Broadband absorption of wave energy by graded arrays of heaving buoys in 3D

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HIGHLIGHTS

Graded arrays of heaving buoys capture over 90% of incident wave energy over a frequency band covering two thirds of usable frequencies, and a wide range of incident wave directions. An array of ten stacks achieves an average absorption of 98% at normal incidence (fig. 3b).

1 INTRODUCTION AND PRELIMINARIES

Heaving buoy-type wave energy converters (WECs) achieve maximum efficiency at resonance, but have a relatively narrow resonance bandwidth compared to ocean waves [1], where periods between 5 and 20 s are feasible for power capture [2]. Although deploying WECs in arrays can improve cost-effectiveness through optimised layouts [3], control strategies will be necessary to achieve high efficiency over broad frequency bands [1, 2]. Typically, the power take-off (PTO) parameters are adjusted according to predicted sea states, which requires accurate wave prediction capabilities, is sensitive to model errors, and increases overall cost and complexity [1].

Theories for efficient broadband capture of ocean-wave energy have recently been demonstrated in 2D by grading the resonant properties of WEC-arrays, governed by standard linear theory [4, 5]. The two-fold approach uses local band structures, resulting from carefully graded resonant properties, to create a frequency dependent series of near-resonant responses, which are captured via PTO damping to form a rainbow absorbing array. Put simply, the grading of local resonances is used to prevent the transmission of targeted wave frequencies by creating a rainbow reflecting array, while the PTO damping is chosen to minimise the amplitude of reflected waves [4, 5]. In 2D, the spatially-controlled amplification of wave energy according to frequency has been achieved by grading a surrounding structure of rigid, vertical barriers (98.2% absorption of 4–8 s waves; [4]), and by grading the resonant frequencies of heaving point absorbers via PTO spring terms to create a series of local bandgaps through destructive wave interactions (99% absorption of 10–20 s waves; [5]).

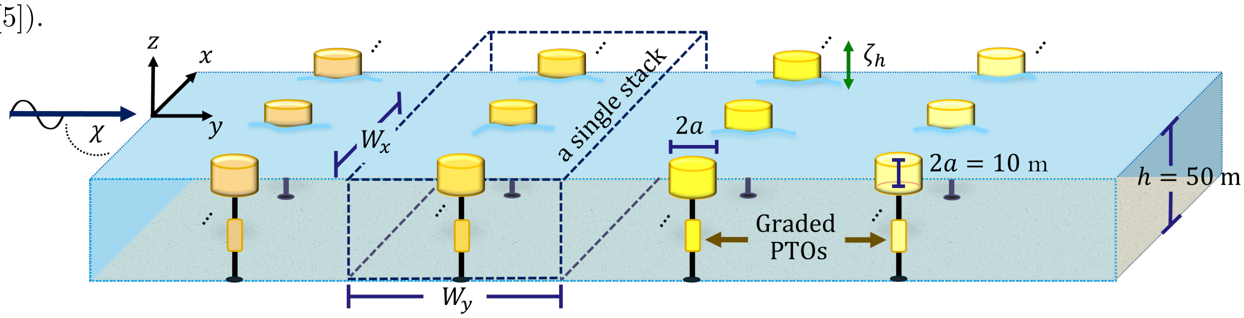

Figure 1: The graded array contains finitely many stacks, each consisting of infinitely many WECs (periodicity W_x). The incident wave (unit amplitude) travels left-to-right at an angle of χ rad to the x-axis.

Here, we extend the approach outlined by [5] to an array of heaving buoys with graded PTOs in a 3D water domain, by arranging the WECs to form diffraction gratings, or *stacks*, with a common periodicity W_x (fig. 1). The midpoint-to-midpoint distance between stacks is denoted W_y (fig. 1). In this formulation, the scattering properties of a finite number of stacks S can be described in terms of a discrete directional spectrum, which depends on the incident angle, frequency, and periodicity [6]. At large wavelengths relative to the periodicity, broadband absorption is obtained by manipulating reflected and transmitted propagating modes, similarly to the 2D model, but with the challenge of greater transmission in 3D, which is exacerbated at practically feasible W_x values.

2 METHODS

Motions are subject to time-harmonic motion with radian frequency ω. A boundary value problem in the frequency domain is solved for each stack following [6], where WECs are modelled as truncated cylinders attached to linear spring–damper PTO mechanisms. The periodic arrangement of infinitely many WECs into stacks enables the multidirectional wavefield between stacks to be defined by a discrete set of (both real and complex) scattering angles ($\chi_{m,n}$), and represented as a plane-wave expansion [6] with surface elevation (in polar coordinates (r,θ))

$$\eta(r,\theta) = \Sigma_n \Sigma_m A^{+}_{mn} \exp(\mathrm{i}k_n r\cos(\theta - \chi_{m,n})) + \Sigma_n \Sigma_m A^{-}_{mn} \exp(\mathrm{i}k_n r\cos(\theta + \chi_{m,n})). \tag{1}$$

The wavenumbers k_n are the real ($n = 0$; propagating mode) and imaginary ($n \geq 1$; evanescent modes) roots to the dispersion equation. The amplitudes $A^{\pm}_{mn}$ are associated with wave modes that propagate or decay in the $\pm\, y$-direction. This formulation provides a means to extend the 2D model [5] to 3D, enabling the scattering properties of stacks to be encapsulated by the transfer matrices

$$\boldsymbol{P_s} = \begin{bmatrix} \boldsymbol{T_s^-} - \boldsymbol{R_s^-}\mathrm{inv}(\boldsymbol{T_s^+})\boldsymbol{R_s^+} & \boldsymbol{R_s^-}\mathrm{inv}(\boldsymbol{T_s^+}) \\ -\mathrm{inv}(\boldsymbol{T_s^+})\boldsymbol{R_s^+} & \mathrm{inv}(\boldsymbol{T_s^+}) \end{bmatrix} \text{ for } s = 1, 2, \ldots, S, \tag{2}$$

which relate incoming waves to outgoing waves. Reflection ($\boldsymbol{R_s^\pm}$) and transmission ($\boldsymbol{T_s^\pm}$) matrices for stack s are assembled from the scattered coefficients for incident waves travelling in the $\pm$ y-direction in a plane-wave expansion for the stack (see [6]). Propagating modes are associated with $\chi_{m,0} \in \mathbb{R}$, and decaying modes with $\chi_{m,n} \in \mathbb{C}$ for $n \geq 0$. On the target frequency band, $\omega \in [0.3, 0.65]\,\mathrm{rad\,s^{-1}}$, all of the stacks considered have a single, real scattering angle, which is consistent with the incident angle, and corresponds to one propagating direction. However, higher order, decaying modes must be included to capture interactions between resonant stacks accurately.

The transfer matrices for individual stacks are iteratively combined to capture the interactions between stacks and obtain the solution for the array. Absorption is calculated from the elements of the reflection and transmission matrices of the array, $\boldsymbol{R}^+$ and $\boldsymbol{T}^-$ respectively, associated with the propagating mode, as $\alpha = 1 - |R|^2 - |T|^2$. The eigenvalues of the transfer matrix for each stack reveal the band structures on a unit cell (box; fig. 1) in a corresponding periodic array. Propagation is supported at frequencies below the stack resonance (determined by the resonant frequency of the constituent WECs), which induces a local bandgap wherein propagation is prohibited.

Rainbow reflection is generated by grading the resonant frequencies of finitely many stacks via the PTO springs from high to low ($\omega \approx 0.65 \to \omega \approx 0.3\,\mathrm{rad\,s^{-1}}$) in the direction of the incident wave so that the resonance of stack s lies in the local bandgap of stack $s+1$. The overlapping of local bandgaps on the corresponding unit cells associated with adjacent stacks, creates an effective bandgap over $\omega \in [0.3, 0.65]\,\mathrm{rad\,s^{-1}}$, where $|T|^2 \approx 0$ and $|R|^2 \approx 1$. Each stack has an associated zero of transmission and peak in reflection.

The incident wave is amplified near a stack with a comparable resonant frequency, before being reflected out of the array, due to the rainbow reflection effect. Rainbow absorption is created by extracting the amplified energy in the rainbow reflecting array using the PTO damper. The PTO damping for each stack is chosen to create a near-zero in reflection and corresponding peak in absorption, starting with the penultimate stack and working towards the front of the array.

3 RESULTS

Fig. 2a shows how the broadband absorption of an array evolves as its stacks are tuned. The array is composed of five stacks, in which the WECs are neutrally buoyant with a radius and draft of 5 m (fig. 1). A relatively close spacing of $W_x = W_y = 25\,\mathrm{m}$ (inter-WEC distance = $3a$) is applied to facilitate greater control of the wavefield, which allows a clearer demonstration of the approach, while maintaining reasonable convergence and numerical stability. Stacks are tuned in order of low-to-high resonant frequencies at normal incidence ($\chi = \pi/2\,\mathrm{rad}$). Relatively low damping is applied to the last stack to achieve $|R|^2 \approx |T|^2 \approx 0$ in combination with the fourth stack at $\omega \approx 0.3\,\mathrm{rad\,s^{-1}}$. The resulting graded array has an average absorption of $\hat{\alpha} = \frac{1}{0.35}\int_{0.3}^{0.65} \alpha\,\mathrm{d}\omega = 94.1\%$ (fig. 2a).

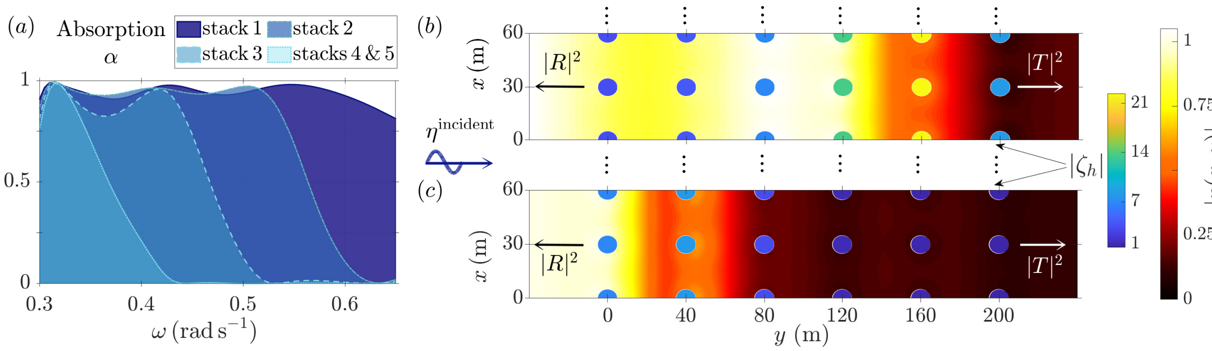

Figure 2: Peaks in (a) absorption are obtained by damping five, graded stacks for $|R|^2 \approx 0$. The average absorption of the array ($W_x = W_y = 25\,\mathrm{m}$) on $\omega = [0.3, 0.65]\,\mathrm{rad\,s^{-1}}$ is $\hat{\alpha} = 0.941$. The surface elevation η (unit amplitude, normal incidence $\chi = \pi/2\,\mathrm{rad}$) and WEC amplitudes ζ_h of a graded array of six stacks ($W_x = 30\,\mathrm{m}$ and $W_y = 40\,\mathrm{m}$; $\hat{\alpha} = 0.949$) is shown at the incident frequencies (b) $\omega = 0.35\,\mathrm{rad\,s^{-1}}$ and (c) $\omega = 0.56\,\mathrm{rad\,s^{-1}}$. Incident waves propagate (left-to-right) until reaching a stack with a comparable resonance, where the amplified energy is extracted by the PTO damper for rainbow absorption.

For fixed WEC spacings (W_x and W_y), the average absorption increases with the number of stacks (fig. 3a). Larger W_x and W_y allow higher transmission and require a greater number of stacks to maintain high efficiency. A good combination of high absorption, practical spacing, and rapid convergence is obtained by grading an array of six stacks when $W_x = 30\,\mathrm{m}$ and $W_y = 40\,\mathrm{m}$ (fig. 2b–c). The rainbow absorbing array allows higher transmission at low frequencies (fig. 2b) compared to high frequencies (fig. 2c) as a result of narrower resonance bandwidths at lower frequencies. However, $|T|^2 < 0.085$ for $\omega = [0.3, 0.65]\,\mathrm{rad\,s^{-1}}$, and can be reduced by a less severe grading (i.e., increasing the number of stacks; fig. 3).

The efficient broadband absorption obtained at normal incidence is maintained over a wide range of incident wave angles, as the stack resonances are unaffected by the incident wave angle. As the incident angle decreases ($\chi \to \pi/8\,\mathrm{rad}$), transmission decreases over the targeted interval (local bandgaps widen in frequency space) but near-zeros in reflection are gradually lost using fixed PTO parameters, which eventually reduces absorption. At larger periodicities (e.g., $W_x = 50\,\mathrm{m}$ versus $W_x = 30\,\mathrm{m}$), the reduction in transmission outweighs the gradual loss of near-zero reflection between $\chi = \pi/2\,\mathrm{rad}$ and $\chi = \pi/8\,\mathrm{rad}$, resulting in (generally) higher absorption (fig. 3a).

4 DISCUSSION AND CONCLUSIONS

The average absorption of a graded array of 10 stacks (fig. 3), with a practically feasible spacing of $W_x = 30\,\mathrm{m}$ and $W_y = 40\,\mathrm{m}$, is less than 2% lower than a 1D array of 10 WECs [5]. The slight difference in absorption is attributed to stacks being weaker reflectors than 2D WECs. Stacks with closely spaced WECs are stronger reflectors and can sustain low transmission over a wider frequency range as a result of broader resonance bandwidths. The separation distances between WECs should remain between three and eight times the WEC radius, as a closer spacing produces slow convergence and numerical instability, while wider spacing decreases efficiency.

In general, higher transmission at larger W_x and W_y distances can be counteracted by increasing the number of stacks. For example, ten stacks achieve $\hat{\alpha} = 0.952$ at $W_x = W_y = 50\,\mathrm{m}$ ($\chi = \pi/2\,\mathrm{rad}$; fig. 3a). However, similar to 1D arrays, the precise control of reflection and transmission zeros is limited by strong interactions between an increasing number of stacks on a fixed frequency interval [5]. Further, increasing W_y causes resonances that arise from interference between waves reflected back and forth between the stacks to move down in frequency, which raises transmission near the upper bound of the target interval, causing absorption to drop.

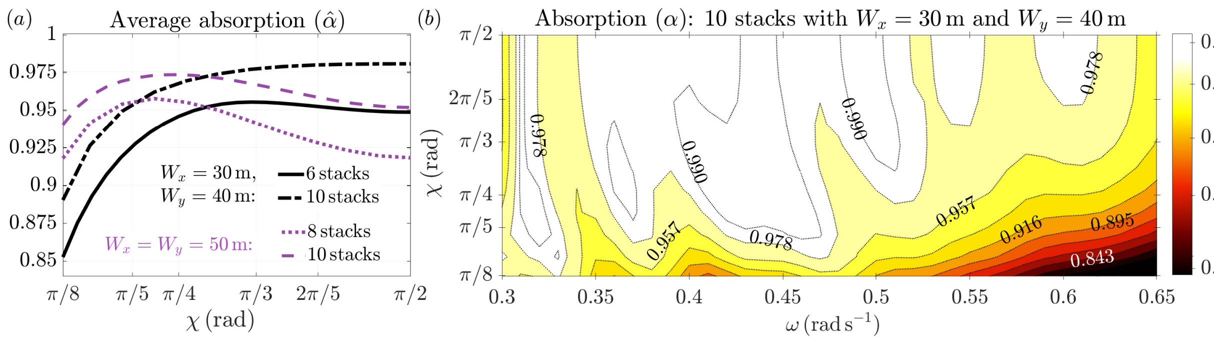

Figure 3: The (a) average absorption on $\omega \in [0.3, 0.65]\,\mathrm{rad\,s^{-1}}$ as a function of the incident angle χ for two spatial configurations of graded arrays. Absorption increases with the number of stacks. An array of (b) 10 stacks ($W_x = 30$, $W_y = 40\,\mathrm{m}$) achieves $\alpha \geq 0.95$ on the majority of the ω–χ domain (average = 0.962).

To remain within the confines of linear theory, the strategies presented are restricted to incident waves of low amplitude, where the amplified WEC motions do not exceed the WEC draft, and break the water surface. Future research directions include approximating non-linearities such as viscous drag, which will reduce WEC motions and therefore efficiency (however, the arrays maintain broadband behaviour; preliminary work), and extending strategies for broadband absorption to arrays that are finite in both the x and y-directions.

In conclusion, 2D arrays of WECs were graded for broadband absorption using the PTO mechanisms. Array designs were presented that capture over 90% of the incident energy from two thirds of usable ocean-wave frequencies using constant-in-time PTO parameters. The grading is robust to a wide range of incident angles, and absorption can exceed 95% with an appropriate choice of spacing. At larger W_x and W_y distances, additional stacks are required for comparable absorption.

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