Predicting Green Water Loads on Structures Using Newtonian Flow Theory

Min Gao[1]*, Paul H. Taylor[1], Scott Draper[1], Hugh Wolgamot[1], Wenhua Zhao[1, 2]
[1] School of Earth and Oceans, The University of Western Australia, Perth, Australia
[2] School of Civil Engineering, The University of Queensland, Brisbane, Australia
E-mail: min.gao@uwa.edu.au

1 INTRODUCTION

A green water event is characterised by a large amount of water overtopping onto the deck of a vessel in extreme conditions. This event, which is a concern for offshore facilities, can cause significant damage to topside structures, injuries to crew on deck, and possibly lead to global failure of vessels. In the current offshore structure design process, guidelines and rules still recommend conducting model tests or using historical data to determine green water loading [1]. There has been extensive research conducted to investigate the green water loads on structures using both physical and numerical modelling [2], as illustrated in Figure 1. These studies have reported various findings related to the green water flow characteristics and the resulting loads on structures. However, these two methods require a lot of resources in terms of time or cost, implying that neither is applicable as an industry design tool.

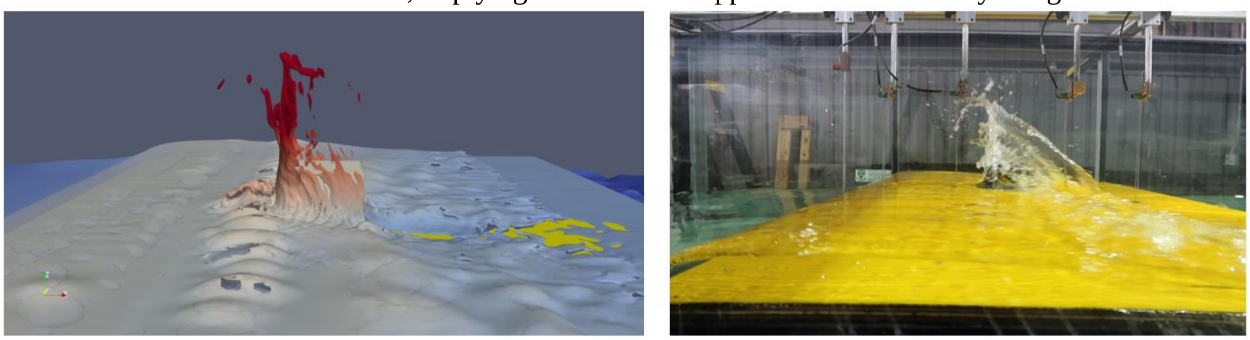

Figure 1: Green water flow impacts on an on-deck cube using CFD (left) and experiment (right) [2].

As green water overtopping has been reported to be similar to dam break flow for certain scenarios [3], studies on shallow water flow incident on a structure can provide valuable insights for investigating green water loads. There have been some efforts to predict the force on a structure exposed to a fast flow [4], [5], [6]. Across current studies, there are mainly three types of method to predict the force on the structure. Methods are either based on the hydrostatic force ($\sim 1/2\rho g h^2 D$), or hydrodynamic force ($\sim \rho u^2 h D$), or the combined form. All three methods rely on empirical coefficients to obtain accurate estimates of the loads. Because of this, extrapolation of these models to different structural shapes or flow conditions is difficult. In our current study, an analytical method based on Newtonian flow theory is proposed to predict the green water loads on topside structures with different configurations.

2 NEWTONIAN FLOW THEORY

Newtonian flow theory is a well-known approach used for hypersonic flow problems to estimate the forces on a craft at hypersonic speeds [7]. Newton theorized that the force on a surface due to a uniform stream of particles arises from the loss of momentum of the particles normal to the surface. For example, if a stream of particles with velocity V_∞ strike a flat surface inclined at an angle θ, Newton assumed that the normal momentum of the particles is transferred to the surface while the tangential component is preserved. Consequently, the particles move along the surface after colliding with the surface, and the change in velocity component normal to the surface is equal to $V\cos\theta$. The mass flux of particles impacting a surface with area A can be estimated simply as $\rho V_\infty A\cos\theta$ (Figure 2). Thus, the time rate of change of momentum of this mass flux in the direction normal to the surface, which equals the normal force exerted on the surface, can be determined as the product of the mass flux and the change in normal velocity:

$$F_n = \rho V_\infty A\cos\theta \times V_\infty \cos\theta = \rho V_\infty^2 A\cos^2\theta \tag{1}$$

Resolving this force in the incident flow direction then results in the predicted streamwise force on the structure:

$$F_x = \rho V_\infty^2 A\cos^3\theta \tag{2}$$

Flow-structure interactions of this type occur when the flow is hypersonic and produce a shock wave that is close to parallel to the plate (such that the deflected flow occupies a thin layer in front of the structure). The high Froude number flows occurring in green water may behave similarly - the upstream flow cannot adjust to the presence of a structure and the vertical projection of a fluid sheet upon impact along the surface plays the role of the flow between the shock and the body surface in hypersonics, allowing the (incompressible) fluid to travel across the impacted surface in a thin layer.

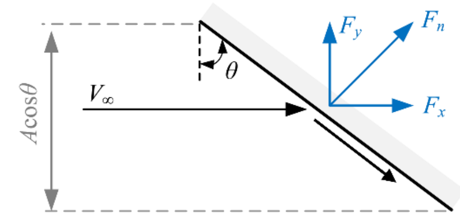

Figure 2: Sketch of Newtonian flow theory.

Taking inspiration from this hypersonic theory, a model is developed to predict the force due to the supercritical green water flow incident on surface mounted structures. Considering a cube with side length *D*, height *H* and an orientation angle θ, shown in Figure 3(a), CFD simulations and experimental results [4] show that the incident flow is diverted upwards or tangentially along the front faces (shown in Figure 3(b)(c)) when it impacts this cube, resulting in almost complete loss of streamwise momentum. Making use of these observations, the entire incident momentum flux is assumed to be converted into a normal force on the two front faces, denoted as *a* and *b*. Resolving this force in the streamwise direction results in:

$$F_{a,x}(t) + F_{b,x}(t) = \rho u^2 hD\sin^3\theta + \rho u^2 hD\cos^3\theta \tag{3}$$

It is important to note that the flow will not impact the entire front face simultaneously for non-normal orientations. Instead, the effective wetted length along each face will increase until the flow has reached all points on the upstream faces. Accounting for this transient leads to

$$F_{a,x}(t) + F_{b,x}(t) = \rho u^2 hDD_{aw}\sin^3\theta + \rho u^2 hDD_{bw}\cos^3\theta \tag{4}$$

where

$$D_{aw} = \begin{cases} \frac{u_f t}{D\cos\theta}, & t < \frac{D\cos\theta}{u_f} \\ 1, & t < \frac{D\cos\theta}{u_f} \end{cases}, D_{bw} = \begin{cases} \frac{u_f t}{D\sin\theta}, & t < \frac{D\sin\theta}{u_f} \\ 1, & t < \frac{D\sin\theta}{u_f} \end{cases} \tag{5}$$

When θ=0, Eq. 4 becomes $F_x(t) = \rho u^2 hD$. This result is similar to the hydrodynamic model formulation. However, a key difference here is that there is no empirical coefficient; the result is equal to the streamwise momentum loss rather than a form drag with an empirical drag or resistance coefficient. The forces on the structure in this scenario are dominated by inertial effects, originating in the loss of momentum from the flow upon impact. This differs fundamentally from drag-dominated scenarios, where forces arise from boundary layer dynamics and vortex shedding around the structure.

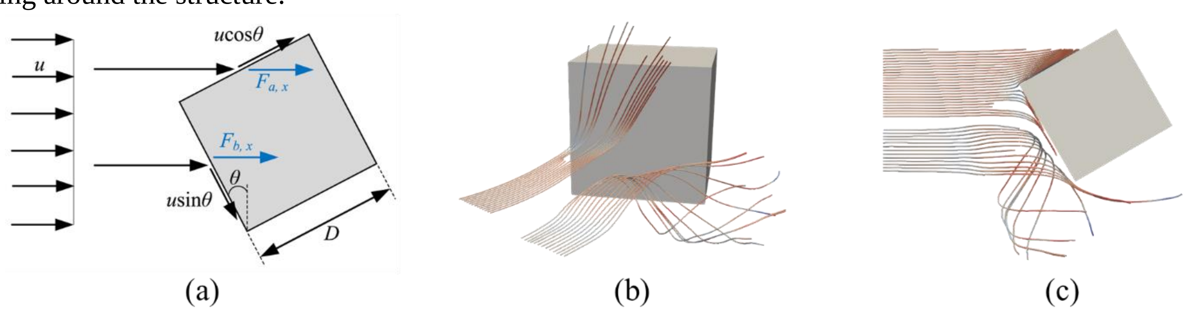

Figure 3: (a) Sketch of uniform flow impacting an orientated cube; Streak lines around a cube in (b) oblique and (c) top view.

Using a similar procedure, the force on a circular cylinder of radius *R=D/2* is also calculated. Discretising the upstream half of the cylinder into small segments, and adopting Newtonian flow theory for each segment (shown in Figure 4) leads to a streamwise contribution on each segment equal to

$$\mathrm{d}F_x = \rho u^2 hR\mathrm{d}\theta\cos^3\theta \tag{6}$$

Because the circular cylinder has a curved surface, when the flow strikes this surface and is assumed to remain in contact with it, the fluid element is also experiencing centripetal acceleration in the radial direction. To balance this centripetal acceleration and keep the fluid element moving along the surface, there must be a positive pressure gradient on the fluid element in the radial direction, which will contribute to the overall force on the structure. Considering this effect, the force on each segment of the cylinder is given by

$$\mathrm{d}F_x = \rho u^2 hR\mathrm{d}\theta\cos\theta\left(\cos^2\theta - \frac{1}{2}\sin^2\theta\right) \tag{7}$$

The second term originates in the centripetal acceleration (the detailed derivation is omitted here, see Anderson [7]). Integrating this over the full upstream half of the cylinder gives

$$F_x(t) = \left(\frac{2}{3} - \frac{1}{6}\right)\rho u^2 hD = \frac{1}{2}\rho u^2 hD \tag{8}$$

This is exactly half of the force on a cube of the same width oriented normal to the flow aligning closely with the results obtained from CFD and experiment. Although some previous studies [5], [8] also note that the force on a circular shape structure is nearly half of that on a square one, it is important to emphasize that this one-half factor here arises not only from the difference in structural geometry but also from the effect of centripetal acceleration in the fluid. Without the centripetal acceleration, the factor would be 2/3.

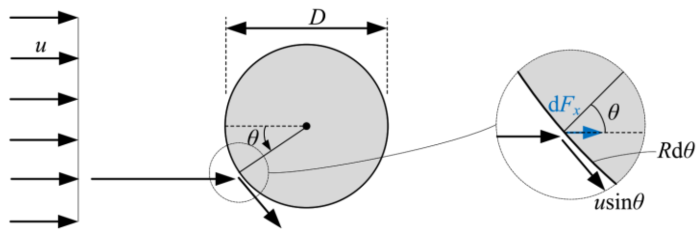

Figure 4: Sketch of uniform flow impacting a circular cylinder.

3 FORCE PREDICTION

To compare the theoretical predictions with the numerical simulations, the undisturbed on-deck flow information at the location of the upstream face of the structure is extracted from a CFD run for a focused wave hitting a rectangular box with flat deck on top to represent u and h. Using these time series, the comparison between the CFD simulated forces and Newtonian flow theory predicted forces are presented in Figure 5 for cubes at four orientation angles θ =0, 15, 30, 45 degrees, 0 being normal incidence onto a cube face and 45 to one edge symmetric upwave.

Overall, the force predictions based on Newtonian flow theory demonstrate good agreement with the CFD simulated forces. The initial rise in force is accurately captured and the predictions remain consistent at later times (> 0.45 s), even with the continuously varying (but supercritical) Froude number. While the model does not account for the slight force increase observed around 0.85s, attributed to the collapse of the vertical sheet, it otherwise replicates the reduction in force reasonably well as the overtopping event concludes. The force prediction method is also applied to a cube oriented normal to the flow but elevated above the deck by a distance δ. Worth noting that these predictions use Eq. 4 but the θ=0 and h replaced with (h-δ). Figure 6 compares the predicted forces with the CFD-simulated forces for a cube at three different elevations.

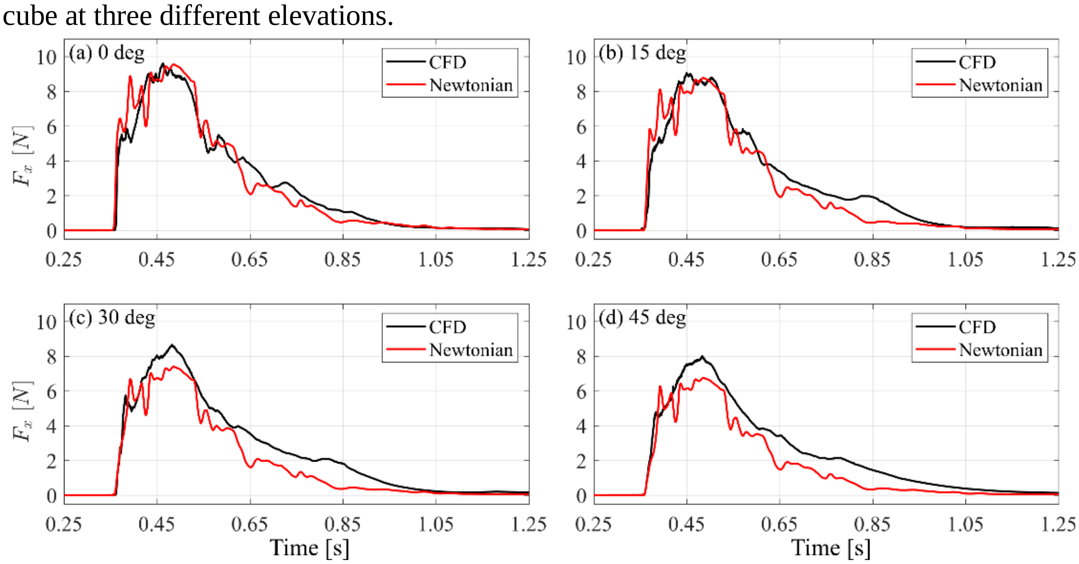

Figure 5: Comparison of simulated and predicted time series of streamwise force on the cube with different angles.

Figure 7 presents a comparison between the predicted forces and CFD-simulated forces for the two cylinders with different diameter (D/h=4, 2). In both cases, the predicted forces are slightly larger than the CFD-simulated forces, with the overestimation becoming more pronounced as the cylinder size decreases.

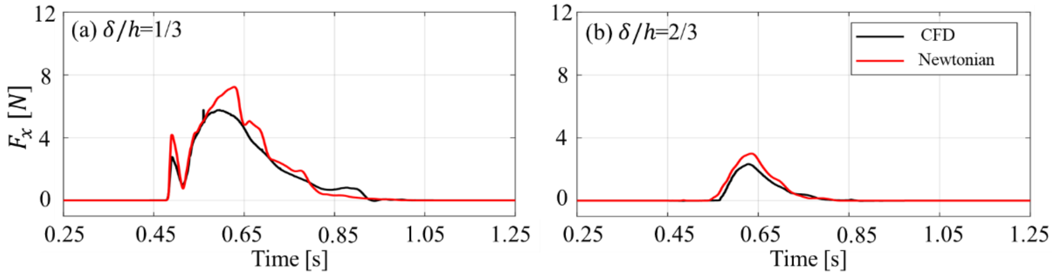

Figure 6: Comparison of simulated and predicted time series of streamwise force on the cube with different elevated height.

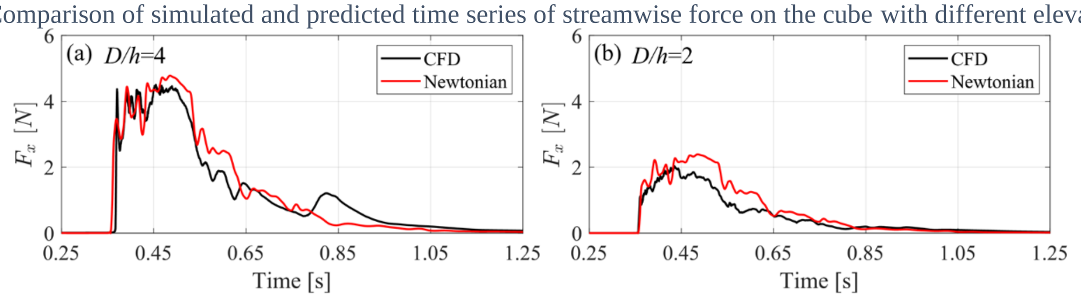

Figure 7: Comparison of simulated and predicted time series of streamwise force on the cylinder with different size.

The inline force on a structure of given shape and orientation depends generally on the following non-dimensional quantities $\frac{F_x}{\rho D u^2 h} = f\left(\frac{uD}{\nu}, \frac{u}{\sqrt{gh}}, \frac{D}{h}, \frac{H}{h}\right)$. Newtonian flow theory assumes that the Froude number is sufficiently high that the flow is supercritical, which appears to be reasonable in this study. Some additional cases have been simulated to check the sensitivity on last two parameters: relative structure width *D/h*, and height *H/h*. Based on the results, the force prediction error is within 10% when *D/h* and *H/h* exceeds 4, respectively, indicating that for relatively larger structures or shallower flows, and relatively taller structures, the force on the structure is independent of these parameters. The theory can also be adapted to structures which do not penetrate the full depth of the fluid layer.

4 CONCLUSIONS

To predict the force on structures subject to supercritical on-deck green water flow, a theoretical model based on the classical Newtonian approximation for hypersonic flow was adapted, replacing the hypersonic assumption with a supercritical Fr≫1. Analytical equations for the force on structures of various angles and sizes were derived. Despite the simplifying assumptions, the model demonstrates reasonable agreement with CFD-simulated results. The predictions are particularly accurate under conditions where the flow is highly supercritical (large Fr), shallow (small *h/D*), and the structure is relatively tall (large *H/h*). While the model has some limitations, it offers an efficient approach for predicting forces on structures without requiring empirical coefficients, making it a practical tool for early-stage design.

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REFERENCES

[1] LR Rules and Regulations for the Classification of Offshore Units.

[2] Gao, M., 2023. Green water loading on topside structures. PhD thesis, The University of Western Australia.

[3] Greco, M., Colicchio, G., & Faltinsen, O. M. (2007). Shipping of water on a two-dimensional structure. Part 2. *Journal of Fluid Mechanics*, *581*, 371-399.

[4] Harish, S., Sriram, V., Schüttrumpf, H., & Sannasiraj, S. A. (2021). Tsunami-like flow induced force on the structure: Prediction formulae for the horizontal force in quasi-steady flow phase. *Coastal Engineering*, *168*.

[5] Arnason, H., Petroff, C., & Yeh, H. (2009). Tsunami Bore Impingement onto a Vertical Column. *Journal of Disaster Research*, *4*(6), 391-403.

[6] Shafiei, S., Melville, B. W., & Shamseldin, A. Y. (2016). Experimental investigation of tsunami bore impact force and pressure on a square prism. *Coastal Engineering*, *110*, 1-16.

[7] Anderson, J. D. (2006). *Hypersonic and high-temperature gas dynamics* (2nd ed. ed.). Reston, Va: American Institute of Aeronautics and Astronautics.

[8] Lee, D., Jeon, M., Nguyen, V. M., & Yoon, H. K. (2016). *An Experimental Analysis of the Impact of Green Water on Offshore Platforms with Green Water Protectors of Various Shapes.* The 26th ISOPE Conference.