# An experimental investigation of nonlinear wave loading on a vertical cylinder – Stokes type expansion and secondary load cycle

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#### 1. INTRODUCTION

Offshore structures, including offshore wind turbine foundations, marine renewable energy device support structures, bridge piers, and floating vessels, are routinely exposed to harsh environmental loads. These frequently drive the design. The physics and statistics of wave-structure interaction are complex and still not fully understood for non-linear loads as experienced in the most severe conditions. This paper analyses loads in focused waves and random sea-states on a bottom-mounted vertical cylinder in the regime relevant to fixed offshore wind turbines. We compare high-quality wave tank measurements, fully nonlinear viscous model simulations, and novel Gaussian Process model predictions based on previous experiments. We consider the Stokes-type force including high-order harmonics. In addition, the secondary load cycle plays a role in the ringing response of the cylinder (e.g. [1, 2, 3]). We also investigate the physics with a novel three-phase decomposition and wavelet analysis.

## 2. METHODS

**2.1 Experimental setup** We perform a new set of experiments in the large flume (76 m long, 4.6 m wide with a constant water depth of 1.8 m) at Kelvin Hydrodynamics Laboratory, the University of Strathclyde. A single bottom mounted surface-piercing vertical cylinder with a radius (R) of 0.2 m was placed 35.3 m away from the wavemaker. Focused wave groups and random waves are generated following the JONSWAP spectrum ( $\gamma = 3.3$ ). Test conditions are given in the figure captions.

**2.2 Numerical simulations** We employ computational fluid dynamics (CFD) models to expand the nonlinear wave loading database with a range of conditions beyond the physical experiments. Two CFD models, OpenFOAM and a Particle-In-Cell solver (PIC), are used for simulations and cross-validations. OpenFOAM is an open-source mesh-based Eulerian model; PIC is an in-house hybrid model developed at Bath and combining a set of Lagrangian particles and an Eulerian grid [4].

2.3 Harmonic components extraction and Stokes type force model We follow the four-phase harmonic extraction method presented in [5, 6]. This method allows the decomposition of nonlinear forces into different harmonics in frequency  $(F_{1,2,3,4,5})$  by repeating the experiments with four different phases (see Eqn 2 in [6] for details) and accurate prediction of high-frequency components based on linear wave force only. Instead of using linear-squared-fitted force coefficients for calculating higher-order forces [5], we further improve the method with a novel Gaussian Process model, which is trained on the results from three previous experiments [5, 6, 7]. This Stokes-GP model is then validated against the new (unseen) experiments.

### 3. RESULTS - STOKES FORCE MODEL

The performance of the Stokes-type force model is evaluated for two types of wave fields: focused wave groups and random seas. We are primarily interested in the high-frequency harmonics, which are very important for violent wave cases [5].



Figure 1: Stokes type model predictions of nonlinear forces for (a): total inline forces, (b): higher order harmonics of a wave group with  $Ak_n = 0.12$ ,  $Rk_n = 0.13$  and  $Dk_n = 1.16$ .



Figure 2: Stokes type model predictions of nonlinear forces of random time series for (a): total inline forces, (b): force exceedance plot with  $H_s = 0.22m$  and  $T_p = 2.77s$ , (c): force difference between Stokes-GP model and measured force.

We present the total inline force time histories for a wave group in Figure 1 (a), where the prediction from the Stokes-GP prediction model agrees well with the measured force. Both numerical models also compare well with experimental data, here the PIC method is slightly better. We then separate the total force into different higher-frequency components up to the 5<sup>th</sup> in Figure 1 (b). The fitted force based on the Stoke-type force model is also presented in red. The extracted higher-order harmonics agree well with the measured forces except for the  $3^{rd}$ , where the Stoke-type force model seems to deviate from the measured force. This deviation is also observed in previous studies [8], and can be attributed to extra nonlinear physics. We have also applied the same Stokes-GP prediction model to random time series separated into wave groups based on the force envelope. The force predictions for the peaks of the random time series are shown in Figure 2, where the Stokes-GP model can predict the force exceedance well.

#### 4. RESULTS - IMPACT FORCES

In this section, we further investigate the nonlinear forces looking beyond the Stokes-type expansion – to the secondary load cycle and the associated structural responses using both three-phase decomposition and wavelet analysis.

**4.1 Three-phase decomposition** We modify the four-phase decomposition method discussed in previous studies to predict the harmonic forces  $(F_{1,2,3,4,5})$  based only on three of the four phases in the decomposition method. The idea is that local high-frequency forcing beyond the Stokes expansion form



Figure 3: Three-phase decomposition prediction of total nonlinear forces for (a): a steep wave group without ringing  $(Ak_p = 0.17)$ , (b): a wave group with ringing effect associated with secondary load cycle  $(Ak_n = 0.18)$ .



Figure 4: Wavelet analysis of inline force series for (a): three phase decomposition prediction, (b): measured inline force, (c): the difference between (a) and (b).

only occurs for the 4<sup>th</sup> phase experiment (taken here as the 0 deg case  $\mathbb{F}_0$ ). Then we can re-construct what the total force time history of  $\mathbb{F}_0$  phase would be if the additional local forcing hadn't occurred. The three-phase harmonic extraction (using  $\mathbb{F}_{90,180,270}$ ) can be written as:

$$AF_{1} - A^{4}F_{4} + A^{5}F_{5} + O\left(A^{6}\right) = -\frac{1}{4}\left(\mathbb{F}_{90} + 2\mathbb{F}_{180} + \mathbb{F}_{270} + \mathbb{F}_{90}^{H} - \mathbb{F}_{270}^{H}\right)$$

$$A^{2}F_{2} - A^{4}F_{4} + O\left(A^{5}\right) = -\frac{1}{2}\left(\mathbb{F}_{90} + \mathbb{F}_{270}\right)$$

$$AF_{1} - 1A^{3}F_{3} + A^{5}F_{5} + O\left(A^{7}\right) = \frac{1}{2}\left(-\mathbb{F}_{90}^{H} + \mathbb{F}_{270}^{H}\right)$$
(1)

We obtain direct identification of the 'ringing' excitation beyond the higher-order harmonics from the additional nonlinear forces beyond the Stokes-type model, as shown in Figure 3 (b), where the additional secondary load cycle and structural responses are separated from the Stokes harmonics. For a steep wave group without significant dynamic response in Figure 3 (a), the proposed three-phase decomposition method re-creates the total harmonic force well.

4.2 Wavelet Impact analysis We further investigate the spatial-temporal energy distribution of a wave group with wavelet analysis. In Figure 4 (a), we show the wavelet spectrum of the three-phase 'non-ringing' predicted force, which shows the clear structure of higher-order harmonics. The

measured force (in Figure 4 (b)), however, shows clear additional impacts, which happen in a very short time duration so over a wide range of frequency. The differences between these two (in Figure 4 (c)) demonstrate the secondary load cycle effect: minor differences in the linear range but significant energy increase at the high-frequency range, which is superficially similar to the energy increase reported during slamming impacts [9].

**4.3 Secondary load cycle** We finally investigate this apparent impact in the opposite direction of wave propagation direction associated with the secondary load cycle. From the synchronised video, we observe a clear splashing of water from a head-on collision of disturbances around the back of the cylinder at the impact time. The detailed investigation of this is still in progress and we will report more at the workshop at the time of the presentation.



Figure 5: Synchronised video with measured surface elevation and force

#### 5. CONCLUSIONS

- 1. The previously presented Stokes-type force expansion captures much of the measured forces.
- 2. An improved fitting method is used to predict the force time history in new tests.
- 3. We confirm an additional load component 'the Grue and Huseby secondary load cycle', associated with a violent splash behind the cylinder.

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