Numerical validation of the restoring stiffness for flexible floating body

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Introduction

There have been lot of discussions in the past concerning the correct expression for the hydrostatic restoring of deformable bodies. The most common formulations were discussed in [2]. In that work, exactly the same formulation of the hydrostatic restoring stiffness matrix, has been obtained by three very different methods. However, some problems were reported regarding its application to the evaluation of the internal loads. In [1] the problem was revisited, and a new formulation was proposed. The proposed formulation seems to be fully consistent with respect to all the aspects of the linear wave body interaction problem, including the evaluation of the internal loads. The fundamental difference between the formulation proposed in [1] and the formulations proposed in [2] is related to the proper accounting for the coordinate systems in which the body deformations are defined and in which the external forces are expressed. The main point is that both the normal vector and the mode shape vector do not change in the body fixed coordinate system while the gravity force vector does, and this fact seems to not be correctly accounted for in the previous work discussed in [2]. However, in spite of the demonstration made in [1] the proposed expression for the restoring stiffness seems to not be fully accepted by the community, and the purpose of the present work is to validate the proposed expressions for restoring stiffness numerically.

Description of the flexible body motions and deformations

The motion of the flexible floating body is described by six rigid body motions and the flexible deformation modes around the rigid body position. Reference is made to Figure 1 where the two relevant coordinate systems (earth fixed and body fixed) are also introduced.

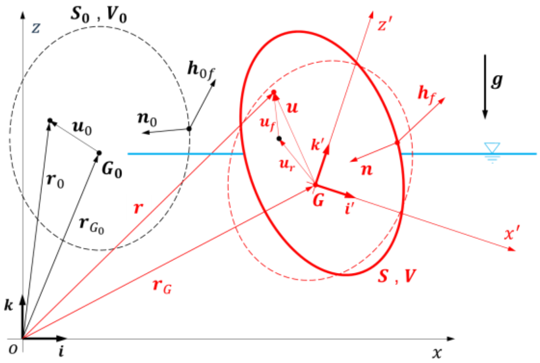


Figure 1: Motion of the flexible body and the different coordinate systems.

The total displacement $\{\boldsymbol{u}\}$ of the point attached to the body is decomposed into its global rigid body part $\{\boldsymbol{u}_r\}$ and its generalized deformation part $\{\boldsymbol{u}_f\}$ so that the instantaneous position in the earth fixed coordinate system becomes:

$$\{\boldsymbol{r}\} = \{\boldsymbol{r}_G\} + \{\boldsymbol{u}\} = \{\boldsymbol{r}_G\} + [\boldsymbol{A}]\{\boldsymbol{u}'\} = \{\boldsymbol{r}_G\} + [\boldsymbol{A}]\left(\{\boldsymbol{u}'_r\} + \{\boldsymbol{u}'_f\}\right) \tag{1}$$

The body deformation vector $\{\boldsymbol{u}'_f\}$ is represented as a sum of the N_f modal contributions described by their space dependent mode shapes $\boldsymbol{h}'_{fi}(\boldsymbol{u}'_r) = h'_{fix'}\boldsymbol{i}' + h'_{fiy'}\boldsymbol{j}' + h'_{fiz'}\boldsymbol{k}'$ and their time dependent modal amplitudes $\chi_{fi}(t)$:

$$\{\boldsymbol{u}'_f(\boldsymbol{u}'_r, t)\} = \sum_{i=1}^{N_f} \chi_{fi}(t)\{\boldsymbol{h}'_{fi}(\boldsymbol{u}'_r)\} = [\boldsymbol{\hbar}'_f]\{\boldsymbol{\chi}_f\} \tag{2}$$

where $[\boldsymbol{\hbar}'_f]$ is the $3 \times N_f$ matrix which columns contain the 3 components of the mode shape vectors and $\{\boldsymbol{\chi}_f\}$ is the vector of the corresponding modal amplitudes.
When the problem is linearized, it is possible to introduce the concept of the generalized modes by rewriting the rigid body motion in its modal form. Within that concept, the instantaneous position of the point attached to the body is given by:

$$\{\boldsymbol{r}\} = \{\boldsymbol{r}_{G_0}\} + \{\boldsymbol{u}_0\} + \left\{\boldsymbol{r}_G^{(1)}\right\} + [\boldsymbol{\theta}^{(1)}]\{\boldsymbol{u}_0\} + [\boldsymbol{\hbar}'_{0f}]\left\{\boldsymbol{\chi}_f^{(1)}\right\} = \{\boldsymbol{r}_0\} + [\boldsymbol{\hbar}'_0]\{\boldsymbol{\xi}\} \tag{4}$$

where $\left\{\boldsymbol{r}_G^{(1)}\right\}$, $\{\boldsymbol{\theta}^{(1)}\}$ and $\left\{\boldsymbol{\chi}_f^{(1)}\right\}$ are the modal amplitudes (rigid and flexible) and they are collected altogether in vector $\{\boldsymbol{\xi}\}$.

External loading in calm water and the restoring stiffness matrix

The calm water case is of concern only, so that the total external loading $\{\boldsymbol{\mathcal{F}}'\}$ is composed of the gravity loading $\{\boldsymbol{\mathcal{F}}^{g\prime}\}$ and the hydrostatic pressure loading $\{\boldsymbol{\mathcal{F}}^{hs\prime}\}$. The gravity force is independent of the body instantaneous position, and it always acts in the direction of the acceleration of gravity. On the other hand, the direction of the pressure forces is defined by the normal vector on the wetted body surface so that its direction changes when body moves. These two facts have important consequences on the description of the external loading in the respective coordinate systems (earth fixed and/or body fixed). After the linearization, it is common to write the external loads in the following form:

$$\{\boldsymbol{\mathcal{F}}'\} = \left\{\boldsymbol{\mathcal{F}}^{g\prime(0)}\right\} + \left\{\boldsymbol{\mathcal{F}}^{p\prime(0)}\right\} + \left\{\boldsymbol{\mathcal{F}}^{g\prime(1)}\right\} + \left\{\boldsymbol{\mathcal{F}}^{p\prime(1)}\right\} = \left\{\boldsymbol{\mathcal{F}}^{g\prime(0)}\right\} + \left\{\boldsymbol{\mathcal{F}}^{p\prime(0)}\right\} - ([\boldsymbol{C}]^g + [\boldsymbol{C}]^p)\{\boldsymbol{\xi}\} = \left\{\boldsymbol{\mathcal{F}}^{\prime(0)}\right\} - [\boldsymbol{C}]\{\boldsymbol{\xi}\} \qquad (5)$$

where the matrix $[\boldsymbol{C}]$ is called the restoring stiffness matrix.
The following expressions are obtained for the external loading of the i-th mode :

$$F_i^{g\prime(1)} = \quad g \iiint_{V_0} \left[\{\boldsymbol{h}'_{0i}\}^T\left(\left[\boldsymbol{\theta}^{(1)}\right] + \nabla \boldsymbol{u}'_f\right) + \left\{\boldsymbol{u}'_f\right\}^T [\nabla \boldsymbol{h}_{0i}]^T\right] \{\boldsymbol{k}\}\rho_m \mathrm{d}V_0 \qquad (6)$$

$$F_i^{p\prime(1)} = \quad -\rho g \iint_{S_0} \left[z^{(1)}\{\boldsymbol{h}'_{0i}\}^T + z^{(0)}\left(\{\boldsymbol{h}'_{0i}\}^T\left[\nabla \boldsymbol{u}'_f - \left[\nabla \boldsymbol{u}'_f\right]^T\right] + \left\{\boldsymbol{u}'_f\right\}^T [\nabla \boldsymbol{h}'_{0i}]^T\right)\right] \{\boldsymbol{n}_0\}\mathrm{d}S_0 \qquad (7)$$

where i goes from 1 to $N = 6 + N_f$, $z^{(0)}$ is the initial vertical position of the point attached to the body, and:

$$z^{(1)} = \sum_{j=1}^{N} \xi_j \, h'_{0jz} \quad , \quad \left[\boldsymbol{\theta}^{(1)}\right] = \sum_{j=1}^{6} \xi_j [\nabla \boldsymbol{h}'_{0j}] \quad , \quad \left\{\boldsymbol{u}'_f\right\} = \sum_{j=7}^{N} \xi_j \{\boldsymbol{h}'_{0j}\} \qquad (8)$$

The elements of the restoring stiffness matrix can be deduced as follows:

$$C_{ij}^g = \quad g \begin{cases} \iiint_{V_0} \{\boldsymbol{k}\}^T \left[\nabla \boldsymbol{h}'_{0j}\right]\{\boldsymbol{h}'_{0i}\}\rho_m \mathrm{d}V_0 & , \quad j = 1,6 \\ \iiint_{V_0} \{\boldsymbol{k}\}^T\left(\{\boldsymbol{h}'_{0i}\}\nabla \boldsymbol{h}'_{0j} + [\nabla \boldsymbol{h}'_{0i}]\{\boldsymbol{h}'_{0j}\}\right)\rho_m \mathrm{d}V_0 & , \quad j > 6 \end{cases} \qquad (9)$$

$$C_{ij}^p = \quad -\rho g \begin{cases} \iint_{S_0} h'_{0jz}\{\boldsymbol{h}'_{0i}\}^T\{\boldsymbol{n}_0\}\mathrm{d}S_0 & , \quad j = 1,6 \\ \iint_{S_0} \left[h'_{0jz}\{\boldsymbol{h}'_{0i}\}^T + z^{(0)}\left(\{\boldsymbol{h}'_{0i}\}^T\left(\nabla \boldsymbol{h}'_{0j} - \left[\nabla \boldsymbol{h}'_{0j}\right]^T\right) + \{\boldsymbol{h}'_{0j}\}^T[\nabla \boldsymbol{h}'_{0i}]^T\right)\right]\{\boldsymbol{n}_0\}\mathrm{d}S_0 & , \quad j > 6 \end{cases} \qquad (10)$$

Numerical validation of the restoring stiffness matrix

Test case

The numerical example which we chose for the validation is the rectangular barge from [3] and only six modes of motions are considered: heave, pitch and the first four vertical bending modes i.e. (ξ_3 , ξ_5 , ξ_7 , ξ_8, ξ_9 , ξ_{10}).

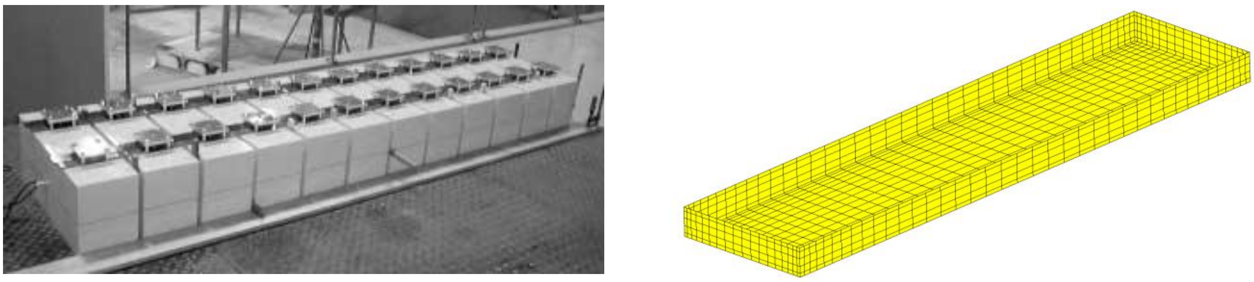


Figure 2: Hydroelastic barge.

Methodology

The validation of the restoring stiffness matrix is performed by comparing the results obtained from equations 9 and 10 to the restoring coefficients obtained by direct calculations. In direct calculations, one can integrate the pressures, and gravity loads on the deformed hydrodynamic model and project them on the modal basis. Once the modal total force is obtained when deforming the hydrodynamic model on each mode, the restoring coefficients for each deformation mode can be expressed as the ratio between the resulting modal forces and the applied modal amplitude.

For exemplification purposes, Figure 3 depicts the considered barge, which is subjected to hydrostatic pressure and gravity loads when it is deformed considering only the first vertical bending mode. The hydrostatic force acting on the body is evaluated numerically using equation 11. Similarly, the loading induced by the acceleration of gravity is evaluated as shown in equation 12.

$$F_i^{Hs} = \quad -\rho g \iint_S z\{\boldsymbol{h}_i'\}^T\{\boldsymbol{n}'\}\mathrm{d}\tilde{S} \tag{11}$$

$$F_i^{G} = \quad g \iiint_V \{\boldsymbol{h}_i'\}^T\{\boldsymbol{k}\}\rho_m \mathrm{d}V \tag{12}$$

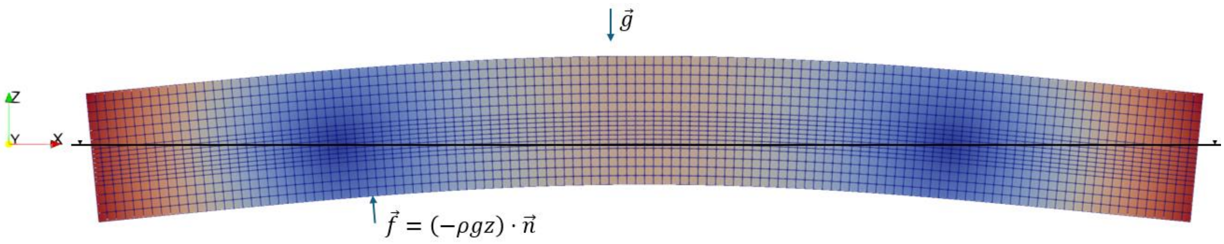


Figure 3: Illustration of the first vertical mode

The modal total force is evaluated at various modal amplitudes to determine the influence of the modal amplitude on the restoring coefficient, as depicted in Figure 4. However, as the restoring matrix proposed in equations 9 and 10 is linearized, the directly calculated modal restoring coefficients at small modal amplitudes will be used for comparison purposes hereafter.

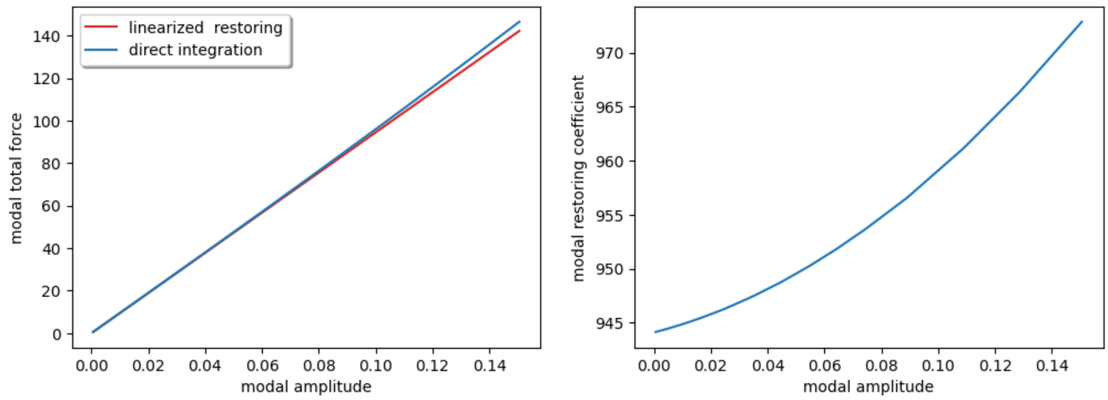
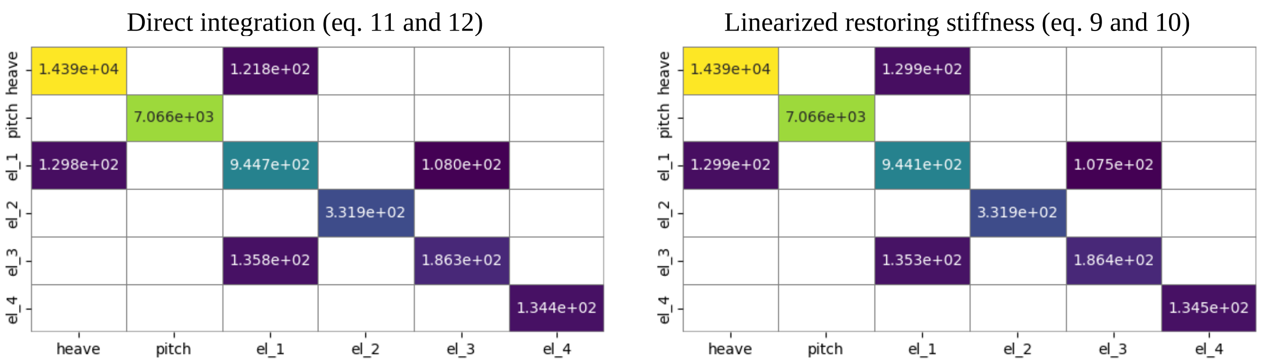


Figure 4: Variation of the modal total force for the first elastic mode at various modal amplitudes

Results

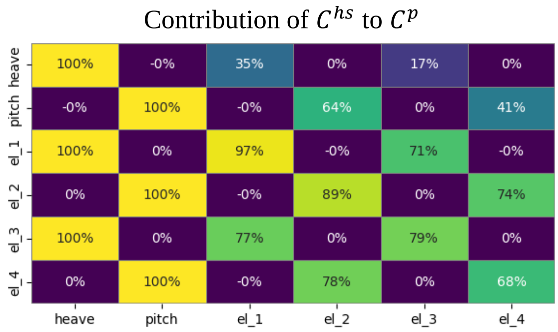
Based on the methodology described in the preceding sections, the restoring stiffness is evaluated by direct integration and using the linearized formulation proposed in equations 9 and 10 for all six motions. The numerical results are summarized hereafter.



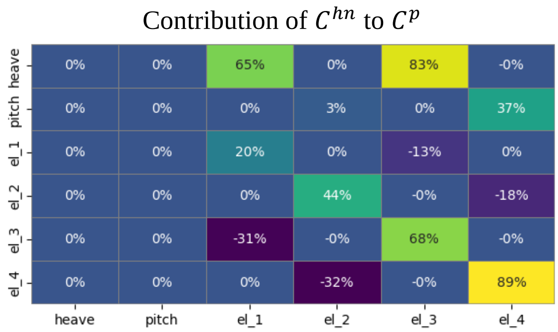
From the numerical results depicted above, it can be observed that the restoring coefficients evaluated by the two methods are in perfect agreement apart from the coupling term between heave and the first elastic mode, where a relative error of 6% is observed. This is most probably related to numerical errors in the direct integration approach.

Furthermore, in order to better understand the importance of different components in the pressure part of the linearized restoring stiffness matrix, for $j > 6$, equation 9 can be split into three components, as follows: $C_{ij}^{p} = C_{ij}^{hs} + C_{ij}^{hn} + C_{ij}^{hh}$. Where C_{ij}^{hs} represents the pressure variation contribution, C_{ij}^{hn} is the contribution due to the change of normal on the deformed model, and C_{ij}^{hh} represents the mode variation contribution. The three terms are evaluated numerically and their contribution as percentage of the total C_{ij}^{p} is presented below.

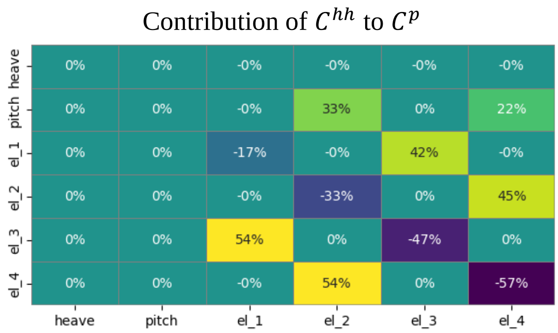
$$C_{ij}^{hs} = -\varrho g \iint_{S_0} h'_{0jz} \{\boldsymbol{h}'_{0i}\}^T \{\boldsymbol{n}_0\} dS_0$$



$$C_{ij}^{hn} = -\varrho g \iint_{S_0} z^{(0)} \{\boldsymbol{h}'_{0i}\}^T \left(\nabla \boldsymbol{h}'_{0j} - \left[\nabla \boldsymbol{h}'_{0j}\right]^T\right) \{\boldsymbol{n}_0\} dS_0$$



$$C_{ij}^{hh} = -\varrho g \iint_{S_0} z^{(0)} \{\boldsymbol{h}'_{0j}\}^T [\nabla \boldsymbol{h}'_{0i}]^T \{\boldsymbol{n}_0\} dS_0$$



Discussions & conclusions

The previously proposed methodology for evaluating the generalized restoring stiffness of deformable bodies is verified and validated through direct calculations. Numerical results show that the restoring stiffness matrix can be consistently evaluated accounting for the effect of pressure variation as well as for the normal and mode variations for both the rigid and elastic modes. Moreover, it is shown that for the considered flexible barge, the normal and mode variation contributions play a significant role to the total restoring stiffness matrix. Their importance is about 50% for the coupling terms between the rigid and elastic modes, and about 25% for the terms of the restoring matrix between the elastic modes.

References

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