A new approach to evaluate second-order wave loads on three-dimensional floating bodies

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Introduction

The second-order wave loads can manifest across a broad range of wave frequencies and are prone to approaching the resonance frequencies associated with the motion of floating structures. This phenomenon can give rise to issues such as low-frequency slow drift and high-frequency springing. The assessment of wave loads on large-scale offshore structures predominantly relies on a combination of potential-flow theory and perturbation expansion techniques. The complete second-order wave excitation loads, referred to as $\mathbf{F}^{(2)}$, can be divided into two parts. That is

$$\mathbf{F}^{(2)} = \mathbf{F}_q^{(2)} + \mathbf{F}_p^{(2)}. \tag{1}$$

$\mathbf{F}_q^{(2)}$ pertains to the quadratic terms in the hydrodynamic pressure and the correction terms arising from the application of perturbation expansion; $\mathbf{F}_p^{(2)}$ is closely related to the pressure terms owing to the second-order velocity potential.

In comparison to linear wave loads, the computation of second-order wave loads presents greater challenges in achieving converged results. A primary factor contributing to this difficulty is the necessity to resolve the velocity and acceleration of fluid particles, which are represented as spatial derivatives of the velocity potential across various orders. It is well established that the velocity and acceleration of fluid particles exhibit pronounced singularities in regions characterized by abrupt geometric changes, such as at tips and edges, thereby complicating accurate calculations. For the time-independent mean drift wave loads, calculation formulas that mitigate the impact of these singularities have been proposed, including the far- and middle-field formulas[1, 2, 3]. The works[1, 2, 3] primarily concentrate on $\mathbf{F}_q^{(2)}$, whereas our research expands on this by incorporating the complete loads, including both $\mathbf{F}_p^{(2)}$ and $\mathbf{F}_q^{(2)}$. The oscillatory nature of wave loads induces resonant motion of floating bodies. As a result, the advancement of computational methods capable of addressing the aforementioned singularities associated with second-order wave loads that exhibit temporal oscillation is of considerable significance. This study seeks to contribute to this field.

Calculation of the second-order wave forces using a novel approach

This study will examine the scenario of an object in translational motion, with a particular emphasis on the computation of wave force. Then, $\mathbf{F}_q^{(2)}$ can be represented as follows:

$$\mathbf{F}_q^{(2)} = -\rho \iint_{S_b} \left[\frac{1}{2}\left(\nabla\Phi^{(1)}\right)^2 + \nabla\frac{\partial\Phi^{(1)}}{\partial t}\cdot\boldsymbol{\Xi}^{(1)} \right]\mathbf{n}\mathrm{d}s + \frac{1}{2}\rho g \oint_{\Gamma_w}\left(\zeta^{(1)}\zeta^{(1)} - 2\zeta^{(1)}\Xi_3^{(1)}\right)\mathbf{n}\mathrm{d}l. \tag{2}$$

In Eq. (2), the hulls, characterized by vertical sides at the waterline, are concerned. In addition, the superscripts (1) and (2) indicate quantities of the first and second order with respect to wave steepness; $\Phi^{(1)}$, $\boldsymbol{\Xi}^{(1)}$ and $\zeta^{(1)}$ represent the first-order velocity potential, translational displacement, and water-surface elevation, respectively; S_b is the mean wetted body surface; Γ_w is the mean waterline; $\mathbf{n} = (n_1, n_2, n_3)^T$ is the normal vector on the body surface, with the positive orientation directed outward from the fluid domain. A control domain, denoted as Ω_c, is defined around the body. The boundaries of Ω_c are represented by S_b, S_c, and S'_f. Here, S_c is the control surface at a specified distance from the body, while S'_f is the free-surface area that is enclosed between the intersection line of S_b and S_c with the water surface. By using Gauss's theorem within Ω_c, Eq. (2) can be reformulated as follows:

$$\begin{aligned}\mathbf{F}_q^{(2)} = &-\rho \iint_{S_b}\left(\nabla\frac{\partial\Phi^{(1)}}{\partial t}\cdot\boldsymbol{\Xi}^{(1)}\right)\mathbf{n}\mathrm{d}s + \frac{1}{2}\rho g\oint_{\Gamma_w}\left(\zeta^{(1)}\zeta^{(1)} - 2\zeta^{(1)}\Xi_3^{(1)}\right)\mathbf{n}\mathrm{d}l \\ &-\rho\left\{\iint_{S_b}\left(\frac{\partial\Phi^{(1)}}{\partial n}\nabla\Phi^{(1)}\right)\mathrm{d}s + \frac{1}{2}\iint_{S_c\cup S'_f}\left[2\frac{\partial\Phi^{(1)}}{\partial n}\nabla\Phi^{(1)} - \left(\nabla\Phi^{(1)}\cdot\nabla\Phi^{(1)}\right)\mathbf{n}\right]\mathrm{d}s\right\}.\end{aligned} \tag{3}$$

In contrast to Eq. (2), the calculation of the quadratic term associated with the fluid velocity across the body surface is no longer required in Eq. (3).

Subsequently, we will conduct calculations on $\mathbf{F}_p^{(2)}$, which can be represented in the following form:

$$\mathbf{F}_p^{(2)} = -\rho \iint_{S_b} \left(\frac{\partial \Phi_I^{(2)}}{\partial t} + \frac{\partial \Phi_D^{(2)}}{\partial t} \right) \mathbf{n} \mathrm{d}s, \tag{4}$$

in which, $\Phi_I^{(2)}$ and $\Phi_D^{(2)}$ denote the second-order incident and diffraction potentials, respectively. An auxiliary spatial potential, referred to as ψ_k (where k = 1, 2, 3), is introduced. ψ_k satisfies a no-flow condition on the horizontal sea bed. On the boundary S_b, ψ_k satisfies the following boundary condition:

$$\frac{\partial \psi_k}{\partial n} = n_k. \tag{5}$$

In addition, ψ_k also needs to satisfy specific free-surface and far-field boundary conditions, which will be elaborated upon subsequently. By employing Green's second theorem across the entire fluid domain, we can derive:

$$\iint_{S_b} \left(\Phi_I^{(2)} + \Phi_D^{(2)} \right) n_k \mathrm{d}s = \iint_{S_b} \left(\Phi_I^{(2)} n_k - \psi_k \frac{\partial \Phi_I^{(2)}}{\partial n} \right) \mathrm{d}s + \iint_{S_b} \psi_k B^{(2)} \mathrm{d}s - \iint_{S_f \cup S_\infty} \left(\Phi_D^{(2)} \frac{\partial \psi_k}{\partial n} - \psi_k \frac{\partial \Phi_D^{(2)}}{\partial n} \right) \mathrm{d}s. \tag{6}$$

in which, S_∞ denotes a cylindrical surface situated in the far field, with its radius approaching infinity; $B^{(2)}$ signifies the body-surface forcing term. Using Eqs. (4) and (6), we proceed with the following decomposition:

$$\mathbf{F}_p^{(2)} = \mathbf{F}_I^{(2)} + \mathbf{F}_D^{(2)} + \mathbf{F}_f^{(2)} + \mathbf{F}_b^{(2)}, \tag{7}$$

in which,

$$F_{I,k}^{(2)} = -\rho \iint_{S_b} \frac{\partial \Phi_I^{(2)}}{\partial t} n_k \mathrm{d}s; \quad F_{D,k}^{(2)} = \rho \iint_{S_b} \psi_k \frac{\partial^2 \Phi_I^{(2)}}{\partial t \partial n} \mathrm{d}s; \quad F_{f,k}^{(2)} = \rho \iint_{S_f \cup S_\infty} \left(\frac{\partial \Phi_D^{(2)}}{\partial t} \frac{\partial \psi_k}{\partial n} - \psi_k \frac{\partial^2 \Phi_D^{(2)}}{\partial t \partial n} \right) \mathrm{d}s; \quad F_{b,k}^{(2)} = -\rho \frac{\partial}{\partial t} \left(\iint_{S_b} \psi_k B^{(2)} \mathrm{d}s \right). \tag{8}$$

In Eq. (8), the subscript k denotes the force component in the k-th direction. The computations for $F_{I,k}^{(2)}$ and $F_{D,k}^{(2)}$ resemble those of the first-order problem. In the context of the time-domain computation for $F_{f,k}^{(2)}$, the incorporation of a damping layer on the free surface allows for a reduction in the computational domain to a confined free-surface area surrounding the body.

For a body in translational motion, the surface integral involving $B^{(2)}$ in Eq. (8) can be represented as follows:

$$\iint_{S_b} \psi_k B^{(2)} \mathrm{d}s = -\iint_{S_b} \left\{ \psi_k \left[\left(\mathbf{\Xi}^{(1)} \cdot \nabla \right) \nabla \Phi^{(1)} \cdot \mathbf{n} \right] \right\} \mathrm{d}s. \tag{9}$$

Following Cong et al. (2020)[4], by applying Stokes' theorem and Gauss' theorem, $F_{b,k}^{(2)}$ can be expressed as:

$$\begin{aligned} F_{b,k}^{(2)} = {} & \rho \oint_{\Gamma_w} \left[\psi_k \frac{\partial}{\partial t} \left(\nabla \Phi^{(1)} \times \mathbf{\Xi}^{(1)} \right) \right] \cdot \mathrm{d}\mathbf{l} + \rho \iint_{S_b} \left[n_k \frac{\partial}{\partial t} \left(\nabla \Phi^{(1)} \cdot \mathbf{\Xi}^{(1)} \right) \right] \mathrm{d}s \\ & + \rho \frac{\partial}{\partial t} \left\{ \iint_{S_c \cup S_f'} \left[\frac{\partial \Phi^{(1)}}{\partial n} \left(\nabla \psi_k \cdot \mathbf{\Xi}^{(1)} \right) + \frac{\partial \psi_k}{\partial n} \left(\nabla \Phi^{(1)} \cdot \mathbf{\Xi}^{(1)} \right) - \left(\nabla \Phi^{(1)} \cdot \nabla \psi_k \right) \left(\mathbf{n} \cdot \mathbf{\Xi}^{(1)} \right) \right] \mathrm{d}s \right\}. \end{aligned} \tag{10}$$

In the preceding discussion, we have reformulated the expressions for $\mathbf{F}_q^{(2)}$ and $\mathbf{F}_p^{(2)}$, resulting in formulas that exclude the square terms associated with the derivatives of the velocity potential over S_b, as well as the higher-order spatial derivative terms over S_b. This modification has partially alleviated the effects of the singularity in fluid particle velocity at critical locations, such as at tips and edges. Nevertheless, the revised expressions still incorporate integral terms that exhibit a linear relationship with the fluid particle velocity over S_b, indicating that the singularity's influence has not been entirely eradicated. However, it is noteworthy that the body-surface integrals in Eqs. (3) and (10) can cancel each other out. Then, we define $\mathbf{F}_{bq}^{(2)} = \mathbf{F}_b^{(2)} + \mathbf{F}_q^{(2)}$, allowing for the complete second-order wave excitation force to be presented in the following form:

$$\mathbf{F}^{(2)} = \mathbf{F}_I^{(2)} + \mathbf{F}_D^{(2)} + \mathbf{F}_f^{(2)} + \mathbf{F}_{bq}^{(2)}. \tag{11}$$

in which,

$$F_{bq,k}^{(2)}=\rho\frac{\partial}{\partial t}\left\{\oint_{\Gamma_w}\left[\left(\frac{\partial\boldsymbol{\Xi}^{(1)}}{\partial t}\cdot\mathbf{n}\right)\Xi_3^{(1)}-\frac{\partial\zeta^{(1)}}{\partial t}\left(\boldsymbol{\Xi}^{(1)}\cdot\mathbf{n}\right)\right]\psi_k\mathrm{d}l\right\}+\frac{1}{2}\rho g\oint_{\Gamma_w}\left(\zeta^{(1)}\zeta^{(1)}-2\zeta^{(1)}\Xi_3^{(1)}\right)n_k\mathrm{d}l$$
$$+\rho\oint_{\Gamma_w}\Phi\left(\frac{\partial\boldsymbol{\Xi}^{(1)}}{\partial t}\times\mathrm{d}\mathbf{l}\right)\cdot\mathbf{e}_k-\frac{\rho}{2}\iint_{S_c\cup S_f'}\left[2\frac{\partial\Phi^{(1)}}{\partial n}\left(\nabla\Phi^{(1)}\cdot\mathbf{e}_k\right)-\left(\nabla\Phi^{(1)}\cdot\nabla\Phi^{(1)}\right)n_k\right]\mathrm{d}s \tag{12}$$
$$+\rho\frac{\partial}{\partial t}\left\{\iint_{S_c\cup S_f'}\left[\frac{\partial\Phi^{(1)}}{\partial n}\left(\nabla\psi_k\cdot\boldsymbol{\Xi}^{(1)}\right)+\frac{\partial\psi_k}{\partial n}\left(\nabla\Phi^{(1)}\cdot\boldsymbol{\Xi}^{(1)}\right)-\left(\nabla\Phi^{(1)}\cdot\nabla\psi_k\right)\left(\mathbf{n}\cdot\boldsymbol{\Xi}^{(1)}\right)\right]\mathrm{d}s\right\},$$

in which, $\mathbf{e}_k$ is the unit vector in the k-th direction. Eqs. (8), (11), and (12) are applicable for computation of second-order wave forces in the time domain, remaining unaffected by the singularity associated with fluid particle velocity.

Subsequently, the second-order bi-chromatic incident waves with frequencies ω_1 and ω_2 are considered. In this context, the second-order wave forces can be expressed as:

$$\mathbf{F}^{(2)}=\sum_{j=1}^{2}\sum_{l=1}^{2}\left(\mathbf{F}_{jl}^{(2)+}+\mathbf{F}_{jl}^{(2)-}\right), \tag{13}$$

in which, the superscripts "+" and "–" denote the force components associated with the sum frequency and difference frequency, respectively. Similar to Eq. (11), the following decomposition can be made:

$$\mathbf{F}_{jl}^{(2)\pm}=\mathbf{F}_{I,jl}^{(2)\pm}+\mathbf{F}_{D,jl}^{(2)\pm}+\mathbf{F}_{f,jl}^{(2)\pm}+\mathbf{F}_{bq,jl}^{(2)\pm}. \tag{14}$$

Under steady state, it is possible to isolate the temporal components. That is

$$\mathbf{F}_{(I)(D)(f)(bq),jl}^{(2)+}=\mathrm{Re}\left[A_jA_l\mathbf{f}_{(I)(D)(f)(bq),jl}^{(2)+}\mathrm{e}^{-\mathrm{i}(\omega_j+\omega_l)t}\right]; \tag{15a}$$

$$\mathbf{F}_{(I)(D)(f)(bq),jl}^{(2)-}=\mathrm{Re}\left[A_jA_l^*\mathbf{f}_{(I)(D)(f)(bq),jl}^{(2)-}\mathrm{e}^{-\mathrm{i}(\omega_j-\omega_l)t}\right]. \tag{15b}$$

In Eq. (15), the superscript * signifies the complex conjugate. $\mathbf{f}_{jl}^{(2)\pm}$ denotes the amplitude of the second-order sum- and difference-frequency wave force under the action of bi-chromatic incident waves with unit amplitude, commonly referred to as the wave force quadratic transfer function (QTF). It satisfies

$$\mathbf{f}_{jl}^{(2)\pm}=\mathbf{f}_{I,jl}^{(2)\pm}+\mathbf{f}_{D,jl}^{(2)\pm}+\mathbf{f}_{f,jl}^{(2)\pm}+\mathbf{f}_{bq,jl}^{(2)\pm}, \tag{16}$$

in which, $\mathbf{f}_{I,jl}^{(2)\pm}$, $\mathbf{f}_{D,jl}^{(2)\pm}$, $\mathbf{f}_{f,jl}^{(2)\pm}$, and $\mathbf{f}_{bq,jl}^{(2)\pm}$ are the QTFs of the wave force components. They can be expressed as:

$$\mathbf{f}_{I,jl}^{(2)\pm}=\mathrm{i}\omega_{jl}^{\pm}\rho\iint_{S_b}\phi_{I,jl}^{(2)\pm}n_k\mathrm{d}s;\quad \mathbf{f}_{D,jl}^{(2)\pm}=-\mathrm{i}\omega_{jl}^{\pm}\rho\iint_{S_b}\psi_k\frac{\partial\phi_{I,jl}^{(2)\pm}}{\partial n}\mathrm{d}s;\quad \mathbf{f}_{f,jl}^{(2)\pm}=\frac{\mathrm{i}\omega_{jl}^{\pm}\rho}{g}\iint_{S_f}q_{jl}^{(2)\pm}\psi_k\mathrm{d}s. \tag{17}$$

In Eq. (17), $\omega_{jl}^{\pm}=\omega_j\pm\omega_l$; $\phi_{I,jl}^{(2)\pm}$ and $q_{jl}^{(2)\pm}$ represent the second-order complex spatial incident potentials of the sum- and difference-frequency, as well as the free-surface forcing term. In the sum- and difference-frequency problems, the radiation potential induced by the simple harmonic body motion with the frequency $\omega_{jl}^{\pm}$ in the k-th direction is used as ψ_k. The calculations for $\mathbf{f}_{I,jl}^{(2)\pm}$ and $\mathbf{f}_{D,jl}^{(2)\pm}$ can be conducted directly and are relatively straightforward. The calculation of $\mathbf{f}_{f,jl}^{(2)\pm}$ should be conducted over the entire free surface, adhering to the weak scattering assumption of the second-order diffraction potential. It can be achieved through a combination of numerical discretization, characteristic function expansion, and asymptotic expansion[5]. $\mathbf{f}_{bq,jl}^{(2)\pm}$ can be expressed in the following form:

$$f_{bq,jl,k}^{(2)+}=H_{bq,jl,k}^{(2)+}+H_{bq,lj,k}^{(2)+};\quad f_{bq,jl,k}^{(2)-}=H_{bq,jl,k}^{(2)-}+H_{bq,lj,k}^{(2)-*}, \tag{18}$$

in which,

$$H_{bq,jl,k}^{(2)+}=\frac{\rho g}{4}\left\{\oint_{\Gamma_w}\left(\frac{1}{2}\eta_l^{(1)}-\xi_{l,3}^{(1)}\right)\eta_j^{(1)}n_k\mathrm{d}l-\frac{1}{g}\iint_{S_c\cup S_f'}\left[\frac{\partial\phi_j^{(1)}}{\partial n}\left(\nabla\phi_l^{(1)}\cdot\mathbf{e}_k\right)-\frac{1}{2}\left(\nabla\phi_l^{(1)}\cdot\nabla\phi_j^{(1)}\right)n_k\right]\mathrm{d}s-\frac{\mathrm{i}\omega_l}{g}\oint_{\Gamma_w}\phi_j^{(1)}\left(\xi_l^{(1)}\times\mathrm{d}\mathbf{l}\right)\cdot\mathbf{e}_k\right\}$$
$$-\frac{\mathrm{i}\omega_{jl}^{+}\rho}{4}\xi_l^{(1)}\cdot\left\{\iint_{S_c\cup S_f'}\left[\frac{\partial\phi_j^{(1)}}{\partial n}\nabla\psi_k+\frac{\partial\psi_k}{\partial n}\nabla\phi_j^{(1)}-\left(\nabla\phi_j^{(1)}\cdot\nabla\psi_k\right)\mathbf{n}\right]\mathrm{d}s+\oint_{\Gamma_w}\left(\mathrm{i}\omega_j\eta_j^{(1)}-\mathrm{i}\omega_l\xi_{j,3}^{(1)}\right)\psi_k\mathbf{n}\mathrm{d}l\right\}; \tag{19a}$$

$$H_{bq,\,jl,\,k}^{(2)-}=\frac{\rho g}{4}\left\{\oint_{\Gamma_w}\left(\frac{1}{2}\eta_l^{(1)*}-\xi_{l,\,3}^{(1)*}\right)\eta_j^{(1)}n_k\mathrm{d}l-\frac{1}{g}\iint_{S_c\cup S_f'}\left[\frac{\partial\phi_j^{(1)}}{\partial n}\left(\nabla\phi_l^{(1)*}\cdot\mathbf{e}_k\right)-\frac{1}{2}\left(\nabla\phi_l^{(1)*}\cdot\nabla\phi_j^{(1)}\right)n_k\right]\mathrm{d}s+\frac{\mathrm{i}\omega_l}{g}\oint_{\Gamma_w}\phi_j^{(1)}\left(\boldsymbol{\xi}_l^{(1)*}\times\mathrm{d}\mathbf{l}\right)\cdot\mathbf{e}_k\right\}$$
$$-\frac{\mathrm{i}\omega_{jl}^{-}\rho}{4}\boldsymbol{\xi}_l^{(1)*}\cdot\left\{\iint_{S_c\cup S_f'}\left[\frac{\partial\phi_j^{(1)}}{\partial n}\nabla\psi_k+\frac{\partial\psi_k}{\partial n}\nabla\phi_j^{(1)}-\left(\nabla\phi_j^{(1)}\cdot\nabla\psi_k\right)\mathbf{n}\right]\mathrm{d}s+\oint_{\Gamma_w}\left(\mathrm{i}\omega_j\eta_j^{(1)}+\mathrm{i}\omega_l\xi_{j,\,3}^{(1)}\right)\psi_k\mathbf{n}\mathrm{d}l\right\}. \tag{19b}$$

Eqs. (16) to (19) represent refined formulas employed to determine the QTF of the complete wave forces, devoid of the influence of singularities associated with fluid particle velocities over the body surface.

Results and discussion

The numerical analysis is conducted utilizing Eqs. (16) to (19) in conjunction with a higher-order boundary element method. For bodies characterized by smooth surfaces, such as hemispheres, the results based on the present method are consistent with the published results[5, 6]. A comprehensive presentation of these results will be provided at the workshop. A truncated cylinder characterized by a radius of a = 1 m and a draft of $2a$ is then specifically examined. The water depth is $5a$. The cylinder experiences single degree of freedom motions, specifically surge, in response to wave action. In the computation, the parameters N_θ = 10, N_z = 12, and N_r = 6 are employed to regulate the mesh discretization of the body surface across the annular, vertical, and radial dimensions. It is noteworthy that mesh discretization is necessary solely within one-quarter of the body surface area. For the velocity potential in the remaining regions, the symmetry characteristics of the Green's function is used to construct a transformation matrix, thereby facilitating the solution process without the necessity of addressing the boundary integral equations directly.

In the computation of wave force QTFs, it is essential to perform mesh discretization on the control surface. For this analysis, a cylindrical surface along with a circular bottom is selected as the control surface, characterized by a radius of $1.5a$ and a draft of $2.5a$. The mesh discretization applied to the control surface employs the same control parameters as those used on the body surface. Tables 1 and 2 illustrate the calculation results of $\mathbf{f}_{bq,\,jl}^{(2)\pm}$ and $\mathbf{f}_{jl}^{(2)\pm}$ under various frequency combinations. Notably, $\mathbf{f}_{bq,\,jl}^{(2)\pm}$ becomes increasingly important with the increase in the sum of ω_j and ω_l, gradually dominating the total force.

Table 1. Sum-frequency QTF results for different combinations of (ω_j, ω_l), where the units of ω_j and ω_l are rad/s.

	(1.0, 0.7)	(1.3, 1.0)	(1.6, 1.3)	(1.9, 1.6)	(2.2, 1.9)
$\lvert f_{bq,\,jl,\,1}^{(2)+}/(\rho g a)\rvert$	0.1163	0.1908	0.3018	0.4233	0.5408
$\lvert f_{jl,\,1}^{(2)+}/(\rho g a)\rvert$	1.1395	0.8129	0.5485	0.3435	0.2179

Table 2. Analogous results to those in Table 1 but for difference-frequency problem

	(1.0, 0.7)	(1.3, 1.0)	(1.6, 1.3)	(1.9, 1.6)	(2.2, 1.9)
$\lvert f_{bq,\,jl,\,1}^{(2)-}/(\rho g a)\rvert$	0.0913	0.0962	0.1047	0.1166	0.1323
$\lvert f_{jl,\,1}^{(2)-}/(\rho g a)\rvert$	0.1232	0.0308	0.0171	0.0427	0.0646

This article mainly discusses the wave forces acting on bodies undergoing translational motion. The proposed approach is also applicable to bodies in rotational motion and the wave moments acting on them. Additionally, it is applicable to hulls that are non-wall-sided at the waterline. Given the constraints on the article's length, the derivation process will not be detailed here and will be presented at the workshop.

Reference

1. Maruo H (1960) The drift of a body floating on waves. J Ship Res 4:1–10
2. Newman JN (1967) The drift force and moment on ships in waves. J Ship Res 11:51–602.
3. Chen X B (2007) Middle-field formulation for the computation of wave-drift loads. J Eng Math 59(1): 61-82.
4. Cong P W, Teng B, Chen L F, et al. (2020) A novel solution to the second-order wave radiation force on an oscillating truncated cylinder based on the application of control surfaces. Ocean Eng 204: 107278.
5. Teng B, Cong P W (2017) A novel decomposition of the quadratic transfer function (QTF) for the time-domain simulation of non-linear wave forces on floating bodies. Appl Ocean Res 65: 112-128.
6. Kim M H, Yue D K P (1990) The complete second-order diffraction solution for an axisymmetric body Part 2. Bichromatic incident waves and body motions. J Fluid Mech, 1990, 211: 557-593.