Modeling of wave breaking in short-crested seas

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1 Introduction

Energy dissipation plays a fundamental role in the modeling of wave breaking within nonlinear potential flow solvers. It involves two steps: the first one is the wave breaking detection, and the second one is the removal of energy induced by the detected breaking event. For the wave-breaking onset, the ratio of the fluid velocity and local crest phase speed, $B = U/C$, allows accurate detection of the breaking events. The key point for this kinematic criterion B is the evaluation of the crest phase speed, which is challenging in multidirectional seas. [1] proposed a so-called Spatial Hilbert Transform Method (SHTM) and argued that the SHTM allows an accurate and efficient computation of the local crest phase speed in short-crested sea states. Once the breaking wave is detected, adequate management of the energy dissipated during wave breaking is indispensable and pivotal. The present study proposes a novel energy dissipation model for multi-directional sea states, by adding viscous terms to the free surface kinematic and dynamic boundary conditions. The resulting breaking model is implemented in the nonlinear wave solver HOS-NWT, which is a computationally efficient open-source Numerical Wave Tank (NWT) based on the High-Order Spectral (HOS) method, extensively validated for non-breaking waves [2]. This enhanced NWT is validated against experiments in an ocean wave basin.

2 Dissipation model

The kinematic and dynamic free surface boundary conditions of the HOS model, accounting for a viscous term, yields

$$\frac{\partial \eta}{\partial t} = \left(1 + |\nabla_h \eta|^2\right) W - \nabla_h \phi_s \cdot \nabla_h \eta + 2\nu_{eddy} R \nabla_h^2 \eta, \tag{1}$$

$$\frac{\partial \phi_s}{\partial t} = -g\eta - \frac{1}{2} |\nabla_h \phi_s|^2 + \frac{1}{2} \left(1 + \mid \nabla_h \eta \mid^2\right) W^2 + 2\nu_{eddy} R \nabla_h^2 \phi_s \tag{2}$$

where (η, ϕ_s) are the free surface elevation and potential respectively, ∇_h is the horizontal gradient, W represents the vertical fluid velocity at the free surface, g denotes the gravity acceleration, ν_{eddy} is the eddy viscosity, and R represents a spatial ramp function dependent on the horizontal coordinate $\mathbf{x} = (x, y)$. The additional viscous term in Eq.(2) can be interpreted as an equivalent pressure, $p_f(\mathbf{x}) = -2\rho\nu_{eddy} R \nabla_h^2 \phi_s$, with ρ is the fluid density. The power dissipated by this extra term can be evaluated by integrating on the free surface $p_f \partial\phi/\partial n$ over the wave-breaking region, S_{br}, with $\partial\phi/\partial n$ the normal fluid velocity at the free surface.

$$P_b = \int_{S_{br}} p_f \frac{\partial \phi}{\partial n} dS = -2\rho\nu_{eddy} \int_{S_{br}} R \nabla_h^2 \phi_s \frac{\partial \eta}{\partial t} dS \tag{3}$$

From an ocean wave physical point of view, this power corresponds to the energy dissipation rate $P_b = \int_{l_{cl}} \epsilon dl$, in which ϵ is the dissipation rate per unit transversal length, and l_{cl} is the transverse spatial extent of the wave breaking crest curve. Thus, the eddy viscosity for a given breaking event can be evaluated as:

$$\nu_{eddy} = -\frac{\int_{l_{cl}} \epsilon dl}{2\rho \int_{S_{br}} R\nabla_h^2 \phi_s \frac{\partial \eta}{\partial t} dS} \tag{4}$$

The energy dissipation rate per unit transverse length of the breaking crest can be computed by $\epsilon = b\rho C^5/g$. b represents the breaking strength, which is taken in this first work as a constant $b = 0.05$, and C is the local crest phase speed computed by SHTM in the present study. Besides, from the instantaneous free surface elevation signals, the SHTM allows us to extract the local wavenumber vector $\mathbf{k}(\mathbf{x}, t)$ and angular frequency $\omega(\mathbf{x}, t)$ with which the dissipation duration and area will be determined.

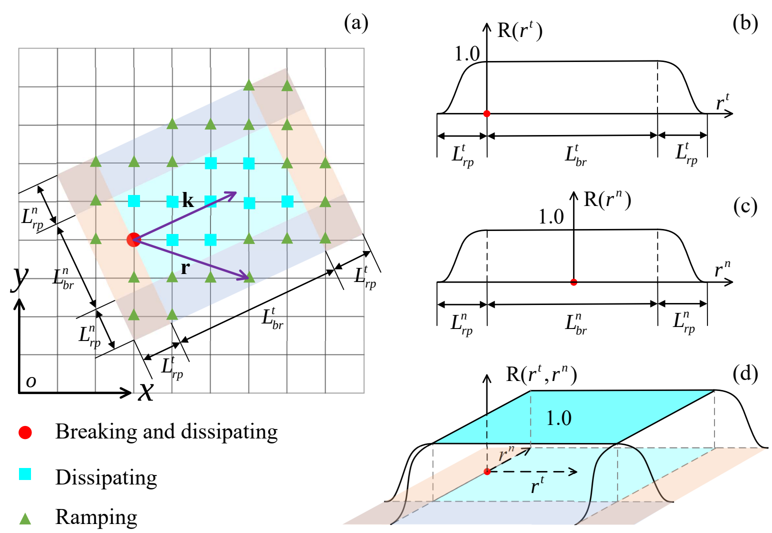


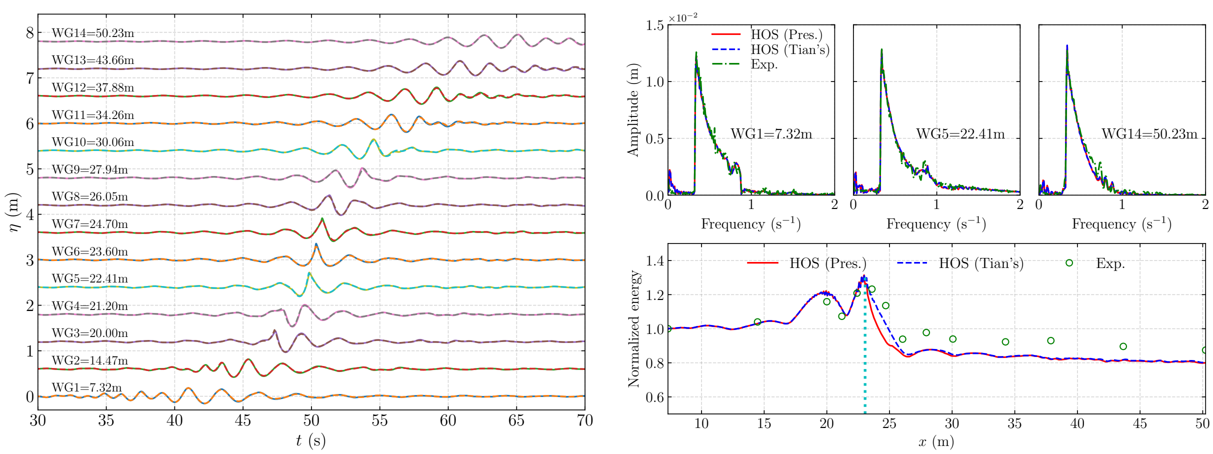
Figure 1: (a) Schematic of the ramping and dissipating region of a single breaker. (b,c) Cubic ramp function in the tangential, and normal wave propagation direction. (d) Mixed cubic ramp function with two horizontal directions.

Figure 1 sketches the dissipation region assuming a wave is detected as breaking at time t_{br} and location $\mathbf{x}_{br} = (x_{br}, y_{br})$, which is named as a breaker. To define the dissipation region, the breaker is detected as the red circle marked in Fig.(1a), in which the wave propagates with the direction of the local wavenumber vector $\mathbf{k}(\mathbf{x}_{br}, t_{br})$. As the cyan shadow region is shown in Fig.1(a), in the wave propagation direction, the dissipation length is determined by $L_{br}^t = 2\pi / \|\mathbf{k}(\mathbf{x}_{br}, t_{br})\|$; in the normal direction, the dissipation width is determined by the mesh size $L_{br}^n = \min\{\Delta x, \Delta y\}$. To prevent numerical instabilities, a spatial ramp function is introduced, and the length of the ramp region in the tangential and normal direction of the wavenumber vectors are given by $L_{rp}^t = 0.25 L_{br}^t$ and $L_{rp}^n = 0.25 L_{br}^n$, respectively. Building a local coordinate system at the wave breaking point allows us to define the ramp function and select the dissipation region easily. To achieve this, a local vector is introduced as

$\mathbf{r} = (\mathbf{x} - \mathbf{x}_{br})$, which establishes the local system in which the tangential direction is given by $r^t = \mathbf{r} \cdot \mathbf{k} / \|\mathbf{k}\|$ and normal direction $r^n = \mathbf{r} \times \mathbf{k} / \|\mathbf{k}\|$. Based on this local coordinate system, we can implement a cubic ramp function individually in the tangential and normal directions of the wavenumber vector, *i.e.* $R(r^t)$ and $R(r^n)$ as displayed in Fig.(1b,c), respectively. The total ramp function for the entire dissipation region is then given by $R(r^t, r^n) = R(r^t)R(r^n)$ as exemplified in Fig.(1d). Besides, the dissipation duration is determined by the local angular frequency, namely $T_{br} = 2\pi/\omega(\mathbf{x}_{br}, t_{br})$. Once the dissipation region is selected as illustrated above and the local crest phase speed specified as $C(\mathbf{x}_{br}, t_{br})$, it allows us to utilize Eq.(4) to determine the value of the eddy viscosity, via which the dissipation is applied during T_{br} over the selected dissipation region. Note that, if there are overlapping due to different breakers, the largest one among these superimposed eddy viscosity values is taken.

3 Results and Conclusion

The proposed breaking model is first validated for unidirectional sea states with a single strong breaking wave (plunging breaker) generated by energy-focusing for which experimental results are available [3]. The case selected exhibits a constant steepness spectrum with $\epsilon_n = 0.0051$, a central frequency $f_c = 0.6$Hz, and a frequency band $\delta f = 0.54$Hz. A comparison with experiments of wave probe signals at different locations is performed in Fig.(2a), in which an excellent agreement is observed.



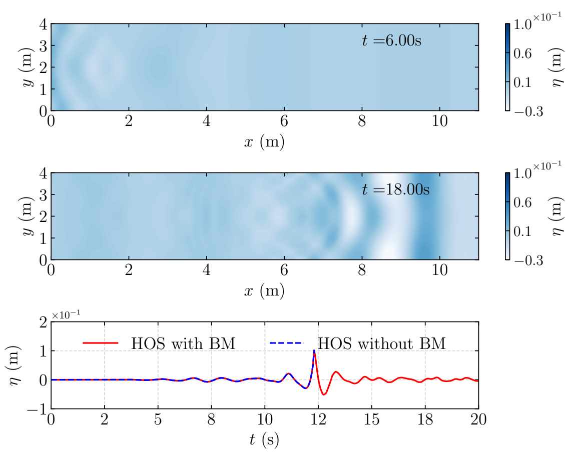
(a) Surface elevation

(b) Frequency-amplitude spectra & energy variation

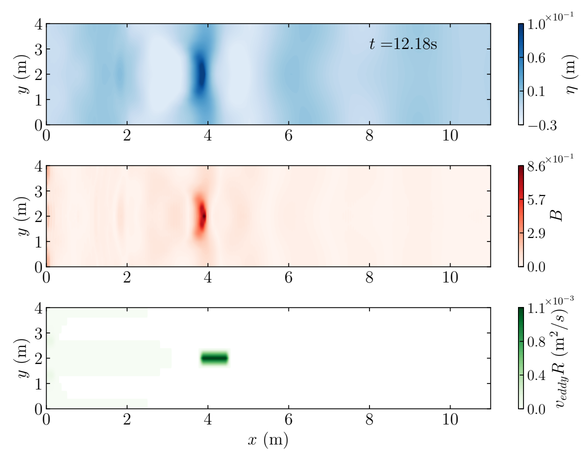
Figure 2: (a) Comparison between HOS with new breaking model (– – –) and experiments (——) at different wave gauges. (b) Comparisons of the frequency-amplitude spectra at various wave gauges and spatial evolution of normalized energy between the proposed model, Tian's model, and experiments (vertical dotted line indicates breaking location).

Comparisons of the frequency-amplitude spectra at three wave gauges and normalized energy variation in space (refer [3] for the detailed description) between the present new model, previous breaking model using Tian's parametrization for eddy viscosity [4], and experiments are displayed in Fig.(2b). It is found that the present model shows comparable dissipation to Tian's model, whereas the present one is straightforward to implement in short-crested seas.

Correspondingly, as an illustration of the present model in short-crested seas, a single breaker, generated through frequency and directional focusing wave with a central frequency $f_c = 1.08$ Hz, global focusing steepness $S = 0.26$ and bandwidth $\delta f = 0.789$ Hz as detailed in [1], is configured in HOS-NWT. A comparison of surface elevation at the breaking location $\mathbf{x}_{br}$ between the HOS with and without present Breaking Model (BM), is shown in the bottom graph of Fig.(3a). This indicates that the one without BM fails to simulate after the breaking event, while the other one equipped with BM remains stable. Besides, these two graphs at the upper part in Fig.(3a) display the surface elevation contours before and after the breaking. Figure(3b) shows the contours of the surface elevation, breaking threshold, and the eddy viscosity of the instant at which the breaking event is initially detected. During the workshop, more detailed comparisons will be presented between the HOS-NWT simulation, equipped with the newly developed BM, and experiments in short-crested seas.



(a) $\eta(\mathbf{x}, 6)$, $\eta(\mathbf{x}, 18)$, $\eta(\mathbf{x}_{br}, t)$



(b) $\eta(\mathbf{x}, t_{br})$, $B(\mathbf{x}, t_{br})$, $\nu_{eddy}R(\mathbf{x}, t_{br})$

Figure 3: (a) Top and middle: surface elevation contours before and after the breaking; bottom: surface elevation at the breaking location. (b) Surface elevation, breaking threshold, and eddy viscosity contour during the breaking.

REFERENCES

[1] Wang, Y., and Ducrozet, G. Computing local crest phase speed in multi-directional sea states through the spatial hilbert transform, 2024. Manuscript submitted Ocean Engineering.

[2] Ducrozet, G., Bonnefoy, F., Le Touzé, D., and Ferrant, P. 2012. *A modified high-order spectral method for wavemaker modeling in a numerical wave tank. European Journal of Mechanics-B/Fluids 34*, 19–34.

[3] Seiffert, B. R., and Ducrozet, G. 2018. *Simulation of breaking waves using the high-order spectral method with laboratory experiments: wave-breaking energy dissipation.* Ocean Dynamics 68(1), 65–89.

[4] Tian, Z., Perlin, M., and Choi, W. 2012. *An eddy viscosity model for two-dimensional breaking waves and its validation with laboratory experiments.* Physics of Fluids 24(3), 036601.