Three-dimensional directional focused wave by the HLIGN theory

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Highlights

- A three-dimensional High-Level Irrotational Green-Naghdi (HLIGN) theory is developed to simulate three-dimensional directional focused waves.
- The HLIGN model is validated in comparison with the experimental observations. The results well reproduce the main characteristics of a three-dimensional focused wave.

1 Introduction

The research of extreme ocean waves holds significant importance for engineers engaged in maritime environment design, especially in the investigation of rogue wave phenomenon. There are various causes for the generation of rogue waves, among which the phase focusing method is considered a significant mechanism for rogue wave occurrence (Kharif et al., 2008).

For the experimental studies on focused waves, Baldock et al. (1996) investigated the physical characteristics of focused waves in two dimensions. After that, Johannessen and Swan (2000) conducted experimental research on three-dimensional directional focused waves. These results have been extensively applied for comparison and validation with other numerical results.

With the advancement of numerical techniques, some researchers have focused on numerical approaches to simulate focused waves and compare the results with the aforementioned experiments. For two-dimensional focused waves, some researchers analyzed the physical characteristics of waves (Zhao et al., 2020; Duan et al., 2023). However, in the oceans, three-dimensional directional focused waves are more likely to occur. Yet, due to strong nonlinearity of waves in extreme sea conditions and computational efficiency issues in three-dimensions, research on this problem is ongoing. Ai et al. (2014) developed 3D non-hydrostatic model to study three-dimensional directional focused waves. They validated algorithms by comparing numerical results with laboratory observation by Johannessen and Swan (2000).

As a strong nonlinear wave model, the High-Level Irrotational Green-Naghdi (HLIGN) model is a viable option for addressing the significant issue of three-directional focused waves. The HLIGN equation was originally derived by using Hamilton's principle (Kim et al., 2001). It has been well validated and applied in two-dimensional problems (Zhao et al., 2015). However, except for Zhao et al. (2019) investigating wave diffraction using IGN-2 model, the three-dimensional HLIGN model has been rarely used.

Considering the strong nonlinearity of the three-dimensional focused waves, the first motivation of this paper is using High-Level IGN model to conduct research on three-dimensional directional focused waves. Another motivation is to compare the results of the HLIGN model with the experimental observations by Johannessen and Swan (2000) to understand the accuracy of the HLIGN model better.

2 Three-dimensional HLIGN model

The fluid is assumed to be incompressible and inviscid. The HLIGN model is based on the assumption that the flow is irrotational. In three-dimensions, the velocity field (u, v, w) is represented by the stream function $\boldsymbol{\psi}$ which could be given in vector form (ψ^u,ψ^v), where the horizontal components in the x and y directions are given by

$$(u,v)=\boldsymbol{\psi}_{,z} \tag{1a}$$

$$w=-\nabla\cdot\boldsymbol{\psi}\,. \tag{1b}$$

∇ is the gradient operator and the subscript after comma indicates the partial differentiation with respect to corresponding variable. $\boldsymbol{\psi}$ is expressed by the following polynomials for the HLIGN model.

$$\boldsymbol{\psi}(x,y,z,t)=\sum_{m=1}^{K}\boldsymbol{\psi}_m(x,y,t)\,f_m(\gamma) \tag{2}$$

where $f_m(\gamma)=\gamma^{2m-1}$, $\gamma=(z+h)/(\eta+h)$ and $\boldsymbol{\psi}_m$ are the unknown stream function coefficients which will be calculated as part of the solution. K indicates the level of the IGN model, for example it could be IGN-5 when K = 5.

Then, the surface elevation η (x, y, t) , the surface velocity potential $\hat{\phi}(x,y,t)$ and the unknown time derivatives of the stream function coefficients $\boldsymbol{\psi}_m$ could be expressed as follows (Kim et al., 2001):

$$\frac{\partial\eta}{\partial t}+\sum_{m=1}^{K}f_m(1)\nabla\cdot\boldsymbol{\psi}_m=0\,, \tag{3a}$$

$$\frac{\partial\hat{\phi}}{\partial t}=-\nabla\cdot\frac{\partial E_k}{\partial(\nabla\eta)}+\frac{\partial E_k}{\partial\eta}-g\eta\,, \tag{3b}$$

$$f_m(1)\nabla\hat{\phi}=-\nabla\frac{\partial E_k}{\partial\left(\nabla\cdot\boldsymbol{\psi}_m\right)}+\frac{\partial E_k}{\partial\boldsymbol{\psi}_m},\quad(m=1,2,\ldots,K)\,, \tag{3c}$$

where E_k is the kinetic energy. The algorithm to solve the HLIGN equations will be discussed at the workshop.

3 Numerical results

In this section, we simulated three-dimensional directional focused waves in finite water depth by use of the HLIGN model. The experimental data by Johannessen and Swan (2000) are used to validate the HLIGN model. The difference of the results between the HLIGN model and the non-hydrostatic model results from Ai et al. (2014) are considered.

The simulation conditions for the three-dimensional focused waves follow those given in Johannessen and Swan (2000). The wave basin spans 11m in the x-direction and 25m in the y-direction, respectively. The water depth is 1.2m. In the computational domain, one side of the flow field serves as the wave generation boundary while the opposing side acts as the wave absorption boundary, with the remaining two sides being the wall boundaries. The flow field is the finite water depth from the bottom $z=-h(x,y)$ to the free surface $z=\eta\,(x,y,t)$. The origin of the coordinate system is at the still-water surface on the wave generation boundary side. The schematic of the numerical wave flume is depicted in Fig. 1.

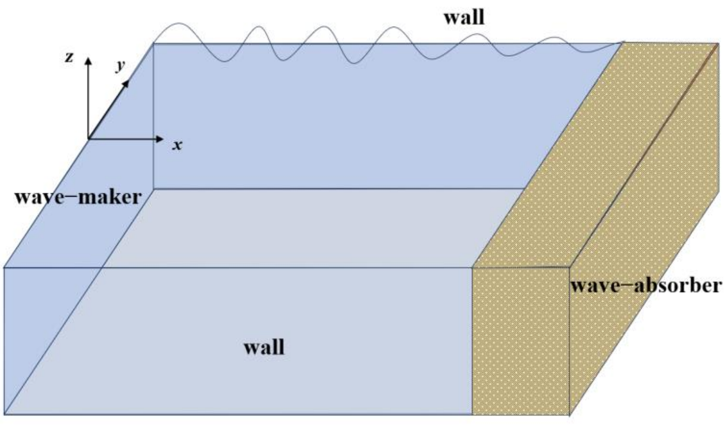

Figure 1 The sketch of wave flume

For the wave-maker boundary condition, the surface elevation has to be expressed the same way as in Johannessen and Swan (2000) to generate the three-dimensional directional focused waves:

$$\eta(x,y,t)=\sum_{n=1}^{N}a_n\sum_{m=1}^{M}b_m\cos\{k_n[(x-x_f)\cos\theta_m+(y-y_f)\sin\theta_m]-\omega_n(t-t_f)\}\,, \tag{6a}$$

$$\sum_{n=1}^{N}a_n=A,\quad\sum_{m=1}^{M}b_m=1,\quad b_m=B\cos^s\frac{\theta_m}{2}\,, \tag{6b}$$

where N and M are the number of frequency components and the direction components per frequency (M=91 in Eq. 6a), respectively. k_n and ω_n are the wave number and frequency, respectively. θ_m is the wave direction varying from $-\pi/4$ to $\pi/4$ with equal spacing. "s" is spreading parameter and B is a normalizing coefficient. The directional focused waves are related to the focused location (x_f , y_f)= (5.5m,12.5m) and focused time t_f .

Taking case D0493 in Johannessen and Swan (2000) as an example, case D0493 is the case where D is the frequency- amplitude spectrum ($\frac{53}{64}\text{Hz} \le f \le \frac{80}{64}\text{Hz}$), the spreading parameter s=4 and the input amplitude-sum A=93mm. The focal time must be sufficient for the shortest regular wave component to pass through the theoretical focal position; that is $t_f \ge l_f / c_{g\min} = 21.9$ s , where $c_{g\min}$ is the group velocity of the shortest regular wave component. Considering the three-dimensional characteristics, l_f is the distance from the farthest position on the wave-maker to the focal point. Thus, we choose $t_f = 25s$ as the focal time. Through self-convergence tests, in this case, IGN-5 was used as the High-Level IGN model to simulate the focused wave. Fig.2 shows the three-dimensional focused wave profile at the focused time. Because of wave nonlinearity, the actual focused time is delayed. The shift in time is 0.57s.

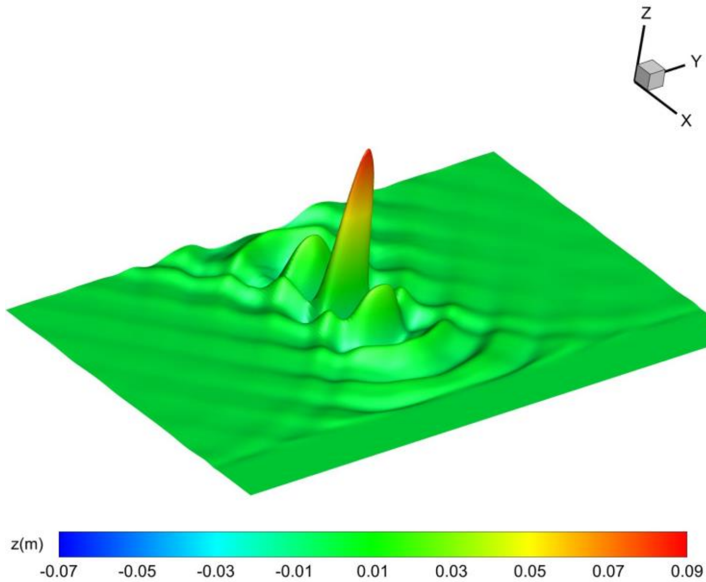

Figure 2 Wave profile of focused waves

In Fig. 2, the three-dimensional focused wave exhibits a symmetric wave field distribution. The right side in the figure demonstrates a good wave-damping effect. Subsequently, the xoz plane at symmetrical profile (y=0) was selected for quantitative analysis.

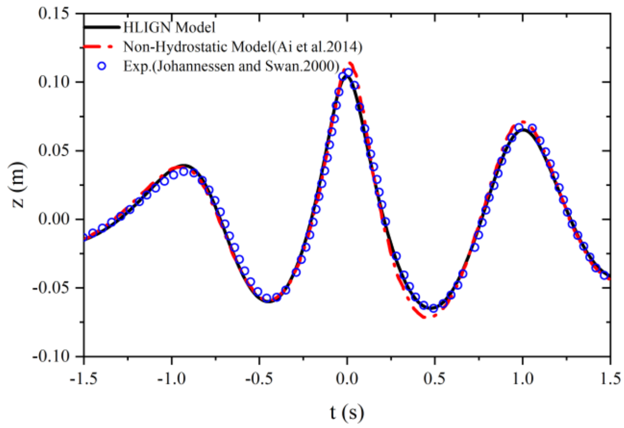

Figure 3 Wave profile at focal position of symmetrical plane in case D0493

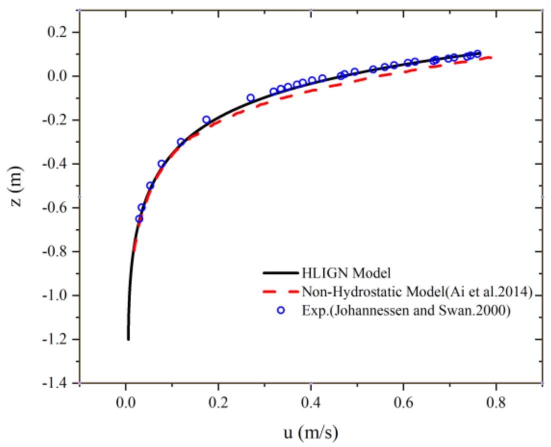

Figure 4 Horizontal velocity distribution beneath the focused crest in case D0493

When analyzing the wave profile, comparison is made among the HLIGN model results, the experimental data by Johannessen and Swan (2000) and the non-hydrostatic model results from Ai et al. (2014). As a kind of nonlinear wave model for wave research, the non-hydrostatic model has been validated on three-dimensional focused wave simulation. We will also do some analysis on the difference of the results between the HLIGN model and the non-hydrostatic model. The comparisons are shown in Fig. 3.

Fig. 3 illustrates the wave profile at the focal position of two nonlinear wave models and experimental data for the case D0493 from Johannessen and Swan (2000), where the x-axis represents time, and the y-axis represents the wave height. Due to the focused position shift caused by the nonlinear character of the wave, for the comparison analysis of wave surface, the numerical results were horizontally shifted along the x-axis, aligning the focused wave crest at x=0. As it is shown in Fig. 3, results from the HLIGN model show better agreement with the experimental data than results of the non-hydrostatic model.

The horizontal velocity beneath the wave crest is presented in Fig. 4. Where the x-axis represents velocity, and the y-axis represents the water depth. In Fig. 4, the HLIGN model results is close to the experimental observations. From the seabed to z=-0.4m, results of the HLIGN model and non-hydrostatic model are approximately same. From z=-0.4m to the wave surface, the results of HLIGN model are closer to the experimental observations compared to the non-hydrostatic model. The results of non-hydrostatic model are slightly higher than the experimental observations.

More information on the three-dimensional HLIGN model and additional numerical results considering other cases will be presented at the workshop.

4 Conclusions

In this paper, we developed a three-dimensional HLIGN model to simulate the three-dimensional directional focused waves. The results of the High-Level IGN model were validated by the experimental observations in case D0493 from Johannessen and Swan (2000). It is shown that the results of HLIGN model have a good agreement with the experimental observations. These results validate HLIGN model and indicate that the HLIGN model is capable of simulating three-dimensional directional focused waves.

Acknowledgments

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