Wave interaction with a periodic array of floating PV system

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1. Introduction

A floating elastic plate functioning as a photovoltaic (PV) system incorporates solar panels onto flexible, floating structures that can adjust to wave dynamics. Kohout et al. [1] investigated the dispersion of surface gravity waves by a configuration of floating elastic plates with changing characteristics. Kohout and Meylan [2] developed a model for wave attenuation involving numerous floating elastic plates in the frequency domain accommodating spring or hinged border conditions. This paper analyses an infinite periodic elastic plate system using the Bloch-Floquet theory. Using linear water wave theory, we compare the Bloch wavenumbers for different edge conditions. Following that, the potential solution is illustrated in the periodic system.

2. Mathematical formulation

The present study illustrates the water wave interaction with a periodic array of moored elastic plates over an impermeable seabed. The elastic plates are considered to be floating on the water's mean-free surface, extending infinitely in the y direction where l is the plate length and d is the uniform spacing between two plates (Fig. 1). Considering the linear theory of water waves, a potential flow-based mathematical model is developed for the motion of homogeneous gravity waves with density ρ. The fluid is assumed to be inviscid as well as incompressible. The fluid's velocity field is likely to be so described as $\vec{v}(x,z,t) = \nabla\tilde{\Phi}(x,z,t)$, where $\tilde{\Phi}$ denotes velocity potential. Following time-harmonic motion, the velocity potential can be written as $\tilde{\Phi}(x,z,t) = Re\{\Phi(x,z)e^{-\mathrm{i}\omega t}\}$ where ω is the angular frequency. With the assumptions bouncy, the velocity potential, Φ, satisfies the governing equation:

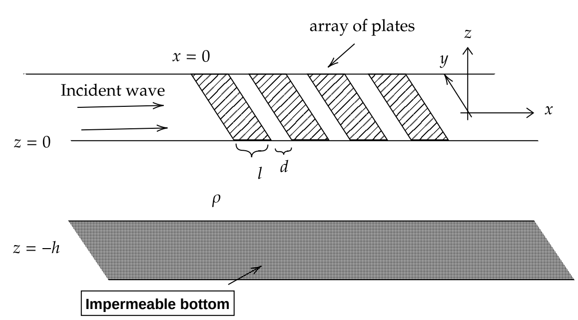

Figure 1: Schematic representation of the physical problem.

$$\Delta\Phi = 0. \tag{0.1}$$

The free surface boundary condition on the non-plate covered regions yields:

$$\left(\frac{\partial}{\partial z} - K\right)\Phi = 0, \quad \text{on } z = 0, \tag{0.2}$$

where $K = \omega^2/g$ with g as the gravitational acceleration.

The linearised condition on the plate-covered surface is given by

$$\left[E\frac{\partial^4}{\partial x^4} + 1 - \epsilon K\right]\frac{\partial\Phi}{\partial z} = K\Phi, \tag{0.3}$$

where $E = \tilde{E}/\rho g$ is the flexural rigidity and $\epsilon = \epsilon_p/\rho$ is connected to the mass-density of the plate.

The plates are considered to be moored at the edges, which results in the boundary conditions,

$$\frac{\partial^3 \Phi}{\partial x^2 \partial z} = 0, \text{ and } \left(E\frac{\partial^4 \Phi}{\partial x^3 \partial z} - E_d \frac{\partial \Phi}{\partial z} \right) = 0, \quad \text{on} \quad z = 0, \ x = 0, l, l + d, \ldots. \tag{0.4}$$

where $E_d = 2(S_m/\rho g)\sin^2(\vartheta)$ is a constant with S_m= mooring line stiffness, ϑ= static mooring line angle ([3]). For $E_d = 0$, the edges convert to the free edges.

The bottom boundary condition is as follows:

$$\frac{\partial \Phi}{\partial z} = 0 \quad \text{on} \quad z = -h. \tag{0.5}$$

2. Periodic array of plates

We plan to consider an infinite system of plates. Since the geometry is periodic, we may invoke the Bloch-Floquet theory which allows us to consider just one periodic element of the array (here we choose the 1st strip, i.e., $D = \{-d/2 < x < l + d/2, -h < z < 0\}$) provided we introduce quasi-periodic boundary conditions on those fluid interfaces which connect one cell to the next. We consider a periodic boundary condition following the Bloch-Floquet theory,

$$\begin{aligned} \Phi(l + d/2, z) &= e^{\mathrm{i}\beta(l+d)}\Phi(-d/2, z), \\ \frac{\partial \Phi}{\partial x}(l + d/2, z) &= e^{\mathrm{i}\beta(l+d)}\frac{\partial \Phi}{\partial x}(-d/2, z), \end{aligned} \tag{0.6}$$

where β is the complex Block wavenumber sought to be obtained.

The solution $\Phi(x, z)$ due to the spatial distribution, can be written as

$$\Phi_1 = \sum_{j=0}^{\infty} \left(A_{j,1} e^{\mathrm{i}k_j\left(x+\frac{d}{2}\right)} + B_{j,1} e^{-\mathrm{i}k_j x} \right) Z_{1,j}(z), \quad (x, z) \in (-d/2, 0) \times (-h, 0), \tag{0.7}$$

$$\Phi_2 = \sum_{j=-2}^{\infty} \left(A_{j,2} e^{\mathrm{i}p_j x} + B_{j,2} e^{-\mathrm{i}p_j(x-l)} \right) Z_{2,j}(z), \quad (x, z) \in (0, l) \times (-h, 0), \tag{0.8}$$

$$\Phi_3 = \sum_{j=0}^{\infty} \left(A_{j,3} e^{\mathrm{i}k_j(x-l)} + B_{j,3} e^{-\mathrm{i}k_j\left(x-l-\frac{d}{2}\right)} \right) Z_{1,j}(z), \ (x, z) \in (l, l + d/2) \times (-h, 0), \tag{0.9}$$

where the eigenfunctions are given by

$$Z_{1,j}(z) = w_{1,j}^{-1/2} \cosh k_j(z + h), \quad \text{and} \quad Z_{2,j}(z) = w_{2,j}^{-1/2} \cosh p_j(z + h), \tag{0.10}$$

with $w_{i,j} = \frac{h}{2}\left(1 + \frac{\sinh(2\gamma_{i,j}h)}{2\gamma_{i,j}h}\right)$ with $\gamma_1 = k, \ \gamma_2 = p$.

The wavenumbers k and p, satisfy the following dispersion relations:

$$\mathcal{D}_1(k) \equiv K - k\tanh(kh) = 0, \quad \text{and} \quad \mathcal{D}_2(p) \equiv K - \mathcal{E}(p)p\tanh(ph) = 0, \tag{0.11}$$

where $\mathcal{E}(p) = (Ep^4 + 1 - \epsilon K)$. $\mathcal{D}_1(k)$ has one pair of real roots of the form $k = \pm k_0$ and infinitely purely imaginary roots $k = \pm k_i$, $i = 1, 2, \ldots$, whereas $\mathcal{D}_2(p)$ produces one pair of real roots, denoted by $\pm p_0$; two pairs of complex conjugate roots, $\pm p_{-1}, \pm p_{-2}$; and an infinite number of purely imaginary roots, $\pm p_i$ for $i = 1, 2, \ldots$.

The eigenfunctions $Z_{1,i}$ are orthonormal as $\int_{-h}^{0} Z_{1,i}(z) Z_{1,j}(z)\,\mathrm{d}z = \delta_{i,j}$. Using the matching conditions Eq. (0.6) along with the orthogonality of the $Z_{1,i}(z)$ results in

$$\left.\begin{aligned} e^{\mathrm{i}\beta(l+d)}\left(A_{i,1} + B_{i,1}e^{\frac{\mathrm{i}k_i d}{2}}\right) &= \left(A_{i,3}e^{\frac{\mathrm{i}k_i d}{2}} + B_{i,3}\right), \\ e^{\mathrm{i}\beta(l+d)}\left(A_{i,1} - B_{i,1}e^{\frac{\mathrm{i}k_i d}{2}}\right) &= \left(A_{i,3}e^{\frac{\mathrm{i}k_i d}{2}} - B_{i,3}\right), \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} A_{i,3} &= A_{i,1}e^{\mathrm{i}\left(\beta(l+d) - \frac{k_i d}{2}\right)}, \\ B_{i,3} &= B_{i,1}e^{\mathrm{i}\left(\beta(l+d) + \frac{k_i d}{2}\right)}. \end{aligned}\right\} \tag{0.12}$$

Using Eq. (0.12) in the continuity of velocity and pressure along with the matching conditions yields,

$$\left(A_{i,1}e^{\frac{\mathrm{i}k_i d}{2}} + B_{i,1}\right) - \sum_{j=-2}^{\infty}\left(A_{j,2} + B_{j,2}e^{\mathrm{i}p_j l}\right)\mathcal{W}_{j,i} = 0, \tag{0.13}$$

$$k_i\left(A_{i,1}e^{\frac{\mathrm{i}k_i d}{2}} - B_{i,1}\right) - \sum_{j=-2}^{\infty} p_j\left(A_{j,2} - B_{j,2}e^{\mathrm{i}p_j l}\right)\mathcal{W}_{j,i} = 0, \tag{0.14}$$

$$e^{\mathrm{i}\beta(l+d)}\left(A_{i,1}e^{-\frac{\mathrm{i}k_i d}{2}} + B_{i,1}e^{\mathrm{i}k_i d}\right) - \sum_{j=-2}^{\infty}\left(A_{j,2}e^{\mathrm{i}p_j l} + B_{j,2}\right)\mathcal{W}_{j,i} = 0, \tag{0.15}$$

$$k_i e^{\mathrm{i}\beta(l+d)}\left(A_{i,1}e^{-\frac{\mathrm{i}k_i d}{2}} - B_{i,1}e^{\mathrm{i}k_i d}\right) - \sum_{j=-2}^{\infty} p_j\left(A_{j,2}e^{\mathrm{i}p_j l} - B_{j,2}\right)\mathcal{W}_{j,i} = 0, \tag{0.16}$$

where $\mathcal{W}_{ij} = \int_{-h}^{0} Z_{1,j} Z_{2,i}\mathrm{d}z$. Now, Eqs. (0.13) to (0.16) along with edge conditions Eq. (0.4) construct a $4(N+2) \times 4(N+2)$ sized system of equations.

3. Numerical results

Tables 1 and 2 presents the Bloch wavenumber β under two different edge scenarios. For free edge case, β encompasses one negative real root β_{-0} along with β_0 as a complex Bloch wavenumber. As N increases, additional purely imaginary modes exists with $(\beta_n - \beta_{n-1}) = \pi/h$ for $n = 1, 2, \ldots$. For moored edge conditions, all the roots shift to the complex plane. Due to the moored edges, the elastic plate transfers more energy into the surrounding medium. It shows that the damping due to the moored edges modifies the β in the complex plane, signifying alterations in energy dissipation.

Figure 2 illustrates the velocity potential fields within the $0 < x/l < 1.5$ range for a plate length of $l/h = 0.1$ along the Bloch wavenumber β_0. The edges are crucial in velocity distribution, signifying wave energy dissipation. The free edge condition facilitates enhanced wave energy transmission, evidenced by increased magnitudes in both cases. The moored edge condition introduces damping, diminishing intensity, particularly in the real component of Φ. The left edge affects an equal vertical velocity distribution along the boundary. In contrast, the right edge has a complex distribution along the boundary.

Free edge condition							
$N=1$	-0.12	$0.12+0.347\mathrm{i}$	$0.08+0.474\mathrm{i}$	$-$	$-$	$-$	$-$
$N=2$	-0.12	$0.12+0.347\mathrm{i}$	$-0.001+0.626\mathrm{i}$	$0.043+0.854\mathrm{i}$	$-$	$-$	$-$
$N=3$	-0.12	$0.12+0.347\mathrm{i}$	$0.626\mathrm{i}$	$-0.002+0.959\mathrm{i}$	$0.001+1.211\mathrm{i}$	$-$	$-$
$N=4$	-0.12	$0.12+0.347\mathrm{i}$	$0.626\mathrm{i}$	$0.959\mathrm{i}$	$-0.001+1.281\mathrm{i}$	$-0.002+1.559\mathrm{i}$	$-$
$N=5$	-0.12	$0.12+0.347\mathrm{i}$	$0.626\mathrm{i}$	$0.959\mathrm{i}$	$-0.001+1.28\mathrm{i}$	$0.001+1.598\mathrm{i}$	$-0.003+1.894\mathrm{i}$
$N=6$	-0.12	$0.12+0.347\mathrm{i}$	$0.626\mathrm{i}$	$0.959\mathrm{i}$	$-0.001+1.28\mathrm{i}$	$0.001+1.598\mathrm{i}$	$0.003+1.914\mathrm{i}$

Table 1: Variation of βl for different N with free edge conditions $E = 1.139h^4,\ \epsilon = 0.096,\ Kh = 1,\ Kl = Kd = 0.1$ and $E_d = 0$.

Moored edge condition							
$N=1$	$-0.129-0.001\mathrm{i}$	$0.109+0.321\mathrm{i}$	$0.087+0.491\mathrm{i}$	$-$	$-$	$-$	$-$
$N=2$	-0.126	$0.117+0.347\mathrm{i}$	$-0.019+0.623\mathrm{i}$	$0.047+0.863\mathrm{i}$	$-$	$-$	$-$
$N=3$	$-0.125-0.001\mathrm{i}$	$0.122+0.35\mathrm{i}$	$-0.012+0.628\mathrm{i}$	$-0.011+0.969\mathrm{i}$	$0.016+1.214\mathrm{i}$	$-$	$-$
$N=4$	$-0.124-0.001\mathrm{i}$	$0.124+0.35\mathrm{i}$	$-0.009+0.628\mathrm{i}$	$-0.007+0.967\mathrm{i}$	$-0.001+1.291\mathrm{i}$	$0.002+1.561\mathrm{i}$	$-$
$N=5$	$-0.124-0.001\mathrm{i}$	$0.125+0.349\mathrm{i}$	$-0.008+0.627\mathrm{i}$	$-0.006+0.965\mathrm{i}$	$-0.001+1.288\mathrm{i}$	$0.005+1.605\mathrm{i}$	$0.001+1.895\mathrm{i}$
$N=6$	$-0.124-0.001\mathrm{i}$	$0.125+0.349\mathrm{i}$	$-0.008+0.627\mathrm{i}$	$-0.006+0.965\mathrm{i}$	$-0.001+1.288\mathrm{i}$	$0.004+1.604\mathrm{i}$	$0.007+1.919\mathrm{i}$

Table 2: Variation of βl for different N with moored edge conditions with $E = 1.139h^4,\ \epsilon = 0.096h,\ Kh = 1,\ Kl = Kd = 0.1$ and $E_d = 0.1l$.

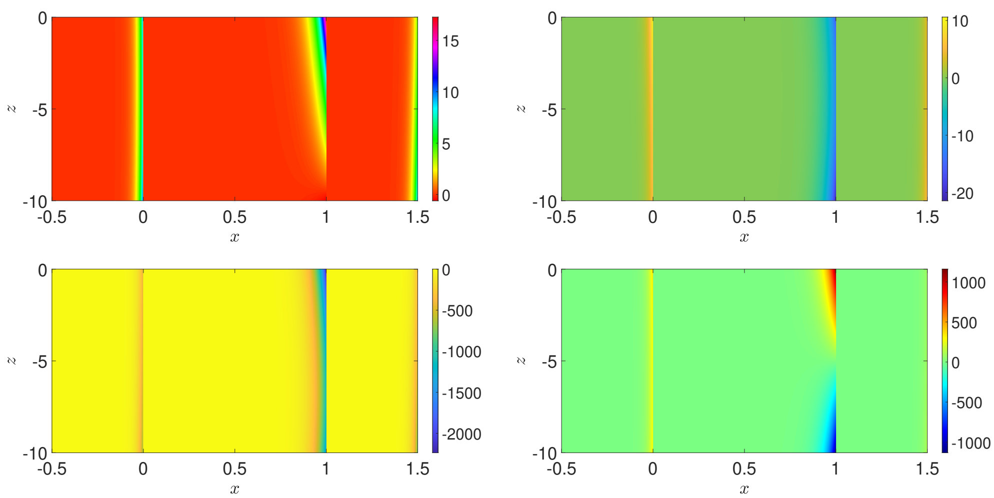

Figure 2: $Re(\Phi)$ (upper two) and $Im(\Phi)$ (lower two) in the periodic cell with free edge condition (left ones), moored edge condition (right ones) and the values $E = 1.139h^4,\ \epsilon = 0.096h,\ Kh = 1,\ Kl = Kd = 0.1$ and $E_d = 0.1l$.

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