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Machine Learning for Computation of Wave Added Resistance

Mostafa Amini-Afshar*, Malte Mittendorf, and Harry B. Bingham

Technical University of Denmark, Department of Civil and Mechanical Engineering,
Section of Fluid Mechanics, Coastal and Maritime Engineering
maaf@dtu.dk, malte.mittendorf@maersk.com, hbbi@dtu.dk

1 Introduction

We present a machine learning model for calculation of wave added resistance. The model training is performed using a large set of *pre-calculated* added resistance curves covering a broad range of ship hulls and operational conditions, i.e. forward speed, draft and relative wave heading. The underlying hydrodynamic model is the classical strip-theory where the wave added resistance is computed according to a modified version of Salvesen's formulation. It is concluded that the developed data-driven model is able to produce a non-linear mapping between a set of operational conditions as well as the ship's main particulars to the wave added resistance coefficient.

2 Background

In both ship design and operation, an efficient and robust computation of the added resistance in seafaring conditions is required. One use case in the field of ship operations is ship *performance analysis.* Here, it is required to subtract the wind and wave added resistances from the total resistance that has been calculated using the vessel's operational data. The difference between this *in-service* calm-water resistance, and the *given* calm-water resistance of the ship for its *newly-built* condition, is in fact an indication of the increased frictional resistance due to marine growth. Performance analysis is usually conducted for quite a number of vessels with only their bulk geometrical data at hand. Similarly, voyage optimization requires a mathematical model for ship's excess resistance in heavy weather in order to determine a realistic fuel penalty used as part of the overall objective function. In addition, the results are required in almost *real time.* Therefore, performing a full added resistance computation, even for a known hull geometry, is practically not viable. Motivated by these challenges, through couple of student projects, we have developed some fast computational tools for estimation of wave added resistance based only on the vessel's bulk geometrical data. In one case, we computed the added resistance by an interpolation method inside a *pre-calculated* database of curves. See for example [1]. Recently, we have employed a machine learning model for fast calculation of wave added resistance [2]. The motivation for revisiting this study is to incorporate results of a more advanced hydrodynamic model as well as considering additional wave heading regimes, i.e. beam-to-following waves. This model shows higher accuracy than our previous interpolation scheme and requires no cumbersome storage of a database for the pre-calculated added resistance curves. Firstly, we describe the underlying hydrodynamic formulations which has been applied for the training of the model. In the following, we present some details of the developed model and the results.

3 The Hydrodynamic Model

We employ the classical Salvesen-Tuck-Faltinsen (STF) strip theory [3]. An open-source MATLAB implementation of this theory, called *DTU_StripTheorySolver*, has been made available by us [4]. In this solver, we have implemented two formulations for wave added resistance [5, 6]. One is based on Maruo's method using the Kochin Function, and the other is according to a modified version of Salvesen's method. Our focus in this abstract is on the modified Salvesen's method based on the following equation

$$R_w = \overline{F_x} = -\frac{\rho}{2}\Re\left\{\int_{S_b}\left[\phi_B\phi^*_{0nx} - \phi_{Bn}\,\phi^*_{0x}\right]\,ds\right\} - \frac{\rho}{4}\Re\left\{\int_{S_b}\left[\phi_B\phi^*_{Bnx} - \phi_{Bn}\,\phi^*_{Bx}\right]\,ds\right\}. \quad (3.1)$$

Here, the wave added resistance R_w is the mean second-order force $\overline{F_x}$ in x-direction, which is obtained by an integration over body surface S_b. The combination of radiation and scattering velocity potentials (the disturbance potential) is denoted by ϕ_B, and ϕ_0 is the incident wave potential, all in the frequency domain. The fluid density is ρ, the subscript n denotes a derivative normal to the body surface, and the subscript x indicates the x-derivative. The asterisks indicate the complex conjugate and $\Re$ takes the real part. The derivation of (3.1) is briefly described in the following.

Generally, in the far-field method, the mean wave drift force is obtained through application of the Reynolds Transport Theorem to the average rate of momentum change inside a closed volume bounded by: the body surface, the free surface, the sea bed and a far-field control surface. The resulting far-field integral can be computed using three methods. One approach is to adopt an *arbitrary* far-field control surface where the fluid kinematics can be conveniently computed. A more robust method is to apply the Kochin function and convert the far-field integrals into their equivalents over the body surface. In the third method, Green's second identity is used to express the far-field integrals in terms of the body surface integrals as shown in Eq. (3.1). Although this equation was derived many years ago, for example by Newman in [7], it has largely remained unnoticed by the community. In 2023 workshop [8], we have *revived* this equation and demonstrated its agreement with the Kochin Function method and the direct pressure integration or the near-filed method. Note that for forward-speed cases where the body is not submerged, one line integral and one free-surface integral should be added to (3.1) depending on the type of linearization adopted (Neumann-Kelvin or double-body). This has been fully derived recently by Kashiwagi in [9].

Now, inside the strip theory, the surface integrals in (3.1) can be approximated by a combination of a line integral over two-dimensional sections and a line integral along the ship length, see [6] for the details. Salvesen in [10], neglects the second integral in (3.1), using a so-called *weak scatterer assumption*, and invokes a *long-wave assumption* to express only the first integral inside the STF strip theory. As a result, his final formulation for added resistance requires no knowledge of the sectional velocity potentials. In [6], we have shown that much more accurate results are achieved if none of these assumptions are adopted. We call this *full* strip-theory rendering of (3.1) the modified version of Salvesen's method.

4 The Data-driven Model

We have developed a multivariate regression model, which approximates a function $f : \mathbb{R}_n \to \mathbb{R}$, where n is the number of dimension of the model's feature space. Herein, deep learning is utilized, as the subsequent models are capable of handling high-dimensional, continuous and non-linear data [11]. The used Deep Neural Network (DNN) is trained on simulated wave added resistance results obtained from the DTU StripTheorySolver. The employed dataset includes 9 scaling variants derived from several parent hulls of tankers and bulk carriers, i.e. block coefficient $C_B \geq 0.8$. The slenderness ratio of the considered hull is $L_{\mathrm{pp}}/\nabla^{1/3} \in [3.42, 7.38]$, whereas the beam-to-length ratio is $B/L_{\mathrm{pp}} \in [0.125, 0.25]$. Furthermore, the operational conditions are defined in terms of the Froude number $\mathrm{Fn} \in [0.0, 0.3]$, the wave encounter angle $\beta \in [0.0, 180]^\circ$, and the non-dimensional wave frequency $\overline{\omega} = \omega\sqrt{L_{\mathrm{pp}}/g} \in [1.45, 4.58]$. The model's feature vector includes the above-mentioned variables (as well as the draft-to-length ratio) and the used target variable is the wave added resistance coefficient $C_{aw} = L_{pp}R_w/\rho g B^2 A^2$. Before training both input and output were normalized, since neural networks are not scale invariant, as opposed to, e.g., tree-based models. The final model architecture is composed of two hidden layers with 64 neurons each. The dataset was split into 80% for model training and 20% for the model assessment. Similar to [2], it is envisioned to perform hyperparameter tuning as well as more advanced feature engineering for increased predictive accuracy.

5 Validation and Results

The model will be validated against *out-of-sample* hull geometries, including a bulk carrier and a product tanker. For this part, the added resistance obtained from the proposed DNN model will be compared with the corresponding value computed *directly* by the strip theory solver using the detailed hull lines. These results will be presented during the workshop.

6 Conclusion

The developed machine learning model will able to *reproduce* the added resistance curves with high accuracy. It requires only the bulk geometrical data (length, breadth, block coefficient, slenderness ratio), and the operational condition (forward speed, draft, relative wave heading). With respect to future work, the model training can be performed using more advanced hydrodynamic models instead of strip theory results, for example slender body theory or panel methods.

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