# Prediction of Parametric Roll Motions and Their Dynamic Properties Using Machine Learning Approach

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## 1 INTRODUCTION

Despite significant advances in ship design and operational techniques, parametric roll accidents persist, demanding more effective prediction methods. Real-time deterministic prediction of parametric roll motion remains challenging due to its highly nonlinear and non-ergodic nature. In this study, a data-driven machine learning approach is introduced to predict parametric roll motions in both regular and irregular waves. Beyond merely comparing time-series predictions, the deterministic, probabilistic, and statistical properties of the predicted roll motion under parametric resonance are investigated, demonstrating the ability of the proposed approach to train and analyze the dynamic properties of these complex nonlinear phenomena.

## 2 THEORETICAL BACKGROUND

### 2.1 Physics of Parametric Roll

The ship's parametric roll arises from time-varying transverse stability, particularly fluctuations in the metacentric height (*GM*). Classically, a simplified one-degree-of-freedom (1-DoF) roll motion equation has been widely used to describe the dynamic behavior of parametric roll [1].

$$\left(I_{44}+A_{44}\right)\ddot{\xi}_4+B_{44}\dot{\xi}_4+Mg\cdot GZ\left(\xi_4,t\right)=0 \tag{1}$$

Here, $I_{44}$ and $A_{44}$ represent the moments of inertia for the ship's mass (*M*) and added mass, respectively, while $B_{44}$ denotes the roll damping coefficient. The derived differential equation can be further simplified using the approximate restoring arm (*GZ*) and *GM* formulas for a given encounter frequency ($\omega_e$).

$$\ddot{\xi}_4+2\delta\dot{\xi}_4+\left[\omega_m^2+\omega_a^2\cos\left(\omega_e t\right)\right]\xi_4=0 \tag{2}$$

$$\omega_m=\sqrt{\frac{Mg\cdot\left(GM_m\right)}{I_{44}+A_{44}}},\omega_a=\sqrt{\frac{Mg\cdot GM_a}{I_{44}+A_{44}}},\delta=\frac{1}{2}\frac{B_{44}}{I_{44}+A_{44}} \tag{3}$$

The derived second-order differential equation can be transformed into a damped Mathieu-type equation, facilitating the evaluation of the ship's parametric resonance instability using the Ince-Strutt stability diagram, which is determined by the characteristics of parametric excitation.

$$\frac{d^2x}{d\tau^2}+c\frac{dx}{d\tau}+\left[p+q\cos\tau\right]x=0 \quad \text{where} \quad p=\left(\frac{\omega_0}{\omega_e}\right)^2,q=\left(\frac{\omega_1}{\omega_e}\right)^2,c=\frac{2\delta}{\omega_e} \tag{4}$$

In irregular waves, parametric roll motions exhibit complex dynamic characteristics. While irregular waves can be considered as ergodic process under the quasi-stationary hypothesis, the parametric roll motions induced by these waves are non-ergodic responses due to their highly nonlinear dynamics [2]. Therefore, identifying parametric roll using probabilistic and statistical approaches is challenging in irregular waves, and an efficient deterministic prediction strategy is highly demanded.

### 2.2 Numerical Approach: Impulse Response Function

The impulse response function (IRF) is a well-established time-domain analysis approach for investigating dynamic systems [3]. This approach is particularly effective for directly calculating wave-induced six-degree-of-freedom (6-DoF) ship responses ($\xi_k$), incorporating memory effects through the convolution integral of the retardation function ($R_{jk}$).

$$\left(M_{jk}+M_{jk}^{\infty}\right)\ddot{\xi}_k\left(t\right)+\int_{-\infty}^{t}R_{jk}\left(t-\tau\right)\dot{\xi}_k\left(\tau\right)d\tau+C_{jk}^{R}\xi_k\left(t\right)=F_{diff,j}\left(t\right)+F_{FK,j}\left(t\right)+F_{rest,j}\left(t\right)+F_{rolldamp,j}\left(t\right) \tag{5}$$

$$R_{jk}\left(t\right)=\frac{2}{\pi}\int_{0}^{\infty}B_{jk}\left(\omega\right)\cos\left(\omega t\right)d\omega \quad \text{and} \quad M_{jk}^{\infty}-\frac{C_{jk}^{R}}{\omega^2}=A_{jk}\left(\omega\right)+\frac{1}{\omega}\int_{0}^{\infty}R_{jk}\left(\tau\right)\sin\omega\tau d\tau \tag{6}$$

Here, $M_{jk}$, $M_{jk}^{\infty}$, and $C_{jk}^{R}$ denote the inertia, infinite-frequency added mass, and radiation-restoring matrices, respectively. $A_{jk}$ and $B_{jk}$ are the frequency-domain added mass and damping coefficients. The Froude-Krylov ($F_{FK,j}$) and restoring forces ($F_{rest,j}$) are calculated considering the temporal variations in the ship's wetted surfaces, whereas the diffraction force ($F_{diff,j}$) is determined using linear strip theory. Based on this weakly nonlinear approach, the nonlinear properties of wave-induced ship responses are accounted for. The linear critical damping formula ($F_{rolldamp,j}$) is introduced to include the effects of viscous roll damping.

## 2.3 Data-Driven Approach: Machine Learning

In this study, a data-driven machine learning (ML) approach is introduced for the deterministic prediction of parametric roll motions. The neural network comprises a long short-term memory (LSTM) with a convolutional neural network (CNN) to predict time-series ship motions ($\xi_k^{(pr)}(t)$) several minutes in advance, based on input spatiotemporal wave field data ($\zeta_{pr}(x,y,t)$). Additionally, LSTM encoder is incorporated to account for the memory effects of past motion records ($\xi_k^{(re)}(t)$) [4].

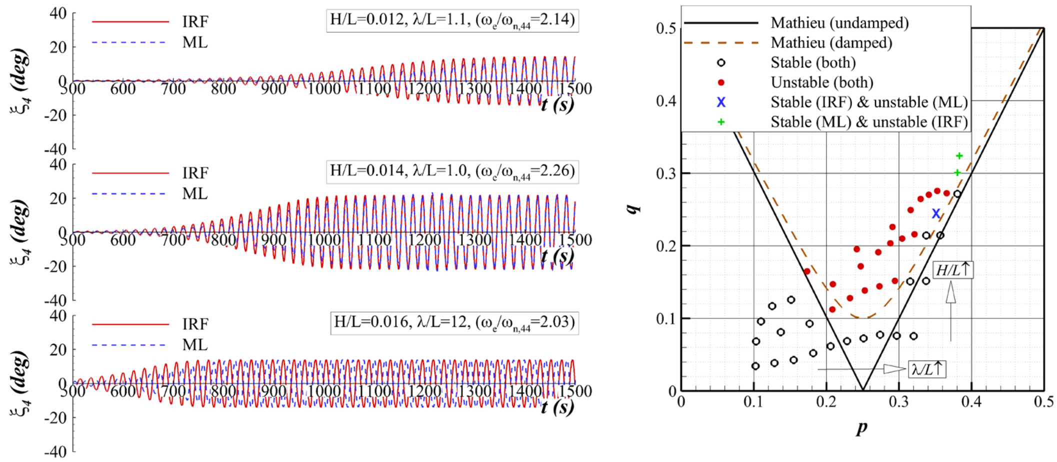
$$\xi_k^{(pr)}\left(t\right)=L^{(d)}\left\{f^{(1,2,3)}\left\{\zeta_{pr}\left(x,y,t\right)\right\};L^{(e)}\left\{\xi_k^{(re)}\left(\tau\right);0\right\}\right\} \text{ where } \tau\in\left[t_0-T_{re},t_0\right) \text{ and } t\in\left[t_0,t_0+T_{pr}\right] \tag{7}$$

Here, $f^{(i)}$ represents $i^{th}$ convolutional layer, $L^{(d)}$ and $L^{(e)}$ indicate LSTM decoder and LSTM encoder layers, respectively. The current time instant is defined as $t_0$, while $T_{pr}$ and $T_{re}$ represent the prediction and memory time windows, respectively. The CNN layers sequentially compress the input wave field feature maps, generating a time series vector that serves as the input for the LSTM decoder. Simultaneously, the LSTM encoder trains the memory effects inherent in past motion records to create a memory vector, which provides the initial conditions for the LSTM decoder. The LSTM decoder combines the compressed wave field vector and the memory vector to produce a time-series prediction of ship motions. Through this integrated neural network structure, the wave-induced ship motion responses can be deterministically predicted, considering both wave excitation and memory effects. Training of the neural network is conducted using time-series datasets of ship motions in various irregular waves, generated via the IRF method.

# 3 RESULTS AND DISCUSSIONS

## 3.1 Parametric Roll in Regular Waves

In this study, the parametric roll motions of container ship under regular head and short-crested irregular waves were analyzed. The speed of the ship was set to 10.0knots, with a still-water metacentric height ($GM_{still}$) of 1.27m. Under these conditions, the natural roll frequency was determined to be 0.2909rad/s.
Figure 1 shows the time series predictions and stability diagram analysis for regular head wave conditions, particularly those with an encounter frequency approximately twice the natural roll frequency. The Ince-Strutt diagram, derived from the linear Mathieu's equations, was established using a perturbation-based approach [5]. Under the parametric resonance, the roll motion rapidly amplified to 10-30 degrees within a few periods and eventually stabilized due to the effects of the nonlinear restoring moment and damping. The ML approach demonstrated its effectiveness not only in predicting the occurrence of parametric resonance but also in accurately estimating the steady-state amplitudes of parametric roll motions.




(a) Time series of roll motions (b) Stability diagram

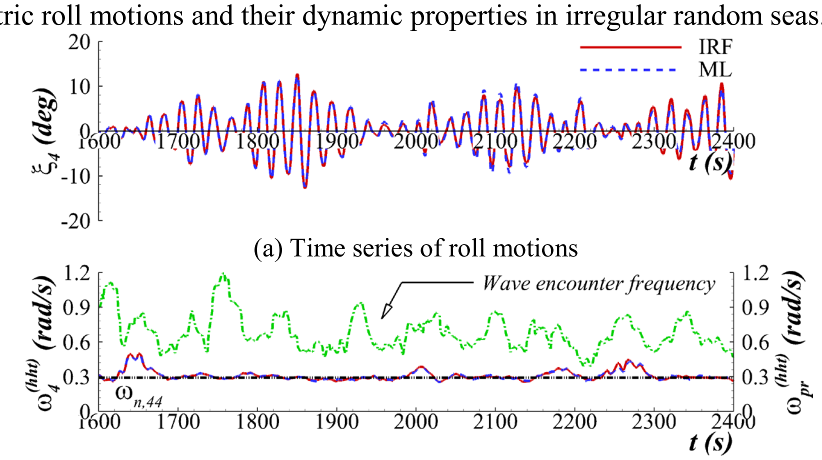
Figure 1 Deterministic predictions and staiblity diagram comparison results of parametric roll in regular waves

## 3.2 Parametric Roll in Irregular Waves

The parametric roll motions in irregular waves were predicted using the proposed ML approach, and their dynamic properties were analyzed. Figure 2 presents the deterministic predictions and their Hilbert-Huang Transform (HHT) analysis results for the ship's roll responses during parametric resonance. Based on the HHT analysis, the temporal evolution of instantaneous frequency ($\omega_k^{(hht)}$) can be examined [6].

$$\omega_k^{(hht)}(t) = \frac{d}{dt}\left[\tan^{-1}\left\{\frac{h_k(t)}{IMF(\xi_k(t))}\right\}\right] \quad \text{where} \quad h_k(t) = P.V.\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{IMF(\xi_k(t))}{t-\tau}d\tau \tag{8}$$

Here, IMF and P.V. represent Intrinsic Mode Function and Cauchy Principal Value, respectively. The spectral properties of roll motions converged to the natural roll frequency and exhibited distinct tendencies compared to the instantaneous frequency of the encounter wave elevation ($\omega_{pr}^{(hht)}$) at the ship's center of gravity, indicating the occurrence of parametric resonance. The proposed ML approach successfully predicted parametric roll motions and their dynamic properties in irregular random seas.

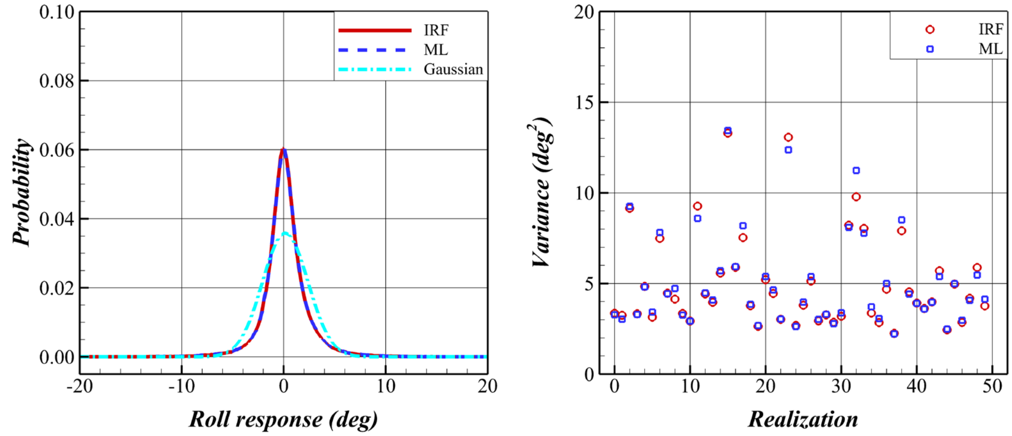



(a) Time series of roll motions

(b) Instantaneous frequency

Figure 2 Determinitic predictions and HHT analysis results of parametric roll in short-crested irregular waves

Finally, the probabilistic and statistical characteristics of the parametric roll motions were compared and presented in Figure 3. Owing to their nonlinearity, the probabilistic distributions of parametric roll responses exhibited non-Gaussian characteristics with higher Kurtosis. Furthermore, the evaluated statistical properties (variances) varied significantly depending on the different realizations (wave seeds), reflecting the non-ergodic nature [2]. The probabilistic and statistical properties evaluated by the IRF and ML approaches showed strong agreement even across different realizations.




(a) Probability of responses (b) Statistical properties

Figure 3 Comparison of probabilistic and statistical properties of parametric roll in short-crested irregular waves

## 4 CONCLUSIONS

The following conclusions were obtained from the present study

- The ML approach effectively predicted the onset and steady-state amplitudes of parametric roll motions in regular head waves.
- The dynamic characteristics of parametric roll motions in irregular waves, including their deterministic, probabilistic, and statistical properties, were demonstrated to exhibit strong agreement between the ML and IRF approaches.

## ACKNOWLEDGEMENT

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