Three alternative linear free-surface boundary conditions for flows around ships

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Highlights: *Computations of the flow velocity created by a simple analytically-defined ship-hull surface with a sharp wedge-like bow and a round stern in the infinite-gravity or zero-gravity limits are presented to illustrate basic numerical and theoretical issues associated with the linearization of the free-surface boundary condition about the 'infinite-gravity flow' around a ship, also called 'double-body flow' around a ship-hull surface and its free-surface mirror image, or the linearization about the 'zero-gravity flow'.*

1. Introduction

The flow around a ship that advances at a constant speed $\mathbf{V} \equiv (V, 0, 0)$ through regular waves or in calm water is commonly evaluated within the framework of potential-flow theory. Moreover, the boundary condition at the free surface in this classical approach is applied at the undisturbed free-surface plane $z = 0$ and is linearized as done by Kelvin and Michell in their pioneering studies of ship waves. This usual Kelvin-Michell linearization of the nonlinear free-surface boundary condition assumes that the velocity $\nabla\Phi \equiv (\Phi_X, \Phi_Y, \Phi_Z)$ of the flow disturbance due to a ship that advances in calm water or through regular waves is significantly smaller than the ship speed V. Thus, the **Kelvin-Michell linearization** is based on the assumption

$$\| \nabla\Phi \| \ll V \ . \tag{1}$$

An alternative linearization of the free-surface boundary condition has been considered, and is used in [1-5] where a detailed review of the related literature can also be found. This alternative linearization is based on the assumption that the flow velocity $\nabla\Phi$ created by a ship is significantly smaller than the velocity $\nabla\Phi^\infty - \mathbf{V}$ of the (apparent total) flow created by the ship in the infinite-gravity limit $g = \infty$, in which the free surface becomes a rigid wall. The 'base-flow' $\nabla\Phi^\infty$ is identical to the unbounded flow around the double-body hull surface $\Sigma^H \cup \Sigma^H_*$ where Σ^H denotes the mean-wetted ship-hull surface and Σ^H_* is its mirror image with respect to the free-surface plane $Z = 0$. Thus, the **double-body linearization** assumes

$$\| \nabla\Phi \| \ll \| \nabla\Phi^\infty - \mathbf{V} \| \ . \tag{2}$$

For common slender ships, the double-body flow velocity $\nabla\Phi^\infty$ is significantly smaller than the ship speed V except in small regions around the points where the ship bow and stern intersect the free-surface plane $Z = 0$, which are stagnation points of the double-body flow. Thus, one has

$$\| \nabla\Phi^\infty \| \ll V \tag{3}$$

and the double-body linearization is practically equivalent to the classical Kelvin-Michell linearization except in *small* regions around the bow and the stern of the ship.

Linearization of the free-surface boundary condition about the double-body flow may arguably be realistic for slow ships with blunt bows. However, this linearization may be ill suited in practice because the double-body flow varies very rapidly in the vicinity of the points where the bow and the stern of a ship intersect the plane $Z = 0$. Reliable numerical evaluation of the *derivatives* of the double-body flow velocity $\nabla\Phi^\infty$ that are required in the free-surface boundary condition associated with linearization about the double-body flow can then be difficult, and a thorough study of the influence of the discretization of the free surface in the vicinities of a ship bow and stern is required to verify that significant numerical inaccuracies do not occur; such thorough numerical studies do not appear to have been reported.

Moreover, the *nonlinear* potential flow at a ship bow greatly differs from the stagnation flow associated with the double-body flow-model. In particular, the analysis given in [6-12] for a ship that steadily advances in calm water shows that the actual (nonlinear) free-surface flow in the vicinity of a ship bow greatly differs from the uniform stream $(-V, 0, 0)$ or the double-body flow $(\Phi^\infty_X - V, \Phi^\infty_Y, 0)$. This result is at variance with *both* the Kelvin-Michell and the double-body flow linearization assumptions (1) or (2). Linearization about the double-body flow, which is nearly identical to the uniform flow $(-V, 0, 0)$ except in the vicinities of the points where the bow and the stern of a ship intersect the free surface as was already noted, is then no better justified than the Kelvin-Michell linearization, which has the huge merit of being based on a uniform flow that is unaffected by numerical inaccuracies.

The Rankine component G^R in the fundamental Rankine-Fourier decomposition $4\pi G = G^R + G^F$ of the

Green function $G(\boldsymbol{\xi},\mathbf{x})$ for potential flow around a ship steadily advancing through regular waves is given by

$$G^R \sim -1/r + 1/r' \ \text{ as } \ r' \to 0 \ \text{ and as } \ r' \to \infty \ \text{ where} \tag{4a}$$

$$r \equiv \sqrt{h^2 + (\zeta - z)^2} \ \text{ and } \ r' \equiv \sqrt{h^2 + (\zeta + z)^2} \ \text{ with } \ h^2 \equiv (\xi - x)^2 + (\eta - y)^2 \ , \tag{4b}$$

as is shown in e.g. [13-15]. Thus, the Green function that satisfies the Kelvin-Michell linear free-surface boundary condition for ship motions in regular waves is asymptotically equivalent, in *both* the near-field limit $r' \to 0$ and the far-field limit $r' \to \infty$, to the Green function associated with potential flows in the *zero*-gravity limit $g = 0$ and the corresponding conditions $\phi = 0$ and $G = 0$ at $z = 0$, rather than the *infinite*-gravity limit $g = \infty$ and the conditions $\phi_z = 0$ and $G_z = 0$ at $z = 0$ as is assumed in the linearization about the double-body flow. This basic theoretical result suggests that linearization based on the assumption

$$\| \nabla \Phi \| \ll \| \nabla \Phi^0 - \mathbf{V} \| \tag{5}$$

where the flow velocity $\nabla \Phi^0$ is associated with the zero-gravity limit $g = 0$ arguably is no less justified than the double-body linearization (2). Indeed, the bow waves created by fast ships with fine bows in the 'overturning bow-wave regime' considered in [9-12] suggest that **linearization about the 'zero-gravity flow velocity'** $\nabla \Phi^0$ might be reasonable. In particular, the nonlinear flow analysis considered in [6,7] for a ship in calm water shows that the bow-wave profile is tangent to the stem line, in accordance with the zero-gravity flow model.

Thus, it evidently is useful to consider the flow velocities

$$\nabla \Phi^\infty \equiv (\Phi^\infty_X, \Phi^\infty_Y, 0) \ \text{ and } \ \nabla \Phi^0 \equiv (0, 0, \Phi^0_Z) \tag{6}$$

created at the free-surface plane $Z = 0$ by a ship hull that has a pointed wedge-like end and a round end. The weakly-singular boundary integral equations given in [16] can be used to evaluate the flow potentials Φ^∞ and Φ^0 with satisfactory accuracy. However, accurate numerical evaluation of the corresponding flow velocities (6) outside the ship is more difficult. To avoid numerical inaccuracies, an analytical expression for the velocity potential Φ of the flow created by a ship that steadily advances along the positive X axis is then used here to evaluate the flow velocities (6) for a simple analytically-defined ship-hull surface Σ^H with a sharp wedge-like bow and a round stern. Specifically, the approximation proposed by Hogner and defined in [17-19] is applied.

2. Illustrative application based on the Hogner slender-ship approximation

Hogner's approximation *explicitly* determines the flow created by a ship that steadily advances in calm water in terms of the Froude number F and n^x, i.e. the speed V and the length L of the ship, and the ship-hull form. Despite its simplicity, Hogner's approximation is realistic and widely useful. In particular, the Hogner approximation is shown in [20] to be useful for hull-form optimization and indeed has been widely applied for that purpose. Hogner's approximation is also useful and has been applied to analyze the influence of wave interferences on the far-field wave pattern of a ship steadily advancing in calm water in [21-25]. Moreover, the Hogner approximation is used to investigate how to filter inconsequential short ship waves in [26-28].

In the *infinite-gravity limit* $g = \infty$, the Hogner flow potential $\Phi \equiv VL\,\phi$ is given by

$$\phi^\infty(\mathbf{x}) = \frac{-1}{4\pi} \int_{\Sigma^H \cup \Sigma^H_*} da(\boldsymbol{\xi}) \, \frac{n^x(\boldsymbol{\xi})}{r} \tag{7a}$$

where r is defined by (4b). Moreover, $\mathbf{x} \equiv \mathbf{X}/L \equiv (x, y, z \le 0)$ denotes a flow field point, $\boldsymbol{\xi} \equiv (\xi, \eta, \zeta)$ is a point of the double-hull surface $\Sigma^H \cup \Sigma^H_*$ and $n^x(\boldsymbol{\xi})$ is the $x-$component of the unit vector $\mathbf{n} \equiv (n^x, n^y, n^z)$ that is normal to $\Sigma^H \cup \Sigma^H_*$ and points outside the double body. At a point $(x, y, z = 0)$ of the plane $z = 0$, the velocity $\nabla_{\mathbf{x}} \phi^\infty(\mathbf{x})$ that corresponds to the Hogner potential (7a) is given by

$$\begin{Bmatrix} \phi^\infty_x(x,y) \\ \phi^\infty_y(x,y) \end{Bmatrix} = \frac{1}{2\pi} \int_{\Sigma^H} da(\boldsymbol{\xi}) \begin{Bmatrix} x - \xi \\ y - \eta \end{Bmatrix} \frac{n^x(\boldsymbol{\xi})}{(h^2 + \zeta^2)^{3/2}} \ \text{ and } \ \phi^\infty_z = 0 \ . \tag{7b}$$

In the *zero-gravity limit* $g = 0$, the Hogner approximation (7a-b) is modified as

$$\phi^0(\mathbf{x}) = \frac{-1}{4\pi} \int_{\Sigma^H} da(\boldsymbol{\xi}) \, n^x(\boldsymbol{\xi}) \left[\frac{1}{r} - \frac{1}{r'} \right] \tag{8a}$$

$$\phi^0_x = 0 \ , \ \phi^0_y = 0 \ \text{ and } \ \phi^0_z(x,y) = \frac{1}{2\pi} \int_{\Sigma^H} da(\boldsymbol{\xi}) \, \frac{-\zeta \, n^x(\boldsymbol{\xi})}{(h^2 + \zeta^2)^{3/2}} \tag{8b}$$

where r and r' are defined by (4b).

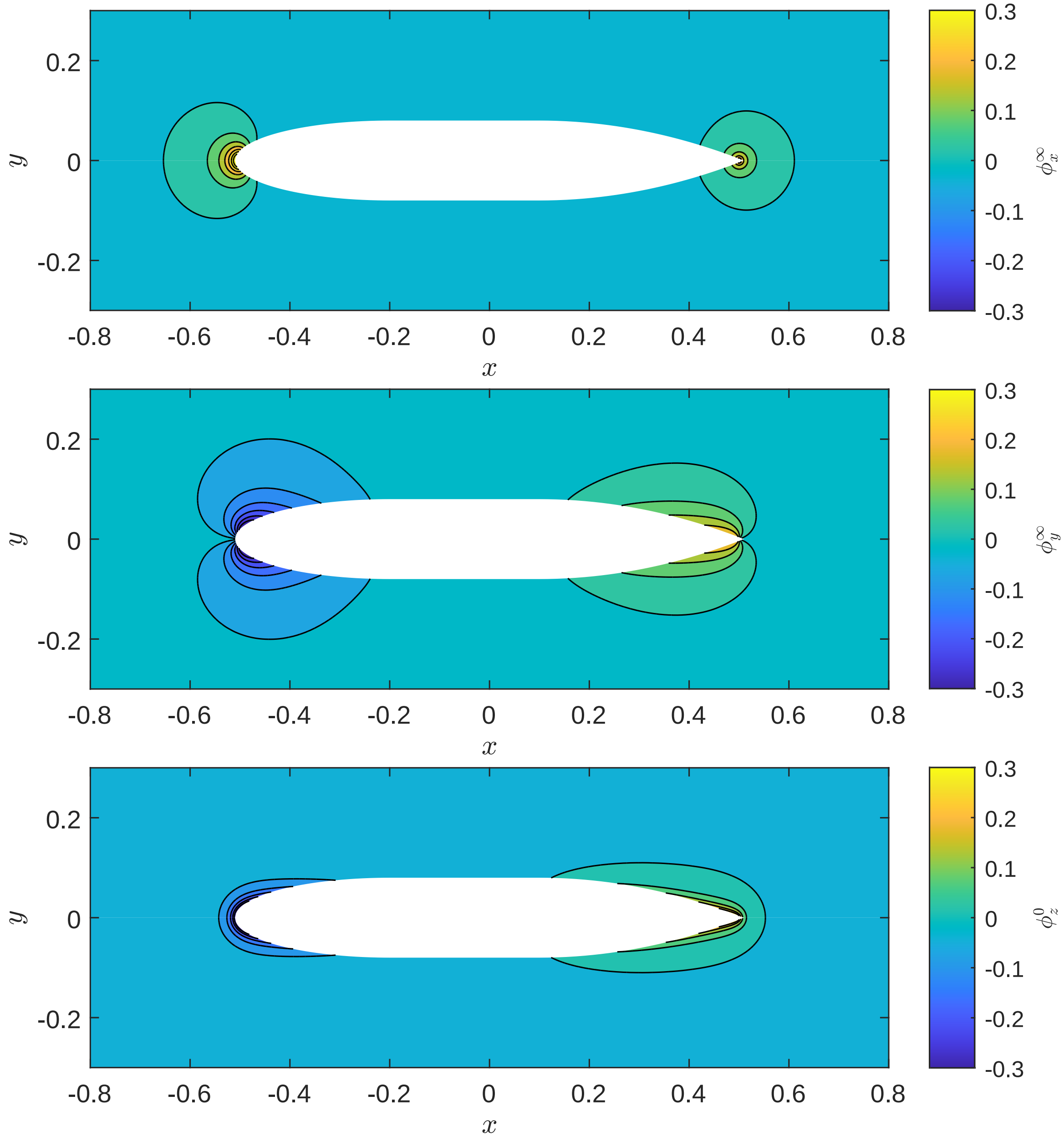

Figure 1: The horizontal flow velocity components ϕ_x^∞ (top row) and ϕ_y^∞ (center row) and the vertical velocity ϕ_z^0 (bottom row) defined by (7b) and (8b) are depicted in the figure for a simple analytically-defined wall-sided hull surface Σ^H that has rectangular framelines, a uniform draft $d \equiv D/L = 0.04$, a beam $b \equiv B/L = 0.14$, a sharp wedge-like bow and a round stern. The figure shows that the (nondimensional) flow velocity components ϕ_x^∞, ϕ_y^∞ and ϕ_z^0 are significantly smaller than 1 (i.e. the ship speed V) except in small regions in the vicinities of the ship bow and stern. In particular, the flow velocity ϕ_x^∞ is negligible except in very small regions centered at the bow and stern, and varies rapidly near these two stagnation points of the double-body flow. Similarly, the vertical flow velocity ϕ_z^0 is negligible except in a thin layer along the ship waterline near the bow and stern.

3. Conclusions

Two conclusions can be drawn from this study: (i) The results depicted in Fig.1 show that a thorough numerical study of the influence of the discretization of the free surface in the vicinity of a ship bow and stern is necessary if the free-surface boundary condition is linearized about a 'base flow' taken as the 'infinite-gravity double-body flow' or as the 'zero-gravity flow'. (ii) The nonlinear analysis of ship bow waves previously reported in [6-12] suggests that these two alternative linear free-surface boundary conditions may not offer benefits over the classical Kelvin-Michell linearization, which evidently is incomparably simpler and ultimately seems likely to be better suited and preferable for practical applications.

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