Towards a second-order force model for floating non-slender structures in fully nonlinear waves

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1 INTRODUCTION

The wave-induced loads on slender bodies can be calculated using nonlinear wave kinematics, through semi-empirical formulas for the inertia and drag components. However, for non-slender floating structures, where diffraction and radiation forces are dominant, there is currently no simple method to account for the forces induced by nonlinear waves explicitly. In the present study, the fixed-body formulation introduced in Ref. [1] is extended to account for body motion. The model uses the wave elevation and velocity potential from the nonlinear wave solver HOS-NWT [2] and calculates the second-order force under the Pinkster approximation, based only on the output of a first-order radiation-diffraction analysis. The extended formulation explicitly includes the 6 DoF position and velocity, allowing force calculation for moving bodies.

2 FORMULATION FOR FIXED BODY

The fixed body formulation was presented in Ref. [1] and is outlined here as a starting point. A global coordinate system $\mathbf{x}_o = [x_o, y_o, z_o]$ is considered, along with an inertial system located at the position of the body at rest $\tilde{\mathbf{x}} = [\tilde{x}, \tilde{y}, \tilde{z}]$. The incident free surface elevation η_I and velocity potential ϕ_I at the still water level (SWL), defined at $z_o = 0$, of a nonlinear wave field can be written as a Fourier series in the wave number space k as

$$\eta_I(x_o, t) = \sum_{\ell} \hat{A}(k_\ell, t)\mathrm{e}^{-\mathrm{i}k_\ell x_o} \quad \text{and} \quad \phi_I(x_o, t) = \sum_{\ell} \hat{B}(k_\ell, t)\mathrm{e}^{-\mathrm{i}k_\ell x_o} \tag{1}$$

where $\hat{A}$ and $\hat{B}$ are time-dependent modal coefficients for η_I and ϕ_I respectively and contain both free and bound wave contributions. The summation covers both negative and positive integers with $k_{-\ell} = -k_\ell$, $\hat{A}_{-\ell} = \hat{A}^*_\ell$ and $\hat{B}_{-\ell} = \hat{B}^*_\ell$. Following standard potential flow theory [3], the second-order wave force on a fixed body will be

$$\mathbf{F} = -\rho \int_{S_0} \phi_t \; \mathbf{n} \; \mathrm{d}S - \frac{1}{2}\rho \int_{S_0} \nabla\phi \cdot \nabla\phi \; \mathbf{n} \; \mathrm{d}S + \frac{1}{2}\frac{\rho}{g} \int_{\mathrm{WL}} \phi_t^2 \left(1 - n_z^2\right)^{-1/2} \mathbf{n} \; \mathrm{d}l \tag{2}$$

where $\phi = \phi_B = \phi_I + \phi_S$ is the total incident and scattered potential, S_0 is the wetted body surface up to the SWL, $\mathbf{n}$ is the vector normal to that surface, n_z is its vertical component and WL is the still waterline. The spatial structure of ϕ_I and η_I at a given time instant is identical to the case of linear waves. This permits the expression of the potential force, as

$$\mathbf{F}_P(t) = -\rho \int_{S_0} \phi_t \; \mathbf{n} \; \mathrm{d}S = \frac{\partial}{\partial t} \sum_{\ell} -\frac{1}{g}\mathbf{X}(k_\ell)\hat{B}(k_\ell, t)e^{-\mathrm{i}k_\ell x_o} = \sum_{\ell} -\frac{1}{g}\mathbf{X}(k_\ell)\hat{B}_t(k_\ell, t)e^{-\mathrm{i}k_\ell x_o} \tag{3}$$

where $\mathbf{F}(k)$ is the force transfer function obtained by radiation-diffraction analysis and $\hat{B}_t$ are the coefficients for the time derivative of the velocity potential, directly provided by HOS-NWT. Ref. [1] showed that it is consistent with the so-called Pinkster's approximation [4]. For all quantities obtained through linear radiation-diffraction analysis, the dispersion relation is employed to associate a wavenumber with a frequency, thus introducing a level of approximation. However, since the time derivative in Eq. (3) is not linearly evaluated in the frequency domain as iω, but explicitly obtained with the nonlinear wave solver as $\hat{B}_t$, the contribution of bound waves is taken into account. The other two terms in Eq. (2) correspond to the quadratic pressure (QP) and the waterline integral (WL). Linear radiation-diffraction analysis provides transfer functions for the potential $\hat{\phi}(k)$ and velocity $\nabla\hat{\phi}(k)$, which can be applied to the coefficients $\hat{B}(k,t)$ at every time instant. Therefore, the integrals in Eq. (2), can be replaced by a double summation, which includes the application of a quadratic transfer function (QTF) on pairs of $\hat{B}(k,t)$ at every timestep,

$$\begin{aligned}\mathbf{F}_{QP}(t) &= -\frac{1}{2}\rho\int_{S_0}\nabla\phi\cdot\nabla\phi\ \mathrm{d}\mathbf{n} = \sum_m\sum_\ell \mathbf{Q}_{QP}(k_m,k_\ell)\ \hat{B}(k_m,t)\hat{B}(k_\ell,t)e^{-\mathrm{i}(k_m+k_\ell)x_o}\\ \mathbf{F}_{WL}(t) &= \frac{1}{2}\frac{\rho}{g}\int_{\mathrm{WL}}\phi_t^2\left(1-n_z^2\right)^{-1/2}\mathbf{n}\ \mathrm{d}l = \sum_m\sum_\ell \mathbf{Q}_{WL}(k_m,k_\ell)\ \hat{B}_t(k_m,t)\hat{B}_t(k_\ell,t)e^{-\mathrm{i}(k_m+k_\ell)x_o}\end{aligned} \tag{4}$$

with the QTFs being,

$$\mathbf{Q}_{QP} = -\frac{1}{2}\rho\int_{S_0}\nabla\hat{\phi}(k_m)\cdot\nabla\hat{\phi}(k_\ell)\ \mathbf{n}\ \mathrm{d}S \quad\text{and}\quad \mathbf{Q}_{WL} = \frac{1}{2}\frac{\rho}{g}\int_{\mathrm{WL}}\frac{\hat{\phi}(k_m)\hat{\phi}(k_\ell)}{\sqrt{1-n_z^2}}\mathbf{n}\ \mathrm{d}l. \tag{5}$$

Given that the double summation is computationally expensive, the eigenvalue decomposition method of the QTFs [5] is employed.

3 EXTENSION TO A MOVING BODY

If the body is free to move, a body-fixed reference frame $\mathbf{x} = [x,y,z]$ is defined and 6DoF motions with respect to $\tilde{\mathbf{x}}$ are considered as $\boldsymbol{\xi} = [\xi_1,\xi_2,\xi_3]$ for translations and $\boldsymbol{\alpha} = [\alpha_1,\alpha_2,\alpha_3]$ for rotations, with the generalized vector $\underline{\boldsymbol{\xi}} = [\boldsymbol{\xi},\boldsymbol{\alpha}]$. Due to the motion of the body, the radiation potential ϕ_R is introduced, along with an additional potential ϕ_M, whose role is to fulfil the second-order body boundary condition and will be discussed at the end of the section. Hence, the total potential will comprise

$$\phi = \phi_B + \phi_R + \phi_M = \phi_{RB} + \phi_M \tag{6}$$

where $\phi_{RB} = \phi_B+\phi_R = \phi_I+\phi_S+\phi_R$. To approximate the force on the instantaneous body surface, the velocity potential and normal vectors need to be Taylor-expanded around the mean body surface. It is noted that since the fully nonlinear incident potential ϕ_I is available, no distinction to orders for the potential itself has been made so far. However, to obtain a second-order consistent expression of the force, products of ϕ, $\boldsymbol{\xi}$ and $\boldsymbol{\alpha}$ of at least second order will be preserved. The position and unit normal vectors then will be necessary up to only the first order and are given below

$$\tilde{\mathbf{x}} = \mathbf{x}+\boldsymbol{\xi}+\boldsymbol{\alpha}\times\mathbf{x}+O(\varepsilon^2) \quad\text{and}\quad \tilde{\mathbf{n}} = \mathbf{n}+\boldsymbol{\alpha}\times\mathbf{n}+O(\varepsilon^2) \tag{7}$$

where ε is the order of magnitude for the leading order incident waves and body motion. Then the Taylor-expanded potential can be written as,

$$\phi(\tilde{\mathbf{x}}) = \phi(\mathbf{x}) + (\boldsymbol{\xi}+\boldsymbol{\alpha}\times\mathbf{x})\cdot\nabla\phi(\mathbf{x}) + O(\varepsilon^3) \tag{8}$$

From the body boundary condition $\tilde{\mathbf{n}}\cdot\nabla\phi(\tilde{\mathbf{x}}) = \dot{\tilde{\mathbf{x}}}\cdot\tilde{\mathbf{n}}$, it can be proven that the additional potential ϕ_M is of second order. Therefore, the second-order force will be

$$\begin{aligned}\mathbf{F} = &-\rho\int_{S_0}\left[\phi_t + (\boldsymbol{\xi}+\boldsymbol{\alpha}\times\mathbf{x})\cdot\nabla\phi_{RB,t}\right](\mathbf{n}+\boldsymbol{\alpha}\times\mathbf{n})\ \mathrm{d}S - \frac{1}{2}\rho\int_{S_0}\nabla\phi_{RB}\cdot\nabla\phi_{RB}\ \mathbf{n}\ \mathrm{d}S\\ &+\frac{1}{2}\rho g\int_{\mathrm{WL}}\left[\frac{1}{g}\phi_{RB,t} - (\xi_3+\alpha_1 y-\alpha_2 x)\right]^2\left(1-n_z^2\right)^{-1/2}\mathbf{n}\ \mathrm{d}l\end{aligned} \tag{9}$$

For the quadratic pressure and waterline integral terms, the analysis is similar to Eq. (4). Taking as an example the former and developing the square for each potential,

$$\mathbf{F}_{QP}^{M}(t) = -\frac{1}{2}\rho \int_{S_0} \nabla\phi_B \cdot \nabla\phi_B \ \mathbf{n} \ \mathrm{d}S - \rho \int_{S_0} \nabla\phi_B \cdot \nabla\phi_R \ \mathbf{n} \ \mathrm{d}S - \frac{1}{2}\rho \int_{S_0} \nabla\phi_R \cdot \nabla\phi_R \ \mathbf{n} \ \mathrm{d}S \tag{10}$$

Therefore, three components appear among which the first is identical to Eq. (4). For the other two, a similar QTF-based strategy is devised. Since the 6DoF radiation velocity transfer functions $\underline{\nabla\hat{\phi}_R}$ are available by the radiation-diffraction output, the impulse response functions $\underline{\mathbf{K}}(\tau)$, where τ denotes the time lag can be constructed. Then the radiation velocities can be evaluated as,

$$\nabla\phi_R = \int_\tau \underline{\mathbf{K}}(\tau) \cdot \underline{\dot{\boldsymbol{\xi}}}(t-\tau)\mathrm{d}\tau \quad \text{with} \quad \underline{\mathbf{K}}(\tau) = \frac{1}{2\pi}\int_\omega \underline{\nabla\hat{\phi}_R}(\omega) e^{-i\omega\tau}\mathrm{d}\omega \tag{11}$$

and the force in the second term can be evaluated as

$$\begin{aligned} \mathbf{F}_{QP}^{RB}(t) &= -\rho \sum_\ell \hat{B}(k_\ell, t) e^{-\mathrm{i}k_\ell x_o} \int_{S_0} \nabla\hat{\phi}_B(k_\ell) \cdot \left(\int_\tau \underline{\mathbf{K}}(\tau) \cdot \underline{\dot{\boldsymbol{\xi}}}(t-\tau)\mathrm{d}\tau \right) \ \mathbf{n} \ \mathrm{d}S \\ &= \sum_\ell \sum_m \hat{B}(k_\ell, t) e^{-\mathrm{i}k_\ell x_o} \ \underline{\mathbf{Q}}_{QP}^{RB}(k_\ell, \tau_m) \ \underline{\dot{\boldsymbol{\xi}}}(t-\tau_m) \end{aligned} \tag{12}$$

where underlined quantities denote vectors of all 6 DoFs. Through similar treatment, the third term of Eq. (10) is expressed as in Eq. (13), while the same approach can also be applied to the waterline integral. It is noted that ϕ_M does not contribute to those two terms, as it is of second order.

$$\mathbf{F}_{QP}^{RR}(t) = \sum_\ell \sum_m \underline{\dot{\boldsymbol{\xi}}}^T(t-\tau_\ell) \ \underline{\mathbf{Q}}_{QP}^{RR}(\tau_\ell, \tau_m) \ \underline{\dot{\boldsymbol{\xi}}}(t-\tau_m) \tag{13}$$

For the potential force, the terms up to the second order will be

$$\mathbf{F}_P(t) = -\rho \int_{S_0} (\phi_{RB,t} + \phi_{M,t}) \ \mathbf{n} \ \mathrm{d}S - \boldsymbol{\alpha} \times \rho \int_{S_0} \phi_{RB,t} \ \mathbf{n} \ \mathrm{d}S - \rho \int_{S_0} (\boldsymbol{\xi} + \boldsymbol{\alpha} \times \mathbf{x}) \cdot \nabla\phi_{RB,t} \ \mathbf{n} \ \mathrm{d}S \tag{14}$$

In the first and second integrals, the contribution from ϕ_B can be calculated through Eq. (3), while the contribution from ϕ_R can be obtained through convolution. The last term can be developed to obtain the following expression for the diffraction and radiation potentials respectively,

$$\begin{aligned} \mathbf{F}_{P3}^{B}(t) &= -\rho \sum_\ell \hat{B}_t(k_\ell, t) e^{-\mathrm{i}k_\ell x_o} \left[\boldsymbol{\xi} \cdot \int_{S_0} \nabla\hat{\phi}_B(k_\ell) \ \mathbf{n} \ \mathrm{d}S + \ \boldsymbol{\alpha} \cdot \int_{S_0} \left(\mathbf{x} \times \nabla\hat{\phi}_B(k_\ell) \right) \ \mathbf{n} \ \mathrm{d}S \right] \\ \mathbf{F}_{P3}^{R}(t) &= -\rho \ \boldsymbol{\xi} \cdot \int_\tau \left(\int_{S_0} \underline{\mathbf{K}}(\tau) \ \mathbf{n} \ \mathrm{d}S \right) \cdot \underline{\dot{\boldsymbol{\xi}}}(t-\tau)\mathrm{d}\tau - \rho \ \boldsymbol{\alpha} \cdot \int_\tau \left(\int_{S_0} \mathbf{x} \times \underline{\mathbf{K}}(\tau) \ \mathbf{n} \ \mathrm{d}S \right) \cdot \underline{\dot{\boldsymbol{\xi}}}(t-\tau)\mathrm{d}\tau \end{aligned} \tag{15}$$

The former term can be directly calculated with known quantities, while the second also degenerates to convolution terms. Regarding the contribution of ϕ_M to the potential force, it can be calculated by determining only its normal derivative using Green's theorem,

$$\mathbf{F}_P^M(t) = -\rho \int_{S_0} \phi_{M,t} \ \mathbf{n} \ \mathrm{d}S = -\rho \int_{S_0} \frac{\partial\phi_{M,t}}{\partial n} \underline{\hat{\phi}_j} \ \mathrm{d}S, \tag{16}$$

where $\underline{\hat{\phi}_j}$ is the 6DoF unit amplitude radiation potential. The normal derivative of ϕ_M can be evaluated through the body boundary condition. This term is not yet included in the results below but is part of our ongoing work. Throughout the abstract, only the derivation of the forces has been demonstrated, but for moments the analysis is analogous.

4 RESULTS

The forces obtained from Eq. (9) are used in a time-domain code [6] to simulate the motions of a containership for which model test results are available [7]. For a design wave and an irregular JONSWAP sea state ($H_s = 0.154$ m, $T_p = 1.74$ m) the surge motion time series are shown in Fig. 1, along with the probability of exceedance (POE) curves for the latter. The numerical results obtained with the proposed force model are labelled as Force-HOS. Moreover, a standard force QTF is applied to the linearized wave amplitudes and the loads are used in the time-domain code. The relevant results are denoted as Hydrostar after the employed solver [8]. It is demonstrated in the two cases investigated, that the vessel's motion is captured more accurately under the proposed approach, which is principally attributed to the calculated second-order motions being implicitly used in Eq. (9), through a fourth-order Runge-Kutta scheme, and the fully nonlinear wave field calculated by the HOS model. The QTF approach with linear waves, although computationally more efficient, evaluates the force through consideration of the linear motions. Thus, when second-order motions are significant, as in the case of the large amplitude slow-drift oscillations of this test, the instantaneous position is under-predicted. The same result is also depicted in the POE plots where Force-HOS shows better agreement with the experimental results.

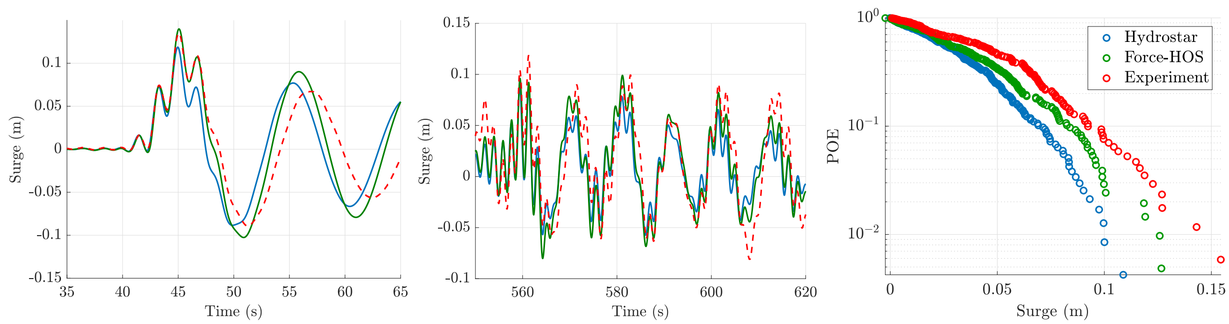

Figure 1: Surge time series in design wave (left), irregular wave (middle) and surge POE (right).

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