Broadband energy attenuation of long-period water waves by a graded array of C-shaped cylinders

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1 INTRODUCTION

Waves that have a wavelength comparable to or larger than ocean structure exhibit stronger energy transmission, and it has been well established that conventional (both bottom-fixed and floating) breakwaters fail to attenuate these long-period water waves effectively [1]. Recently, coastal protection measures based on wave resonance incurred by multiple structures have been proposed to block waves of specific frequencies [2]. These resonance phenomena include Bragg resonance over periodic structures [3] and rainbow trapping for graded arrays [4-8].

Building on the above pioneering research, this work designs a novel breakwater consisting of an array of C-shaped cylinders, aiming to block broad-banded long random waves both effectively and economically. The so-called local resonance is incurred to block a long wave that has a wavelength several times larger than the cylinder diameter, i.e. the C-shaped cylinder is a sub-wavelength resonator. Particularly, the broadband energy attenuation is achieved by grading the cylinder radii (rather than the spacing as widely investigated), such that the local resonance for each spectral component of a random wave would occur alternatively at different spatial locations in the array. The local resonance is associated with water inside the C-shaped cavity oscillating at specific frequency (i.e. the so-called resonant frequency), radiating waves out to interact with incident waves upstream constructively/ destructively [9].

The dimension of each cylinder component in the array is selected carefully based on the so-called Bloch band theory, which was first proposed in solid mechanics [10]. Later, the theory has been applied to evaluate propagation characteristics of water waves over periodic structures, represented by the so-called band diagram [11]. To consider more complex scenarios (*e.g.* truncated C-shaped cylinders), here we extend the model in [12] to a component structure of arbitrary configuration following the approach presented in [13]. Instead of the Helmholtz equation in [12], the model now solves the 3-D Laplace equation and uses the eigenvalue method to obtain the band diagram within the framework of linear wave theory.

This present work carried out a set of physical experiments to evaluate the wave attenuation performance of the proposed 'Resonance Breakwater' for ranges of regular and irregular waves. In particular, spatial distributions of wave energy are measured and analyzed, revealing inherent characteristics and nonlinear behaviors of various resonances that are present. In addition, the effects of *e.g.* the cylinder draft, the wave incident angle and the total number of cylinders in the array are explored.

2 EXPERIMENTAL ANALYSIS

2.1 Experimental design and setup

A set of physical experiments were designed and carried out in a wave flume (69 m × 2 m × 1.8 m) at Dalian University of Technology (DUT), China to verify the proposed concept of 'Resonance

Breakwater'. The water depth was constant at 0.62 m with a Froude scaling of 1:20. In line with actual operational conditions in long wave dominated seas, the breakwater was designed to block waves of a period ranging from 6 s to 10 s (1.3 - 2.2 s at model scale and the results are presented at model scale hereafter) by at least 50%. This is achieved via the band diagram (see Fig. 1 (a)) calculated based on the Bloch band theory as mentioned above. The blue shaded areas in Fig. 1 (a) are band gaps informing the selections of cylinder diameters; four or six cylinders with a grading of 0.05 m in diameter were considered accordingly. The diameter of the first cylinder was 0.4 m. In this work, we kept other cylinder dimensions in the array unchanged, *i.e.* the wall thickness was constant at 0.01 m, and the opening length *ln* = 0.10 m. The local cylinder spacing was 1.2 m ensuring that the Bragg resonance would occur at about 1.3 s, and the periodicity in the transverse direction matches the flume width (1.2 m; the width of a larger section of the flume divided by a longitudinal steal plate). The detailed arrangements see Fig. 1 (a) and Fig. 1 (b).

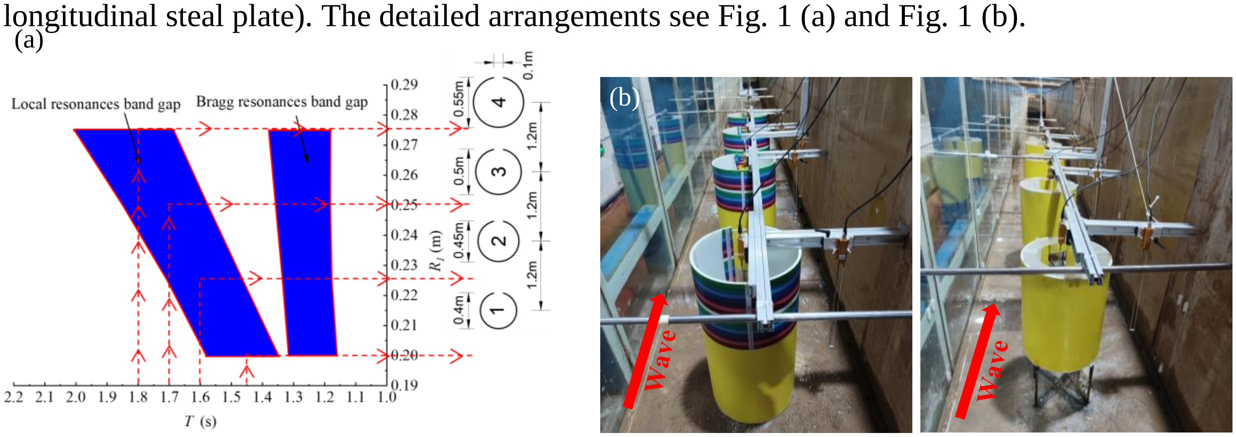

Fig. 1: (a) Band diagram and selection of cylinder diameters; (b) Photographs of the model bottom-mounted or truncated cylinders.

As discussed, a set of regular waves with a wave height of 0.05 m and a period ranging from 1.1 s to 2.3 s at an interval of 0.1 s were generated to interact with the cylinder array. Irregular waves with a peak period ranging from 1.4 s to 2.0 s with an increment of 0.2 s were also tested to investigate the overall spectral structure of the response, further evaluating the broad-banded wave attenuation capability. The P-M spectrum was used and the significant wave height was constant at 0.05 m. In addition, a series of the draft (about 1/3, 1/2, and 2/3 of the water depth) and wave incidence (0°, 45°, 90° with 0° being the opening on the back stagnation points) were considered.

2.2 Broad-banded wave attenuation performance

The overall wave attenuation performance is firstly checked by comparing the wave energy distribution in front of and after the cylinder array, as shown in Fig. 2. For regular waves, the reflection and transmission coefficients were calculated based on the two-point method proposed by Goda and Suzuki [14] - see Fig. 2 (a). We note that the technique of Goda and Suzuki may introduce certain errors in separating incident and reflected waves due to the complex, highly distorted local wave field, but would still be informative when considering the overall trend. For irregular waves, Fig. 2 (b) compares the incident wave spectrum to the transmitted wave spectrum. The former was measured at the location in between the second and the third cylinders without the array in presence (open circle in Fig. 2 (b)) and the latter measured at a location 2.4 m after the cylinder array (red solid circle in Fig. 2 (b)). The orange and the purple shades are the predicted frequency ranges for Bragg resonance and rainbow trapping (resulting from the local resonances occur at different spatial locations), respectively. It can be seen from Fig. 2 that the wave energies in the predicted band gaps are indeed attenuated, leading to small wave transmissions downstream. These observations together with the photos from the experiments (Fig. 3) confirm the energy attenuation capability of the proposed ‘Resonance Breakwater’.

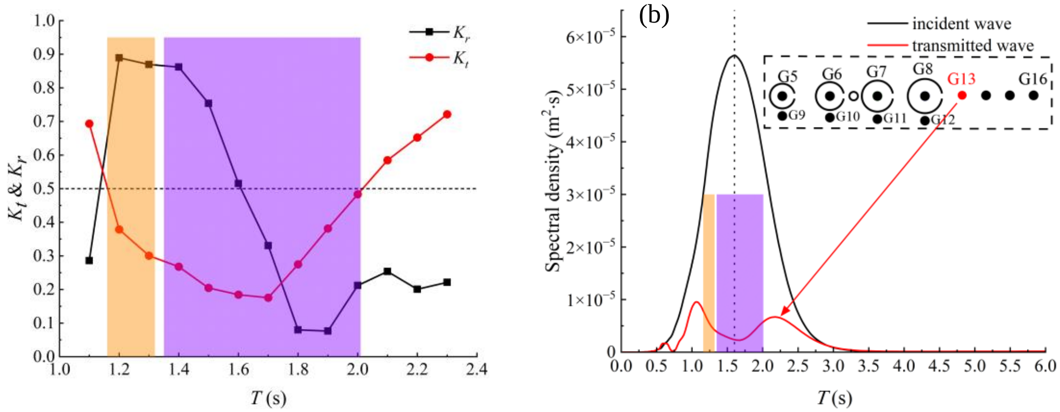

Fig. 2: Example results for the four bottom-mounted C-shaped cylinders. (a) Regular waves with a wave height of 0.05 m; (b) Irregular waves with T_p = 1.6 s and H_s = 0.05 m.

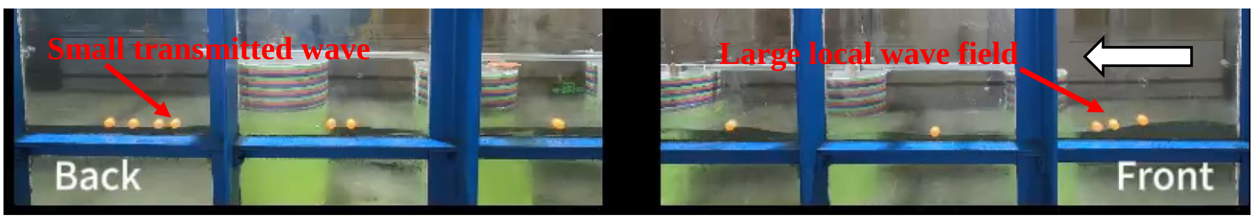

Fig. 3 Photos from the experiments, showing a significant wave energy in front of the cylinder array (right) and a relatively calm water behind the cylinder array (left).

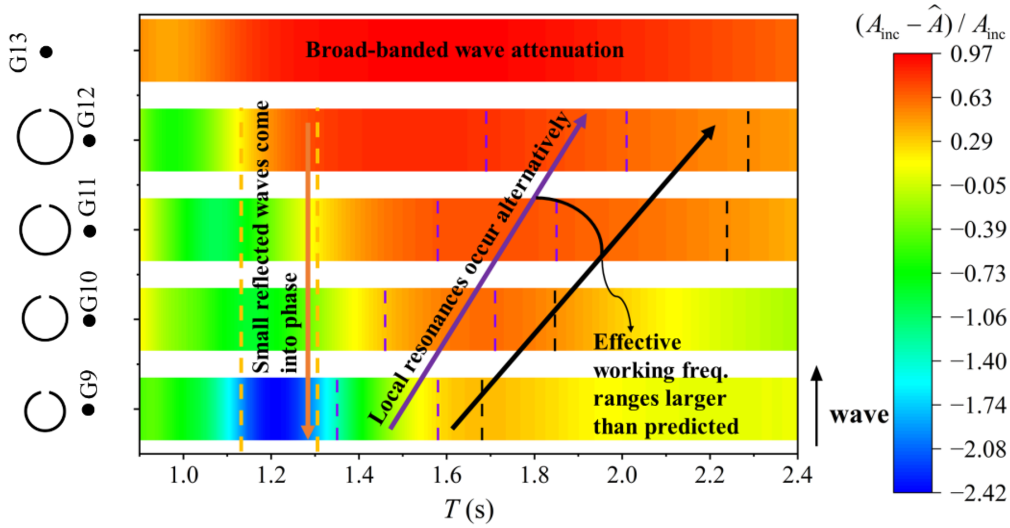

Fig. 4 Spatial distribution of wave energy along and after the array. Positive and negative values indicate the occurrences of wave attenuation and wave amplification, respectively.

To investigate the contributions of each type of resonances that are present and from each cylinder component, Fig. 4 illustrates the spatial distribution of wave energy for an irregular wave with T_p = 1.6 s and H_s = 0.05 m. Four bottom-mounted C-shaped cylinders were considered. Here, A_{inc} represents the incident wave spectrum as in Fig. 2 (b), and $\hat{A}$ are the local wave spectra measured at locations indicated by the symbols shown on the left sketch. That is, the positive and negative values indicate the occurrences of wave attenuation and amplification, respectively. As in Fig. 2, the orange dashed lines in Fig. 4 are the predicted frequency ranges for Bragg resonance and purple dashed lines are the predicted local resonant frequencies for each cylinder component.

As predicted, significant wave attenuations are captured to occur alternatively along the array, corresponding to the mechanism of local resonances occurring alternatively to form the so-called rainbow trapping. In addition, it can be seen that the actual effective working frequency ranges (wave attenuation larger than 0.5; represented by black dashed lines) for each component cylinder are larger than predicted. This may be due to *e.g.* the wave nonlinearity and viscous effects involved, which are not represented by the linear wave theory. These may serve to change the effective length of the neck of the Helmholtz resonator (here the thickness of the C-shaped cylinders) and increase the wave dissipation. Further, it can be seen that for a wave period ranging from about 1.1 s to 1.3 s, small reflected waves from each component cylinder come into phase in front of the array, leading to a strong wave amplification there (also captured by the side wave gauge at 1th cylinder as shown in Fig. 4). This corresponds to the occurrence of (class I) Bragg resonance [2-3]. Finally, the broad-banded wave attenuation is achieved after the array via the synergistic effects among various water resonances that occur alternatively in both spatial and frequency domains, as discussed above. Further results, include the effects of cylinder draft, wave incidence *etc.*, will be presented at the workshop.

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