Advancing the Understanding of Added Resistance in Waves Through Fourier-Kochin Theory

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1 INTRODUCTION

The accurate prediction of added resistance in waves has remained challenging. As such, it is necessary to critically examine the theoretical backgrounds of various methods. Maruo [1] pioneered the successful numerical prediction of added resistance in waves. Naito et al. [2], Kashiwagi [3], Bingham [4] made detailed elaborations and critical examinations of the method. These elaborations are mostly theoretical and mathematics-intensive, and there has been a shortage of visualization of the involved phenomenon. On the other hand, Noblesse et al. [5], Chen [6], and Liang and Chen [7] developed the Fourier-Kochin theory of free-surface flows. In this study, we shall introduce the Fourier-Kochin theory to explain the wave patterns generated by a traveling and pulsating ship, with the goal of advancing our understanding of the far-field method for predicting added resistance in waves.

2 FAR-FIELD WAVES ACCORDING TO FOURIER-KOCHIN THEORY

Consider a ship navigating along a rectilinear trajectory at a constant forward speed U in regular waves with natural frequency ω_0. The fluid is assumed inviscid and incompressible, and the flow is irrotational so that a velocity potential exists. Due to wave excitation, the ship will be undergoing oscillatory motions, thus, leading to the increase of experienced resistance. Following Maruo [1], this added resistance may be expressed as:

$$R_{AW} = \frac{\rho}{8\pi}\left\{\int_{-\frac{\pi}{2}}^{-\alpha_0} + \int_{\alpha_0}^{\frac{\pi}{2}} - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\right\}|H(k_1,\theta)|^2 \frac{k_1[k_1\cos\theta - k\cos\chi]}{\sqrt{1-4\Omega\cos\theta}}\,d\theta + $$
$$+\frac{\rho}{8\pi}\int_{\alpha_0}^{2\pi-\alpha_0}|H(k_2,\theta)|^2 \frac{k_2[k_2\cos\theta - k\cos\chi]}{\sqrt{1-4\Omega\cos\theta}}\,d\theta \qquad (1)$$

where θ is the propagating direction of elementary waves generated by the vessel, and α_0 is the critical angle, with $\alpha_0=\cos^{-1}(1/(4\tau))$ for $\tau > 1/4$ and $\alpha_0=0$ for $\tau \le 1/4$. The complex *Kochin* function $H\left(k_j,\theta\right)$ describes the elementary waves radiated from the ships and $k_j(\theta),\ j=1,2$ are the unsteady wave numbers:

$$k_j(\theta) = \frac{K_0}{2}\frac{1-2\tau\cos\theta \pm \sqrt{1-4\tau\cos\theta}}{\cos^2\theta} \quad \begin{pmatrix} + : \text{j=1} \\ - : \text{j=2} \end{pmatrix} \qquad (2)$$

where $\tau = \omega_e V/g$ is the *Hanaoka* parameter, and $K_0=g/U^2$ is the steady wave number.

In elaborating this method, Naito et al. [1988] decompose the wave systems induced by ship motions into four components by considering the group velocity of generated waves and the dispersioin relation. This explanation can be alternatively elaborated through the Fourier-Kochin theory [5-7] and enhanced by visulizations of the associated wave patterns. For the sake of simplicity, we non-dimensionalize all physical quantities, including the non-dimensional ordinates x≡ (x, y, z); time t, frequency of encounter and ship speed as

$$x = \frac{X}{L};\ t = T\sqrt{\frac{g}{L}};\ f = \omega\sqrt{\frac{L}{g}};\ F = \frac{U}{\sqrt{gL}}. \qquad (3)$$

where L denotes the characteristic length of the disturbance, for example: ship's length, and g is the gravitational acceleration. On the free surface, the linearized free-surface boundary condition to be satisfied by the spatial potential $\phi(\mathbf{x})$ is written as (Newman, 1978):

$$-f^2\phi + 2i\tau\frac{\partial\phi}{\partial x} + F^2\frac{\partial^2\phi}{\partial x^2} + \frac{\partial\phi}{\partial z} = 0,\ on\ z = 0 \tag{4}$$

which gives rise to the dispersion relation expressed as :

$$D = (F\alpha - f)^2 - \sqrt{\alpha^2 + \beta^2} = (Fk\cos\theta - f)^2 - k = F^2\cos^2\theta(k - k^-)(k - k^+) \tag{5}$$

where $(\alpha, \beta) = k(\cos\theta, \sin\theta)$, and

$$k^-(\theta) = \frac{\left(1-\sqrt{1+4\tau\cos\theta}\right)^2}{4F^2\cos^2\theta} \text{ and } k^+(\theta) = \frac{\left(1+\sqrt{1+4\tau\cos\theta}\right)^2}{4F^2\cos^2\theta} \tag{6}$$

The dispersion relation D=0 defines several wavenumber curves. As shown in Figure 1, for τ <1/4 there exists three dispersion curves intersecting with β=0 axis at four values of α. As such, there are three wave systems associated with this scenario, namely, the inner-V waves, outer-V waves, and the ring waves. Specifically, the right and left open wavenumber curves are associated with inner and outer V-shaped wave patterns, respectively, which are composed of transverse waves and divergent waves confined within sector regions. These are the two wave components of k_1 wave systems as defined by Naito et al. [2]. The close wavenumber curve corresponds to ring waves with larger wavelength downstream and smaller wavelength upstream. This corresponds the two wave components of k_2 wave systems as defined by Naito et al. More specifically, the deformed ring waves have been split into two sections: the light blue colored travel upstream and the red colored travel downstream. The definition of the dividing line will be described afterwards.

For τ >1/4, there exists two dispersion curves as the left open wavenumber curve and the close wavenumber curve are merged to form an open wavenumber curve, as shown in Figure 2. This wavenumber curve is divided by a point, at which the wavenumber vector is tangent to the wavenumber curve, into two parts, namely, an open wavenumber curve defined by $k^+(\theta)$ within $\frac{\pi}{2} < \theta \le \pi - \theta_\alpha$, and a closed one defined by $k^+(\theta)$ within $0 \le \theta \le \pi - \theta_\alpha$, where θ_α is expressed as:

$$\theta_\alpha = \tan^{-1}\sqrt{16\tau^2 - 1} \tag{7}$$

As such, there are several wave systems associated with this scenario, namely, the inner-V waves, fan waves (in orange color, evolution of outer V-shaped wave), partial fan waves (in blue), and the partial ring waves (both in red and light blue), as shown in Figure 2. It is noted that the ring wave pattern is no longer closed but turns to "ring-fan waves", which are baptized by Noblesse et al. [5]. In addition, transverse waves in the outer V-shaped wave pattern disappear, and only divergent waves remain in the outer V-shaped waves. The wave components in red travel downstream, while those both in blue and light blue travels upstream, corresponding the two wave components of k_2 wave systems as defined by Naito et al.

The above analysis of unsteady wave systems is useful for understanding Maruo's far field method for predicting added resistance in waves. It is interesting to note that this analysis is purely based on the Hanaoka number τ, which represents the degree of unsteadiness. When scrutinizing the wave patterns, it is observed that the V-shaped waves and ring-fan waves are confined within sector regions between two cusp lines, as defined earlier by Chen [6] using the cusp angle, the angle between cusp line and the track of the source point:

$$\gamma_c = \tan^{-1}\sqrt{1/|6F^2k_c - 1|} \tag{8}$$

where k_c denotes the wavenumber at the inflection point.

Another critical point (α_α, β_α) dividing the left wavenumber curve into two parts corresponds to an asymptote angle expressed as:

$$\gamma_a = \frac{\pi}{2} - \theta_\alpha = \frac{\pi}{2} - \tan^{-1}\sqrt{16\tau^2 - 1} \tag{9}$$

This asymptote is the boundary of fan waves in orange. Further, the phase function is written as

$$\psi = \alpha x + \beta y = kR\cos(\theta - \gamma) \tag{10}$$

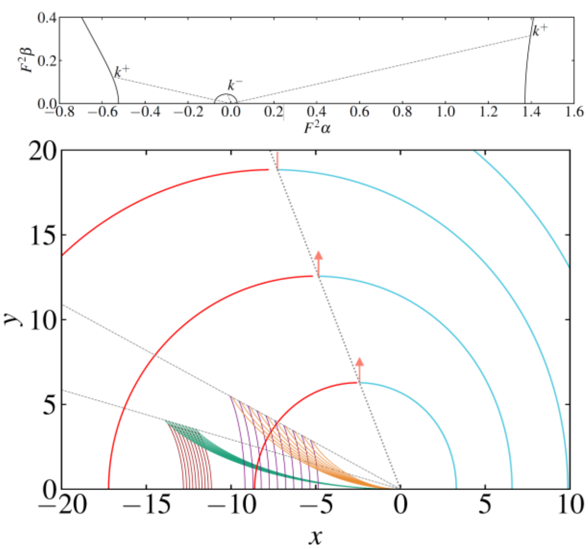

Figure 1: Wavenumber curves for τ=0.2 (top) and the corresponding wave patterns (down)

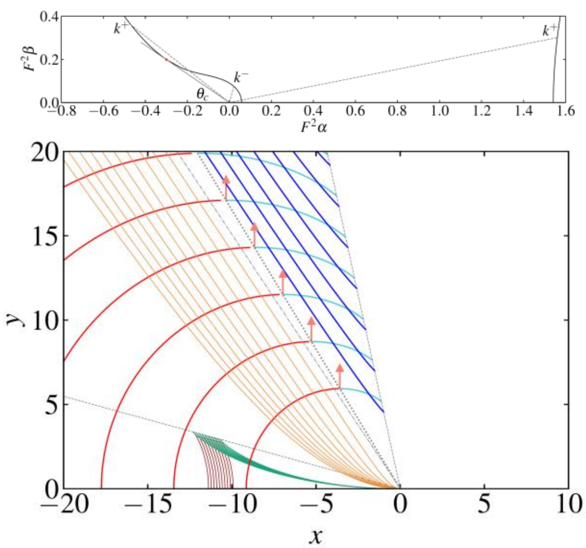

Figure 2: Wavenumber curves for τ=0.3 (top) and the corresponding wave patterns (down)

The stationary phase relation $\psi' = \alpha' x + \beta' y = 0$ in combination with $D'=0 = D_\alpha\alpha' + D_\beta\beta'$ gives rise to

$$xD_\beta - yD_\alpha = 0 = R\|\nabla D\|\sin(\vartheta - \gamma) \tag{11}$$

where $(D_\alpha, D_\beta) = \|\nabla D\|(\cos\vartheta, \sin\vartheta)$. Then we obtain the coordinates on the wave crestlines (iso-phase lines)

$$(x, y) = \psi_n \frac{(D_\alpha,\ D_\beta)}{\alpha D_\alpha + \beta D_\beta} \tag{12}$$

where ψ_n takes account of the radiation condition are given by (Chen, 1999):

$$\psi_n = 2n\pi\,\mathrm{sgn}(D_f)\,\mathrm{sgn}(\alpha D_\alpha + \beta D_\beta) - \mathrm{sgn}(\psi'')\,\pi/4 \tag{13}$$

The phase velocity determined by the dispersion relation is expressed as :

$$\mathbf{v}_p = (v^x{}_p, v^y{}_p) = -(\alpha, \beta)f/k^2 \tag{14}$$

At a point at which the phase velocity is in line with the y-direction, namely, $\alpha = 0$, a line can be defined and the corresponding angle can be expressed as

$$\Theta = \left(D_\beta/D_\alpha\right)_{\alpha=0} = \pi - \frac{1}{2\tau} \tag{15}$$

Following this formula, the angle $\Theta = 111.80^\circ$ and $\Theta = 120.96^\circ$ are derived for $\tau = 0.2$ and $\tau = 0.3$, respectively, as shown in Figures 1 and 2.

The length of the elementary wave in this direction is expressed as:

$$\lambda_\Theta = (2\pi/k)_{\alpha=0} = 2\pi/f^2 \tag{16}$$

3 CONNECTION BETWEEN FAR-FIELD WAVES AND ADDED RESISTANCE

We examine here the real-life cases as encountered in ship opeartions. Noting that full ships operate at about 15 knots and fine ship such as modern contianerships operate at higher speeds of up to 25 knots nowadays, the resultant τ is subsequently in the range of 0.7-3.5. As such, we present a typical case of τ=1.0. For this case, there exist inner V-shaped waves, fan waves, partial fan waves, and partial ring waves, as shown in Figure 3. There is no wave in light blue color, indicating that all the ring wave components travel downstream. All the wave components are squeezed into narrow space. It appears that the wavelength of the blue colors compoents is comparable to those transverse components. More results will be presented at the workshop.

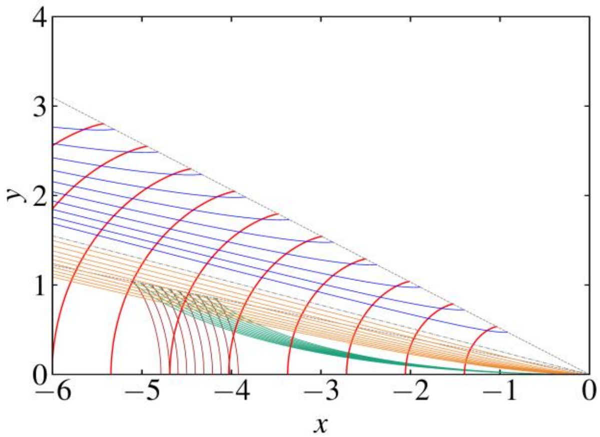

Figure 3: The wave patterns for τ=1.0

4 CONCLUSIONS

In this study, we introduced the Fourier-Kochin theory to explain the wave patterns generated by a traveling and oscillating ship, with the goal of advancing our understanding of the far-field method for predicting added resistance in waves. The generated wave patterns were visualized, and various components were clearly defined to align with the established definitions in classical theory. As part of this process, we derived a concise formula to determine the angle at which the direction of propagation of the elementary wave transitions from upstream to downstream. Moving forward, we aim to further develop this approach to deepen our understanding of the complex problem of added resistance in waves. In doing so, we anticipate that the limitations of relevant numerical methods can be addressed, leading to more accurate predictions in the near future.

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