Phase-manipulation with multiple controlled inputs to enhance investigation of nonlinear hydrodynamic effects

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HIGHLIGHTS

- Phase manipulation techniques are extended to cases where wave and structure motion inputs are independently controlled;
- Self-self and cross-term nonlinear interactions are isolated;
- The techniques are demonstrated on experimental data.

1 INTRODUCTION

Phase manipulation has been used extensively to study 'smooth' potential flow hydrodynamic surface elevations [1], forces [2] and structural motions [3]. In the absence of effects like wave breaking, strong flow separation or friction, this technique allows the Stokes-like harmonics of generalised responses to be extracted by controlling the phases of the incident waves (either wave groups or irregular waves). The technique can be applied experimentally or numerically. Because in a Stokes expansion the leading term in the n^{th} harmonic in frequency scales with the n^{th} power of the wave amplitude, it is important to be able to explicitly separate harmonics.

In this work, we describe how phase manipulation can be extended to cases where there are multiple controlled inputs - in this case waves (controlled by a wavemaker) and structure motions (controlled by an actuator). Higher harmonics of forces or surface elevations in such cases involve terms associated with powers of the wave or motion amplitude, and products of the two. The generalised phase manipulation allows these terms to be explicitly separated from a range of tests involving incident waves and structure motions. For offshore renewable energy structures like wave energy converters or floating wind turbines, it is important to be able to separate these terms as the transfer function from incident surface elevation to linear structural motion may change in time as a result of the changing properties of the power take-off (for example).

2 THEORY

Following [2], we express the surface elevations and motions in a narrow-banded form for ease of explanation.

Hence, the incident surface elevation is:

$$\eta = A(t)\cos(\phi) + A^2(t)(B^+\cos(2\phi) + B^-) + \dots, \tag{1}$$

and similarly the motion is

$$x = X(t)\cos(\psi), \tag{2}$$

where $\phi = \omega t + \varepsilon_W$ and $\psi = \omega t + \varepsilon_M$ and $A(t)$ and $X(t)$ are the slowly varying envelopes.

The resulting surface elevation or hydrodynamic force could be written

$$\begin{aligned} F(t) &= A(t) f_W \cos(\phi + \alpha_W) + X(t) f_M \cos(\psi + \alpha_M) \\ &+ A^2(t)\{f^+_{WW} \cos(2\phi + \alpha_{WW}) + f^-_{WW}\} \\ &+ A(t)X(t)\{f^+_{WM} \cos(\phi + \psi + \alpha^+_{WM}) + f^-_{WM} \cos(\phi - \psi + \alpha^-_{WM})\} \\ &+ X^2(t)\{f^+_{MM} \cos(2\psi + \alpha_{MM}) + f^-_{MM}\} + \ldots \end{aligned} \tag{3}$$

in terms of transfer functions with amplitudes f and phases α.

As in other work, there are many ways to implement the phase manipulation. For wave-only inputs, either 2-phase or 4-phase combinations are generally used, although additional phases can also be employed. In the case of 2-phase combinations the odd and even harmonics are separated, whereas 4-phase combinations produce harmonics separated by 4 (so the linear term appears with the 5$^{\text{th}}$ harmonic in frequency, the 2+ with the 6$^{\text{th}}$, etc).

The simplest approach is analogous to 2-phase combinations where the notation $F_{\beta,\gamma}$ is the response to an input with a phase shift β to the linear wave input and a phase shift γ applied to the linear motion input.

1. linear diffraction terms: $(F_{(0,0)} + F_{(0,\pi)} - F_{(\pi,0)} - F_{(\pi,\pi)})/4 = A(t) f_W \cos(\phi + \alpha_W) + O(A^3)$;
2. linear radiation terms: $(F_{(0,0)} - F_{(0,\pi)} + F_{(\pi,0)} - F_{(\pi,\pi)})/4 = X(t) f_M \cos(\phi + \alpha_M) + O(A^3)$;
3. 2$^{\text{nd}}$-order (2^+, 2^-) wave-motion terms: $(F_{(0,0)} - F_{(0,\pi)} - F_{(\pi,0)} + F_{(\pi,\pi)})/4$
$= A(t)X(t)\{f^+_{WM} \cos(\phi + \psi + \alpha^+_{WM}) + f^-_{WM} \cos(\phi - \psi + \alpha^-_{WM})\} + O(A^4)$;
4. 2$^{\text{nd}}$-order (2^+, 2^-) wave-wave and motion-motion terms: $(F_{(0,0)} + F_{(0,\pi)} + F_{(\pi,0)} + F_{(\pi,\pi)})/4$
$= A^2(t)\{f^+_{WW} \cos(2\phi + \alpha_{WW}) + f^-_{WW}\} + X^2(t)\{f^+_{MM} \cos(2\psi + \alpha_{MM}) + f^-_{MM}\} + O(A^4)$.

However, the approach above does not allow the wave-wave and motion-motion terms to be separated nor (as with typical 2-phase separation) are the sum and difference contributions separated. Considering phase manipulations with 4 phases for each input, 16 realisations are possible. Amongst the combinations of these, the following enable the isolation of the second-order sum terms:

1. wave-wave: $(F_{(0,0)} + F_{(0,\pi)} - F_{(\pi/2,0)} - F_{(-\pi/2,0)} + F_{(\pi,\pi)} + F_{(\pi,0)} - F_{(-\pi/2,\pi)} - F_{(\pi/2,\pi)})/8$
$= A^2(t)\{f^+_{WW} \cos(2\psi + \alpha_{WW})\} + O(A^4)$;
2. motion-motion: $(F_{(0,0)} + F_{(0,\pi)} - F_{(0,\pi/2)} - F_{(0,-\pi/2)} + F_{(\pi,\pi)} + F_{(\pi,0)} - F_{(\pi,-\pi/2)} - F_{(\pi,\pi/2)})/8$
$= X^2(t)\{f^+_{MM} \cos(2\psi + \alpha_{MM})\} + O(A^4)$;
3. wave-motion: $(F_{(0,0)} - F_{(0,\pi)} - F^H_{(0,\pi/2)} + F^H_{(0,-\pi/2)} + F_{(\pi,\pi)} - F_{(\pi,0)} - F^H_{(\pi,-\pi/2)} + F^H_{(\pi,\pi/2)})/8$
$= A(t)X(t)\{f^+_{WM} \cos(\phi + \psi + \alpha^+_{WM})\} + O(A^4)$;

where ^{H} denotes the Hilbert transform. Similar expressions can be used to isolate the difference-frequency or higher-order response terms.

3 RESULTS

We report tests from the wave flume in the Coastal and Offshore Engineering Laboratory at UWA. The tests involved a spun aluminium sphere of diameter 0.25 m attached to a single-axis actuator mounted vertically on the lateral centreline of the 1.5 m wide flume. In the mean position, the sphere was half immersed and was made to follow a set displacement signal (displacement control) when in motion. The actuator was synchronised with the wavemaker and 'wave group' motions imposed on each, such that the sphere motions occurred as the waves passed the location of the sphere. Forces on the sphere and free-surface elevations in the vicinity of the sphere were measured, with a load cell and resistance wave gauges respectively.

Results for free surface elevations 0.5 m up-wave from the sphere are shown in Fig. 1. Figure a) shows the linear response - note that this includes the incident wave. Figure b) illustrates the wave-wave second-order terms, as derived from the expressions above, and from diffraction-only tests. The double-frequency wave packet at around 90 s is due to incident error waves from the wavemaker. Overall, the agreement is highly satisfactory. Figure c) shows the same information for the motion-motion terms, compared to a pure radiation case. The last Figure shows the different components of the second-order surface elevation at this location together. It is apparent that due to phase differences, the total may give a misleading impression of the size of the component terms.

Evidently, there are other ways of obtaining the terms discussed here. The diffraction and radiation tests can be used to reduce the number of combinations needed - however this should be considered to be one possible implementation of generalised amplitude manipulation which offers additional possibilities. It may be important to have redundancy in the number of cases studied. As each wave group is relatively short, the tests can be completed quickly. The required tests can typically be completed in one afternoon. Second-order terms can be explicitly calculated [4]; however, the present method may be useful to confirm theoretical results, extract these terms from fully nonlinear simulations or form a basis for generalisation to higher order where the authors are not aware of methods to calculate these terms separately. Further results and implications will be expounded at the Workshop.

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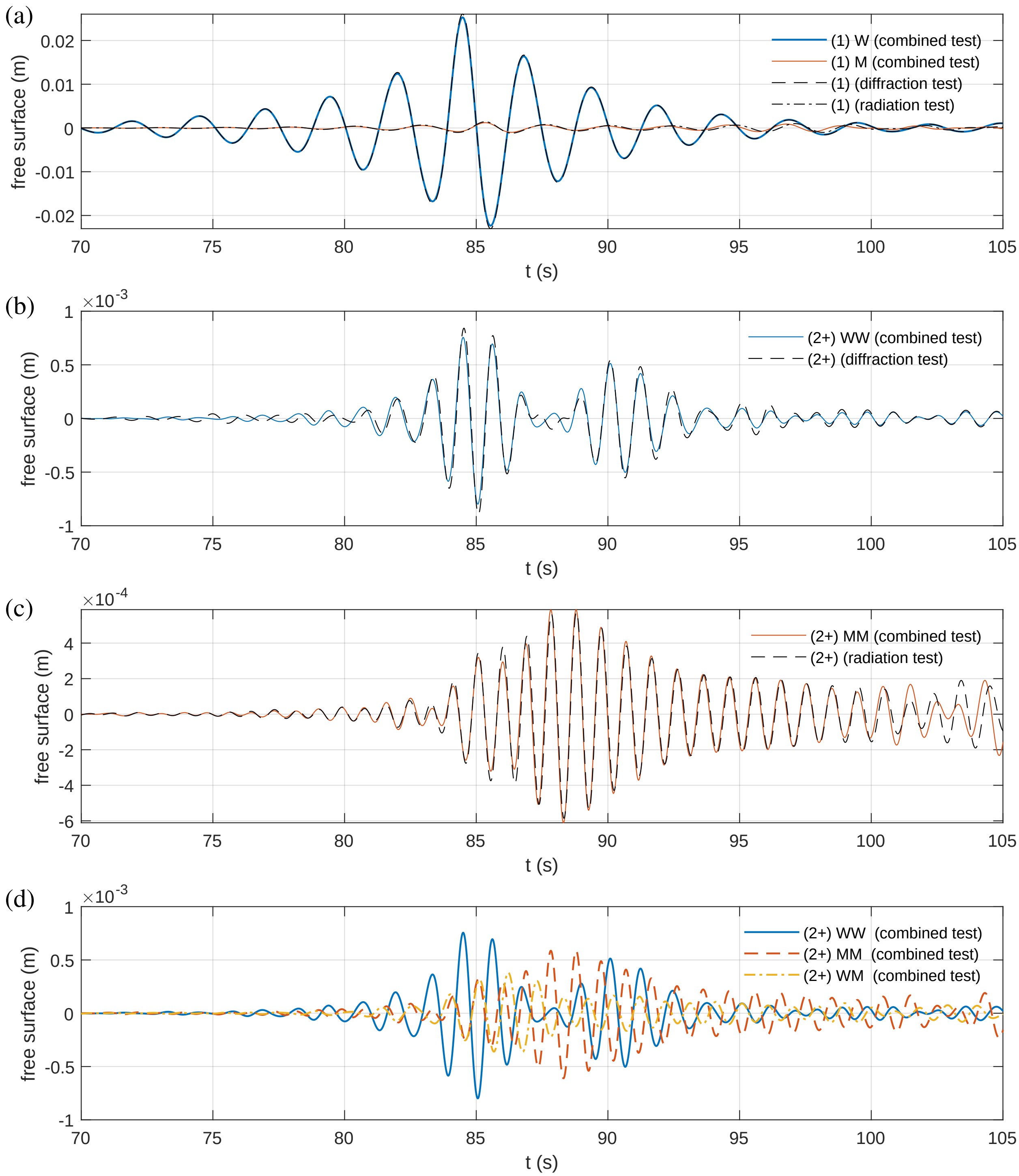

Figure 1: Free surface time series for $f_p = 0.4$ Hz, input linear free surface amplitude at focus $a_f = 0.025$ m and maximum input motion amplitude $a_{m,\max} = 0.08$ m.