Near edge behavior of semi-infinite viscoelastic ice plate leading to failure under incident gravity wave

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HIGHLIGHTS

The oscillations of a semi-infinite viscolaelastic ice plate under incident gravity wave is found by vertical modes expansion in the presence of an intermediate detached section between the free surface and the plate. The length of this section calculated after determination of the point of maximum stresses for semi-infinite plate. The time harmonic solution is constructed and investigated for a wide range of parameters of the problem.

1 INTRODUCTION AND GOVERNING EQUATIONS

The oscillations of a semi-infinite viscoelastic ice plate are considered. The problem is solved in 2D formulation. The oscillations are induced by an incident gravity wave propagating from left to right direction with constant amplitude and frequency. The plate occupies the positive half-axis, while the negative half-axis surface is free one. The water depth is finite and equal to H $(-H < \overline{z} < 0)$. The boundary between the free surface and the ice plate is located at $\overline{x} = 0$, see Fig.1. A key aspect of the problem is the consideration of damping in the oscillations of the viscoelastic plate in the presence of additional finite floating plate or broken ice. The damping is modeled considering the Kelvin-Voigt model of viscoelastic material introducing the so-called retardation time. See [1] for the discussion of application of this model to describe ice behavior.

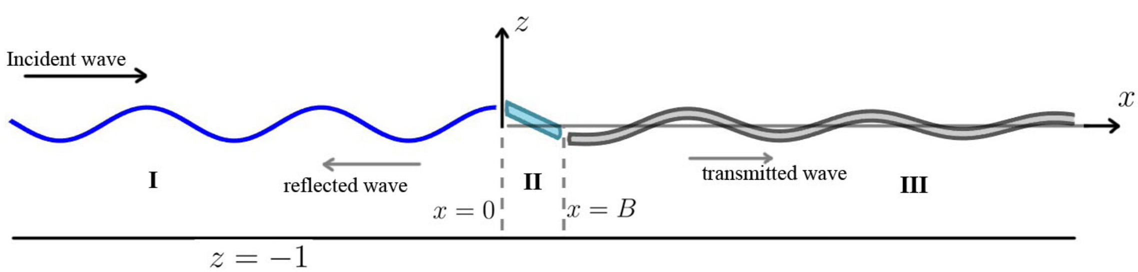

Figure 1: Scheme of the problem in dimensionless variables. Region I: liquid flow with free surface, Region II: liquid flow covered by broken ice or finite plate, Region III: liquid flow covered by semi-infinite viscoelastic plate.

The solution is sought in the form of harmonic oscillations with the frequency of the incident wave. In the first step of solving the problem, the location of the maximum strains is determined for the semi-infinite plate ($B = 0$), which is typically found near the edge. Tkacheva [2] found for a semi-infinite elastic plate $B \approx 0.47L_{ch}$, where L_{ch} is length of a flexural-gravity wave penetrating into the plate with the same frequency. After the maximum found we believe that ice plate broken at this point if this maximum is higher than stresses limit and a "detached" section is introduced. The "detached" section is modeled as (a) broken ice, (b) a finite viscoelastic plate with the same viscoelastic properties, and (c) a rigid plate. In the latter two cases, the free edge conditions is considered. Then, the location of maximum strains in the remaining semi-infinite viscoelastic plate is determined, and this algorithm can be repeated multiple times. The goal of this study is to determine the influence of the problem parameters, particularly the ice viscosity, on the penetration of waves into the plate with the formation of cracks and to predict the location of these cracks and "detachment" of finite pieces from the plate under the described conditions.

The problem is studied within the linear theory of hydroelasticity. At the initial stage on region I with the free surface in Fig. 1 the incident wave is given in the form

$$\overline{\eta}^{in}(\overline{x},\overline{t}) = A\, e^{i(\overline{k}_0\overline{x}-\omega\overline{t})}, \quad \overline{\Phi}^{in} = -i\, A\,\omega\, \overline{f}_0(\overline{z})\, e^{i(\overline{k}_0\overline{x}-\omega\overline{t})} = \overline{\varphi}^{in} e^{-i\omega\overline{t}},$$

where the overbar denotes dimensional functions, variables and wavenumber. Here, $\overline{\eta}^{in}$ is the elevation of the free surface, $\overline{t}$ is time, A is the amplitude, ω is the frequency and $\overline{k}_0$ is the wavenumber of the incident wave. The wavenumber and the frequency are satisfying the dispersion relation for the free surface, $\overline{\Phi}^{in}$ is the corresponding flow velocity potential.

1.1 Formulation without "detached" part

After factoring out $e^{-i\omega\overline{t}}$ and switching to dimensionless variables, the governing equations in region I will be

$$\nabla_2^2(\varphi^{in}+\varphi^{re}) = 0 \quad (x<0,\ -1<z<0), \tag{1}$$

$$(\varphi^{in}+\varphi^{re})_z - \gamma(\varphi^{in}+\varphi^{re}) = 0 \quad (x<0,\ z=0), \tag{2}$$

$$\varphi_z^{in} = \varphi_z^{re} = 0 \quad (x<0,\ z=-1), \quad \varphi^{re} \to -ia_0 f_0(z) e^{i(-k_0x-t)} + o(1) \text{ as } x\to-\infty, \tag{3}$$

where dimensionless functions and variables are denoted without an overbar. Here, φ^{re} is the potential of the wave reflected from the plate, a_0 is its amplitude, $f_0(z) = \cosh(k_0(z+1))/(k_0\sinh(k_0))$, $\nabla_2^2 = \partial^2/\partial x^2 + \partial^2/\partial z^2$, $\gamma = \omega^2 H/g$, and g is the gravitational acceleration. The length scale is H, the time scale is $1/\omega$, the free surface elevation scale is A, and the velocity potential scale is $AH\omega$.

In the initial stage of the solution, the viscoelastic ice plate occupies the right half-axis, corresponding to regions II and III in Fig. 1. In dimensionless variables, the governing equations for the right half-axis are

$$-m\gamma W + \beta(1-i\varepsilon)W_{xxxx} + W = \gamma\Phi \quad (x>0,\ z=0), \quad W_{xx} = W_{xxx} = 0 \quad (x=0) \tag{4}$$

$$\nabla_2^2\Phi = 0 \ \ (x>0,\ -1<z<0), \quad \Phi_z = W \ \ (x>0,\ z=0), \quad \Phi_z = 0 \ \ (x>0,\ z=-1), \tag{5}$$

where W is the ice deflections, Φ is the velocity potential of the fluid flow beneath the plate, m is the ratio of the mass of the ice plate per unit area to the mass of the fluid, β is the dimensionless rigidity of the plate, and ε is the dimensionless retardation time within the Kelvin-Voigt model.

The system of equations (4) – (5) is supplemented by boundary conditions as $x\to\infty$, which depend on the value of ε

$$\begin{cases} \Phi \to 0 \text{ as } x\to\infty \quad (\epsilon \neq 0), \\ \Phi \to s_0 g_0(z) e^{i(\xi_0 x - t)} + o(1) \text{ as } x\to\infty \quad (\epsilon = 0), \end{cases} \tag{6}$$

where s_0 is the amplitude of the transmitted wave, ξ_0 is the wavenumber for the given plate corresponding to the frequency ω, and $g_0(z) = \cosh(\xi_0(z+1))/(\xi_0 \sinh(\xi_0))$.

The final system of equations consists of equations (1) – (6), along with the matching conditions at the vertical boundary between the free surface and the plate

$$\varphi^{in} + \varphi^{re} = \Phi \quad (x = 0,\ -1 < z < 0), \quad (\varphi^{in} + \varphi^{re})_x = \Phi_x \quad (x = 0,\ -1 < z < 0). \tag{7}$$

1.2 Formulation with "detached" part

After determining the location of the absolute maximum strains in the plate, the introduced intermediate section with the "detached" part occupies region II ($0 < x < B$), where B is the length of the "detached" part. The semi-infinite plate now occupies region III. Equations (4) – (5) are rewritten with the substitutions of ($x > 0$) and ($x = 0$) by ($x > B$) and ($x = B$), respectively. Condition (6) remains unchanged. For the "detached" part, its own system of equations is introduced. In the case of broken ice, the system of equations in region II is

$$-m\gamma W^{II} + W^{II} = \gamma \Phi^{II} \ \ (0 < x < B,\ z = 0), \quad \nabla_2^2 \Phi^{II} = 0 \ \ (0 < x < B,\ -1 < z < 0), \tag{8}$$

$$\Phi_z^{II} = W^{II} \quad (0 < x < B,\ z = 0), \quad \Phi_z^{II} = 0 \quad (0 < x < B,\ z = -1), \tag{9}$$

where W^{II} and Φ^{II} are the corresponding deflections and velocity potential of the flow. In the case of modeling an elastic or rigid plate in region II, equation (8) is replaced by an equation and boundary conditions analogous to (4), but for ($0 < x < B$). The matching conditions at the vertical boundaries between the free surface, intermediate "detached" part and the semi-infinite plate are

$$\varphi^{in} + \varphi^{re} = \Phi^{II} \quad (x = 0,\ -1 < z < 0), \quad (\varphi^{in} + \varphi^{re})_x = \Phi_x^{II} \quad (x = 0,\ -1 < z < 0). \tag{10}$$

$$\Phi^{II} = \Phi \quad (x = B,\ -1 < z < 0), \quad \Phi_x^{II} = \Phi_x \quad (x = B,\ -1 < z < 0). \tag{11}$$

The sought functions are ϕ^{re}, Φ^{II}, Φ, W^{II}, and W.

2 METHOD OF THE SOLUTION AND DISCUSSION

The solution to the problem for both formulations are constructed using the so-called vertical mode method [3], or the eigenfunction method [4]. For the first part of the problem, the potentials ϕ^{re} and Φ are represented as sums

$$\varphi^{re} = -i \sum_{n=0}^{\infty} a_n f_n(z) e^{i(-k_n x - t)}, \quad f_n(z) = \frac{\cosh(k_n(z+1))}{k_n \sinh(k_n)}, \tag{12}$$

$$\Phi = -i \sum_{n=-2}^{\infty} s_n g_n(z) e^{i(\xi_n x - t)}, \quad g_n(z) = \frac{\cosh(\xi_n(z+1))}{\xi_n \sinh(\xi_n)}. \tag{13}$$

Here, f_n and g_n are the vertical modes for the free surface and the viscoelastic plate, respectively. The wavenumbers k_n and ξ_n are complex-valued solutions of the corresponding dispersion relations. For the free surface, there is one real root, k_0, and a countable number of purely imaginary roots, k_n, $n \geq 1$. For the elastic plate, there are two complex roots, ξ_{-1} and ξ_{-2}, one real root, ξ_0, and a countable number of purely imaginary roots, ξ_n, $n \geq 1$. For the viscoelastic plate, all these roots become complex with nonzero real and imaginary parts. In this sense, the roots for the viscoelastic plate are similar to those for a porous plate, as computed in [4]. Potentials in the form of (12) – (13) are solutions to the governing equations from the previous paragraph. The principal coordinates a_n and s_n are to be determined.

The vertical modes for the free surface are orthogonal. By truncating the sums in (12) – (13) to the finite upper limit N, substituting these potentials into the conditions (7), multiplying by $f_m(z)$, and integrating the result over z, and taking into account the boundary conditions (4) at $x = 0$, we arrive at a system of $2N + 4$ algebraic equations for the principal coordinates a_n and s_n in the case without "detached" part. Second case is solved by the same method. For the intermediate section in region II, a sum of two similar potentials is introduced to match the solutions in regions I and III at the left, $x = 0$, and right, $x = B$, vertical boundaries. This results in $4N + 6$ algebraic equations in the case of broken ice and $4N + 10$ algebraic equations in the case of a finite viscoelastic plate. For modeling a rigid plate or a plate with variable thickness in region II, the method of normal modes is applied, see [5]. Results of some calculations for one ice plate are shown in Fig.2. These results show that considered problem can be solved and analyzed by described method. Clear difference between cases with and without viscosity is observed. More different examples of calculations will be presented in the workshop.

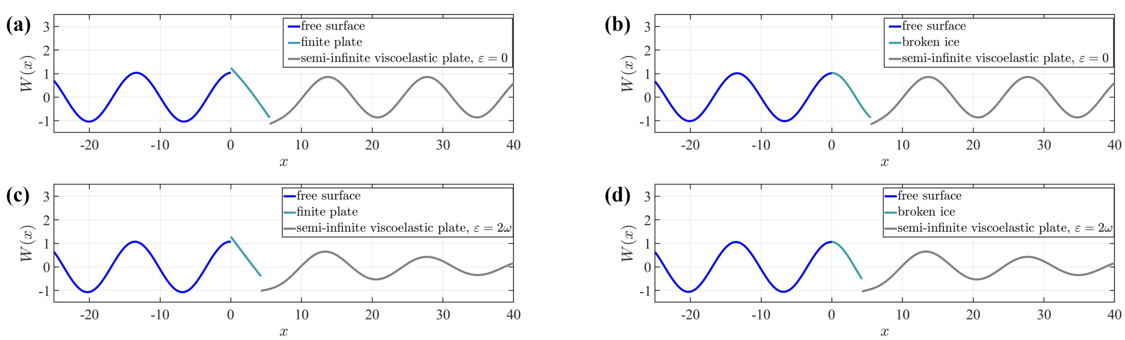

Figure 2: Free surface evaluations and ice deflections. Here (a) and (c) are for a finite plate in region II, (b) and (d) are for broken ice, (a) and (b) without damping, (c) and (d) with damping.

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