Flow over a semicircular obstruction in a channel covered by broken ice

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HIGHLIGHTS: In this work, we investigate the nonlinear interaction between the flow over a semi-circular obstruction on the channel bottom and the broken ice covering the liquid. We provide evidence that the length of the progressive wave increases with the ice thickness; a steady flow may exist for larger obstruction heights when the ice thickness increases.

INTRODUCTION

An ocean region between open sea and dense drift ice is subjected to fragmentation due to storms and waves coming from open waters. Fragmented ice is mobile, affects ice dynamics, promotes overall sea ice melting, and allows waves to propagate in the ice-covered oceans. Although the region of dense ice has been reduced over the last 40 years, the area of MIZ is not decreased as it follows from satellite observations [1]. This can be explained by the fact that ocean waves coming from open waters penetrate into the ice field and break the ice at the same distance, which is mostly governed by the height of the coming waves and the process of its attenuation [2,3]. Paper [4] presents a comprehensive review of mathematical models of ice/liquid interaction in MIZ. They can be divided into two types: the first is the continuum mat models [5] Peters (1950) where the mat is assumed to have certain rheological properties describing the marginal ice zone; and the second is that dealing with a solitary flexible ice floe, followed by interactions of many such floes [6]. Most of the mathematic models mentioned above are based on a linear velocity potential theory.

In this paper, we study the nonlinear progressive wave generated by an obstruction on the bottom of a channel covered by broken ice. We apply the integral hodograph method for solving the two-dimensional nonlinear problem of steady flow over an obstruction on the channel bottom coupled with the mass loading model of the broken ice. The problem is reduced to a system of integral equations, which is solved using the collocation method. The coupling of the broken ice mat and the liquid motion is based on the condition that the ice mat moves along the ice/liquid interface and its acceleration is caused by a force acting from the liquid. Numerical results are presented for a semicircular obstruction, for both the subcritical and the supercritical regime. The effects of wave phase speed (Froude number) and ice thickness are studied.

THEORETICAL ANALYSIS

We consider the steady, two-dimensional potential flow of an inviscid, incompressible liquid in a channel with an arbitrary bottom shape covered by a broken ice mat. Far upstream and downstream, the flow is uniform, with constant velocity $\boldsymbol{U}$ and fixed depth $\boldsymbol{H}$. The liquid is subject to the downward acceleration of gravity $\boldsymbol{g}$, and the thickness of the ice mat is $\boldsymbol{h}$. A Cartesian coordinate system XY is defined with the origin at the bottom and the X-axis along the velocity direction of the incoming flow with a constant velocity $\boldsymbol{U}$. The problem is nondimensionalized relative to the velocity $\boldsymbol{U}$ and depth $\boldsymbol{H}$. The velocity potential ϕ and the stream function ψ are normalized to the product $\boldsymbol{UH}$. The channel bottom is taken to be the $\psi = 0$ streamline, so that the free surface is $\psi = 1$. The shape of the bottom is defined by a function $Y_b(\bar{X})$, or the angle $\beta(S) = dY_b/dX$, which is the slope to the X −axis. The ice/liquid interface is defined by a function $Y(X)$. The obstruction may generate waves extending to downstream infinity. In order to satisfy the radiation condition, we introduce damping regions T_1T_2 downstream, where a term providing wave damping is added in the dynamic boundary condition. The similar problems based on the linear theory were studied by Shishmarev et al. [7] for a body moving under continuous ice at a constant speed, Xue er al. [8] for a load moving along a sheet with a lead, and Zavyalova et al. [9] for a load moving along a channel covered with broken ice.

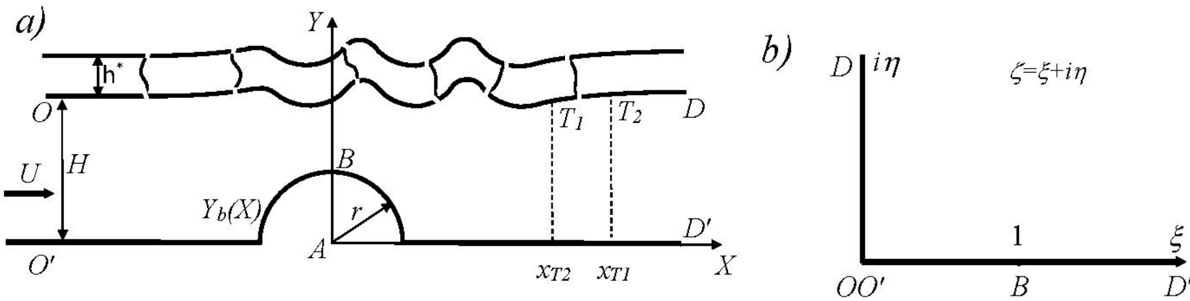

Fig.1 Definition sketch: *a*) the physical plane, and *b*) the parameter plane, or ζ-plane.

Mass-loading model. The ice moving along the interface experiences a vertical acceleration caused by the pressure difference on the upper and lower sides of the ice mat

$$\rho_{ice} h \frac{\partial^2 Y}{\partial t^2} = P_i - P_a, \tag{1}$$

where ρ_{ice} is the ice density, and $\frac{d^2Y}{dt^2} = \frac{d^2Y}{dS^2}U^2$. We assume that the ice mat is not compressible/ extendable, i.e., it moves together with the liquid at infinity with constant speed $\frac{dS}{dt} = U$, although the speed of the liquid on the interface, V, determined from the Bernoulli equation may differ from U. In dimensionless form the Bernoulli equation can be written as follows

$$v^2 = 1 - \frac{2(y-1)}{F^2} - 2\frac{\rho_{ice}}{\rho} h \frac{\partial^2 y}{\partial s^2}, \tag{2}$$

where the Froude number $F = \frac{U}{\sqrt{gH}}$ and ρ is the liquid density.

The dispersion equation for the mass-loading model derived using a linear theory [3] is

$$kH \tanh kH = \frac{\omega^2}{\omega^2 - \rho_{ice}/\rho h \omega^2}, \tag{3}$$

where ω is the wave frequency, and k is the wave number. The wave is steady relative to the obstruction; therefore, $U = \frac{\lambda}{T} = \frac{2\pi}{k}\frac{\omega}{2\pi} = \frac{\omega}{k}$, or $\omega = Uk = Fk\sqrt{gH}$. Then, equation (3) can be rewritten as

$$\frac{kH}{\tanh kH} = \frac{1}{F^2}\left(1 - \frac{F^2 k^2 g}{\omega_b^2 H}\right), \tag{4}$$

where $\omega_b^2 = \frac{\rho g}{\rho_{ice} h}$ is the natural frequency of an ice float of thickness h. The wave number versus the Froude number for various ice thicknesses is shown in figure 2.

Governing expressions. We will derive the complex potential of the flow, $w(z) = \phi(x,y) + i\psi(x,y)$, with z= $x + iy$. For a steady flow, the kinematic conditions on the body surface and the interface mean that the stream function is constant, or $\psi(x, y(x)) = const$, as they both are streamlines. We introduce the first quadrant as an auxiliary parameter plane, or $\zeta -$ plane, and determine two functions, which are the complex potential, $w(\zeta)$, and the function $\omega(\zeta) = -\ln\frac{dw}{dz} + i\theta$. Then, the flow region can be obtained in parameter form as follows:

$$\frac{dw}{dz} = \exp[-\omega(\zeta)], \qquad z(\zeta) = z_0 + \int_0^{\xi} \frac{dw}{d\zeta'} / \frac{dw}{dz} d\zeta', \tag{5}$$

where $z(\zeta)$ is the mapping function.

The region of the complex potential corresponding to the flow region in the physical plane is the infinite strip $-\infty<\phi<\infty$ of unit width, i.e., on the bottom surface $O'D'$, $\psi(x, y_b(x)) = 0$ and at the interface OD, $\psi(x, y(x)) = \frac{Q}{UH} = 1$. Due to the simplicity of the region w, we can find the complex potential by using the conformal mapping technique,

$$w(\zeta) = \frac{1}{2\pi} ln\zeta, \qquad \text{and} \qquad \frac{dw}{d\zeta} = \frac{1}{2\pi}. \tag{6) - (7}$$

The boundary conditions for the complex velocity function, dw/dz, can be written as follows:

$$arg\left(\frac{dw}{dz}\right) = Im\left[ln\left(\frac{dw}{dz}\right)\right] = -\beta_b(\xi), \qquad 0 \le \xi < \infty,\ \eta = 0. \tag{8}$$

$$v(\eta) = \left|\frac{dw}{dz}\right|, \qquad 0 \le \eta < \infty, \quad \xi = 0. \tag{9}$$

where $\beta_b(\xi)$ is the slope of the bottom surface, which is a known function of the coordinate s, $\beta(s)$, on the real axis, and $v = v(\eta)$ is the magnitude of the velocity at the interface OD. The following integral formula gives the solution of the mixed boundary-value problem (8) – (9)

$$\frac{dw}{dz} = v_0 exp\left[\frac{1}{\pi}\int_0^{\infty}\frac{d\beta_b}{d\xi} ln\left(\frac{\xi-\zeta}{\xi+\zeta}\right)d\xi - \frac{i}{\pi}\int_0^{\infty}\frac{d\ln v}{d\eta} ln\left(\frac{i\eta-\zeta}{i\eta+\zeta}\right)d\eta - i\beta_{b0}\right], \tag{11}$$

where $v_0 = v(0)$ and $\beta_{b0} = \beta_b(0)$=0 are the velocity magnitude and angle at upstream infinity (point *O)*. The function $\beta_b(\xi)$ and $v = v(\eta)$ have to be determined from the boundary conditions on the bottom of the channel and at the ice/liquid interface.

RESULTS

For verification purposes we computed subcritical free surface flow, $F = 0.5$, for a semi-circular obstruction of radius $r = 0.2$ on the bottom of the channel. The results for $h = 0$ (the free surface) are compared with those obtained by Forbes & Schwartz (FS) [10] (see figure 3, $h = 0$). The agreement is quite good, except for the end wave in FS, which is the effect of an abrupt calculation region. In our case the calculation region is much longer, and the wave gradually decreases downstream within a damping zone of length 4λ (beyond the region shown in Fig.2). The shape of the free surface for $h = 0$ and radius $r = 0.2$ exhibits a sharp crest and an extended trough, which corresponds to the limit configuration; for larger obstruction radii a steady flow may not exist.

The wave length increases as the thickness of the broken ice increases, which can be seen in Fig.2 for $h/H = 0.2$ and 0.5. This is consistent with the smaller wave number obtained from the dispersion equation in Fig.2. For larger ice thickness, the shape of the interface approaches a sinusoidal curve as in the case of obstructions of a smaller radius. This suggests that a steady flow with an ice sheet may exist for a larger obstruction radius. From Fig.3 it can be seen that the amplitude of the interface depends almost not at all on the ice thickness.

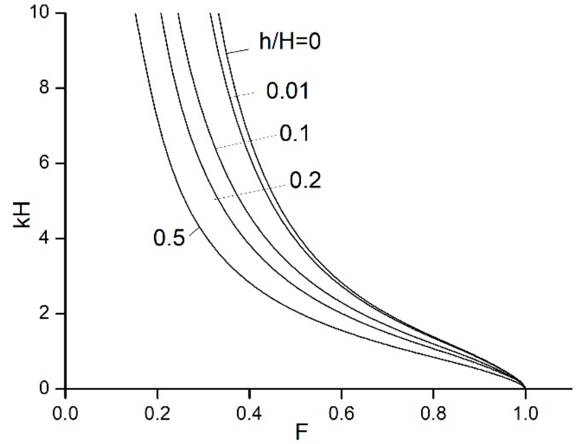

Fig.2. Wave number versus Froude number for various ice thicknesses

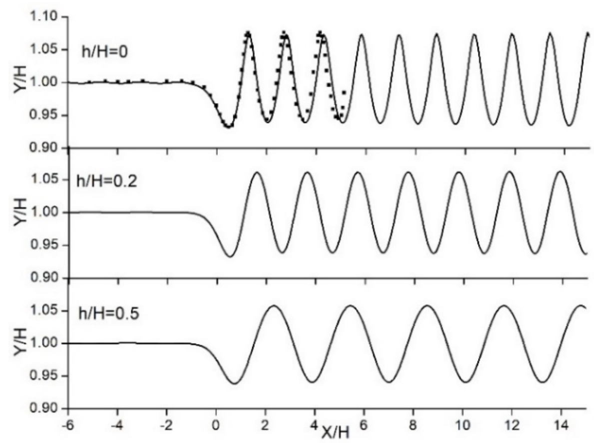

Fig.3. Interface shape for various ice thicknesses: the solid squares correspond to Forbes & Schwartz for the free surface flow with a semi-circular obstruction, r=0.2.

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