The harmonic separation of measured force components on a model jacket from waves and current – phase and symmetry based analysis

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1 Introduction

Jackets are commonly used as support structures for offshore oil and gas platforms and, more recently, for offshore wind turbines and sub-stations. They are fixed, bottom-founded steel structures, with numerous slender cylindrical members. Efficient jacket design requires accurate estimates of the hydrodynamic forces from waves and currents, but this is not fully understood even now. In current design, wave and current horizontal forces are treated with the Morison-equation (Morison et al., 1950),

$$F = \underbrace{(\cdots)\frac{\partial u_{\mathrm{w}}(t)}{\partial t}}_{\text{Inertia}} + \underbrace{(\cdots)\,(u_{\mathrm{w}}(t)+u_{\mathrm{cs}}) \times |u_{\mathrm{w}}(t)+u_{\mathrm{cs}}|}_{\text{Drag}}. \tag{1}$$

Here u_{w} is the wave-induced horizontal fluid velocity that varies in time t, and u_{cs} is the mean current velocity through the jacket. u_{w} is assumed undisturbed by the presence of the structure, while u_{cs} is reduced from the open field steady current u_{c} by 'simple current blockage', a flow effect that reduces drag forces in the absence of waves (Taylor, 1991). Previous studies have also shown that combined waves and current give rise to extra 'wave-current blockage' (Santo et al., 2018a), which gives a greater peak force reduction than simple current blockage alone. While the Morison drag term $u|u|$ is non-analytic, Haritos (2007) noted that

$$(u_{\mathrm{w}}(t)+u_{\mathrm{s}}) \times |u_{\mathrm{w}}(t)+u_{\mathrm{s}}| \approx u_{\mathrm{w}}(t) \times |u_{\mathrm{w}}(t)| + 2|u_{\mathrm{w}}(t)| \times u_{\mathrm{s}}, \tag{2}$$

in his model of the interaction between waves and structural motion $u_{\mathrm{s}}(t)$ on a vertical column. Santo et al. (2018b) used this form to account for motion of a dynamically-responding jacket, showing good agreement with experiments. The approximation is valid when the perturbing component (current u_{cs} and/or structural motion u_{s}) is small relative to u_{w}, so for large waves.

The Morison drag term, which is generally much larger than the inertia term for design wave loads on jackets, then contains force components from waves only ($u_{\mathrm{w}} \times u_{\mathrm{w}}$), from waves and current ($u_{\mathrm{w}} \times u_{\mathrm{cs}}$), and from current ($u_{\mathrm{cs}} \times u_{\mathrm{cs}}$), which is a steady drag component. If u_{w} is driven by a regular wave (i.e., $u_{\mathrm{w}}(t) = a\cos(\phi)$, where a is the fluid velocity amplitude, ϕ is the phase), the wave-only force component can be approximated by a sum of only odd harmonics (Orszaghova et al., 2021), as

$$\mathrm{a}\cos(\phi)\,|\mathrm{a}\cos(\phi)| = a \times a\,((\cdots)\cos(\phi) + (\cdots)\cos(3\phi) + (\cdots)\cos(5\phi) + ..). \tag{3}$$

In contrast, the wave × current term consists of only even harmonics, and these scale with ($u_{\mathrm{cs}} \times a$) as

$$|\mathrm{a}\cos(\phi)|u_{\mathrm{cs}} = u_{\mathrm{cs}} \times a\left((\cdots) + (\cdots)\cos(2\phi) + (\cdots)\cos(4\phi) + \cdots\right). \tag{4}$$

Note that eqn.(4) contains a mean and slowly-varying term in addition to the even harmonics.

We test the hypothesis that the odd (drag) harmonics are dominated by forces from waves, while the even harmonics contain forces from waves × current and current only (with some wave × wave terms related to the vertical motion of the free surface). Here we present the force components from steady, 2nd difference, linear, up to 3rd sum harmonics in frequency – these dominate the total force time histories.

2 Experimental Set-up

Experiments were carried out in the Kelvin Hydrodynamics Laboratory towing tank at the University of Strathclyde, Glasgow. We tested the same jacket model used by Santo et al. (2018a,b), a 1:80 scale model of a 2nd-generation North Sea platform (see Fig. 1 & 2 in Santo et al. (2018a)). The jacket was suspended below a self-propelled carriage via a stiff mounting frame, arranged so that the total in-line horizontal force was measured by a force transducer. The whole jacket model is 1.74 m tall, the distance from the base to still water level was 1.33 m to ensure that the wave crests did not hit the support frame above. The water depth in the tank was 1.8 m. The free surface was measured by resistance-based wave probes mounted on the carriage out to the side of the jacket and on the tank wall. The jacket was towed at constant speed u_c of 0, $\pm$0.07, $\pm$0.14, $\pm$0.28 m/s, with positive current speeds defined as adding to the wave crest kinematics. All current speeds were run with nominally identical NewWave crest- and trough- focused wave groups (Walker et al., 2004) with 0.259 m wave crest amplitude at focus. The groups were based on a JONSWAP spectrum with a peak frequency of 0.52 Hz, $\boldsymbol{\gamma}$ = 3.3 and truncated at 1 Hz. Since the tests were performed with isolated wave groups rather than regular waves, the wave amplitude a in eqn.(3 & 4) is taken as the slowly varying linear envelope, so $a(t)$ and an associated second harmonic difference term. Care was taken to ensure that the waves were as symmetric in time as possible, and that the position of the centre of the model at the instant of wave-group focus always coincided with this focus in space and time.

3 Results and Discussion

Free surface and force measurements for different current speeds are shown in Fig. 1 & 2. The experimental records were highly repeatable, with repeat free surface and force measurements deviating < 2% from the mean. To account for towing the carriage at different speeds, in Fig. 2 & 3, the time axes are scaled as $\tau = t(1 + u_c/c)$, where t is time and c is the phase velocity of the peak wave frequency.

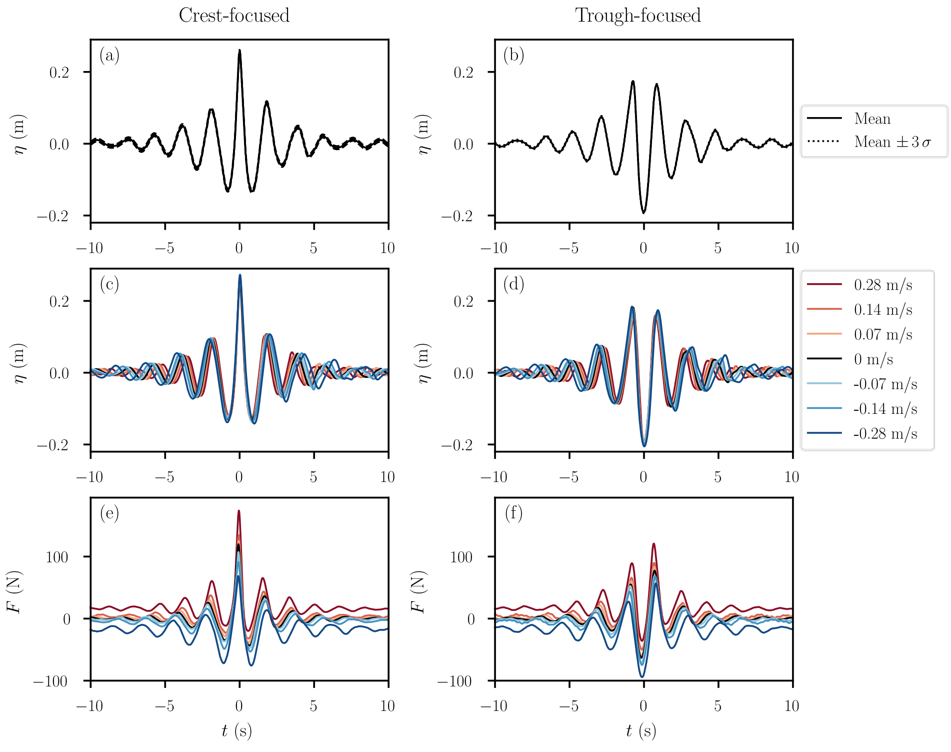

Figure 1: Time-series measurements of crest-focused (left) and trough-focused wave groups (right). **a-b**: Free surface from the fixed wave gauge. **c-d**: Free surface from the carriage-mounted wave gauge at different current speeds. **e-f**: Total force. All time-series are low-pass filtered at 3 Hz.

To decompose the free surface and force time histories into their linear and higher harmonics, we used the two-phase combination method, which combines the crest- and trough-focused time-series to separate odd and even harmonics. Then, the first three harmonics were separated from other odd or even harmonics by digital filtering. The resulting free surface and force harmonics are shown in Fig. 3. Variations in free surface harmonic structure with different currents (Fig. 3 – left panels) are small, so the forces from waves with current should be directly comparable.

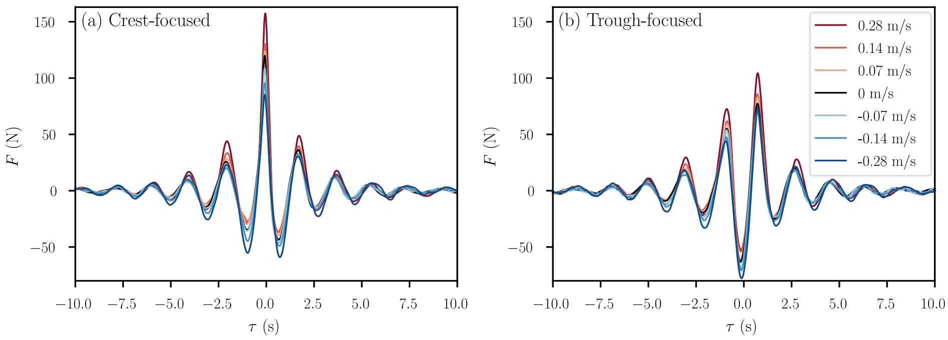

Figure 2: Total force in wave groups: (a) crest-focused, (b) trough-focused. The mean drag force from current has been removed, and the scaled time axis accounts for carriage motion.

The total force time histories (Fig. 2) and the deduced linear force (Fig. 3b) shows slight asymmetry in time mostly due to the presence of the Morison inertia component; this can be extracted by taking the symmetric and skew parts in time (not shown). Force harmonics (Fig. 3 - right panels) are highly nonlinear; while the amplitudes of higher free surface harmonics are less than 1/6 of the linear free surface amplitude, the higher harmonic force components sum to ~ 1/2 the linear force amplitude at the focus time. Linear forces roughly collapse for all current speeds, as do the 3rd harmonic sum forces.

In contrast to the odd harmonics, the even 2nd harmonic sum and difference terms are neatly sorted by current speed. The 2nd and 3rd harmonic sum force harmonics are symmetric about $\tau = 0$, consistent with the loading being drag dominated (Morison drag terms are symmetrical for symmetric wave crests, while the Morison inertia term is $90°$ phase shifted ahead of wave crests and so is skew-symmetric – and higher order potential flow loads are negligible here (Kristiansen and Faltinsen, 2017)). We find that the total 2nd harmonic sum force can be modelled well by the sum of 2 terms - one independent of current and a 2nd term proportional to the current, both functions of τ, and symmetric in time, so consistent with eqn.(4).

The 2nd harmonic difference force is roughly symmetrical in time for zero current, but the additional forces for non-zero current are not symmetric. However, we again find that the effect of the current is modelled well as the sum of 2 terms - one independent of current and time-symmetric, and a 2nd term proportional to the current, so consistent with eqn.(4) but asymmetric with a smaller drag after the passage of the largest crest than before for cases of positive current. We presume this asymmetry is caused by fluid memory effects associated with wake vorticity. This is being explored by wake measurements which will be reported elsewhere (Archer et al., 2024). If so, it is likely to be responsible for the lower extreme force associated with a large wave event with a wavetrain as compared to that from the same wave group but now on otherwise still water, as reported by Santo et al. (2018a).

Overall, these results support our hypothesis: to a reasonable approximation, the influence of current on drag loads is contained in the even (2nd) harmonic terms only. Wave-current blockage effects are visible in the slowly-varying second harmonic difference force. These findings motivate future work to better understand jacket load behaviour, with the aim to analytically estimate these effects from combined wave and current, which are currently not adequately accounted for in the Morison force approximation.

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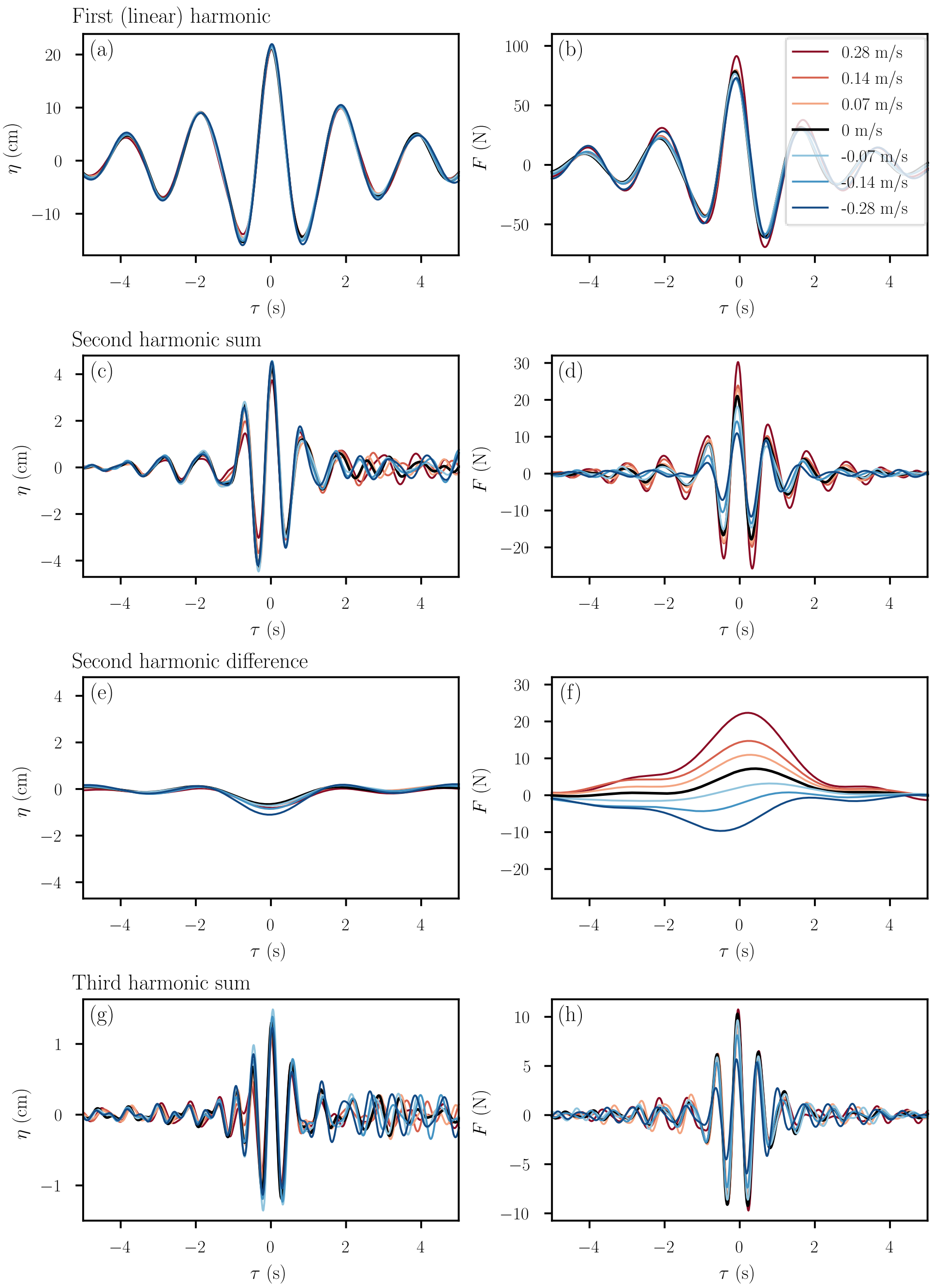

Figure 3: Harmonic time-series of free surface (left) and total force (right). The mean drag force from current is removed from the 2nd harmonic difference force (f) for clarity.