal, *Water-wave scattering by thick vertical barriers*, Journal of engineering mathematics, 35 (1999), pp. 361–384.

[5] S. Kim, I. Kim, and D. You, *Multi-condition multi-objective optimization using deep reinforcement learning*, Journal of Computational Physics, 462 (2022), p. 111263.

[6] P. A. Martin, *Multiple scattering: interaction of time-harmonic waves with N obstacles*, no. 107 in Encyclopedia of Mathematics and its Applications, Cambridge University Press, 2006.

[7] J. P. McGuinness, *Hydrodynamic optimisation of an array of wave-power devices*, PhD thesis, University College Cork, 2018.

[8] C. C. Mei, M. Stiassnie, and D. K.-P. Yue, *Theory and applications of ocean surface waves Part 1: Linear aspects*, World Scientific, 2005.

[9] M. Neshat, B. Alexander, N. Y. Sergiienko, and M. Wagner, *New insights into position optimisation of wave energy converters using hybrid local search*, Swarm and Evolutionary Computation, 59 (2020), p. 100744.

[10] M. K. Ochi, *Ocean Waves: The Stochastic Approach*, no. 6 in Cambridge Ocean Technology Series, Cambridge University Press, 1998.

[11] W. J. Pierson Jr and L. Moskowitz, *A proposed spectral form for fully developed wind seas based on the similarity theory of SA Kitaigorodskii*, Journal of geophysical research, 69 (1964), pp. 5181–5190.

[12] A.-R. Westcott, L. G. Bennetts, N. Y. Sergiienko, and B. S. Cazzolato, *Broadband near-perfect capture of water wave energy by an array of heaving buoy wave energy converters*, Journal of Fluid Mechanics, 998 (2024), p. A5.

[13] B. Wilks, F. Montiel, and S. Wakes, *Rainbow reflection and broadband energy absorption of water waves by graded arrays of vertical barriers*, Journal of Fluid Mechanics, 941 (2022).

[14] W. J. Wiscombe, *Improved Mie scattering algorithms*, Appl. Opt., 19 (1980), pp. 1505–1509.