Fully nonlinear interaction of a periodical wave propagation by a forced oscillating floater in two-layer fluids

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HIGHLIGHTS

- Fully nonlinear numerical wave tank with a forced oscillating floater in two-layer fluids is developed using dual-domain boundary element method.
- Free surface and internal waves are analyzed in terms of two wave modes(barotropic and baroclinic modes), and linear and fully nonlinear computational results are also compared.

1 INTRODUCTION

Observations of internal waves at the interface of two fluids of different densities have been continuously reported and several studies on internal-wave generation and propagation for various stratified fluids have also been published. In a two-layer fluid system, most studies have been for internal solitary waves(ISWs) and they investigated the generated and propagated ISWs using the gravity-collapse approach[1-5]. In addition, there was a numerical study on a fully nonlinear interaction between ISWs and free surface using a multi-domain boundary element method(MDBEM)[6] and another numerical study on ISW generation using a coupled mass source(CMS) method[7]. There have also been several experimental studies on the generation and propagation of internal periodical waves(IPWs)[8,9]. There were several numerical studies on IPWs based mostly on linear theory [7,10].

In this study, free surface and internal periodical waves generated by a heaving floater are analyzed using two-dimensional fully nonlinear numerical wave tank(FN-NWT) technique based on dual-domain boundary element method(BEM). The free surface and internal boundaries are modeled by mixed Eulerian-Lagrangian(MEL) method and full Lagrangian(Material-node) approach, and the elements on these boundaries are updated at each time step. The two-layer fluid system has two different wave modes called barotropic and baroclinic modes. The two wave modes are systematically compared between linear and fully nonlinear NWTs and the wave elevation snapshots and respective-order wave components are extracted by Fast Fourier Transform(FFT).

2 MATHEMATICAL FORMULATIONS

Each fluid domain is assumed to be a potential flow. So, Laplace equation for a velocity potential(ϕ) can be used as the governing equation of each fluid domain as follow Eq. (1). This equation is transformed to boundary integral equation(Eq. 2) by using Green's 2nd identity.

$$\nabla^2 \phi^{(m)} = 0 \qquad [m\text{=1(upper), 2(lower)}] \tag{1}$$

$$\alpha \phi_i^{(m)} = \iint_{\Omega^{(m)}} \left(G_{ij}^{(m)} \frac{\partial \phi_j^{(m)}}{\partial n} - \phi_j^{(m)} \frac{\partial G_{ij}^{(m)}}{\partial n} \right) ds \qquad [m\text{=1, 2}] \tag{2}$$

where $\Omega^{(m)}$ means each domain and m=1, 2 correspond upper and lower domains, respectively; α is the solid angle and it has 0.5 on the boundary; Green's function is defined as $G_{ij} = -\frac{1}{2\pi} \ln R_1$ and R_1 is the distance between source and field points. To generate waves, a body boundary condition can be defined using a body velocity.

$$\frac{\partial \phi^{(1)}}{\partial n} = V \cdot \hat{n} \qquad \text{on a body} \tag{3}$$

in which, $V(=Y\omega\cos(\omega t)$, Y is a forced oscillation amplitude) is a floater velocity. Eqs. (4) and (5) are nonlinear dynamic and kinematic free surface boundary conditions. To represent the movement of free surface boundary, the mixed Eulerian-Lagrangian(MEL) method is used. The free surface nodes are rearranged using total derivative($\frac{\delta}{\delta t} = \frac{\partial}{\partial t} + \vec{v}_1 \cdot \nabla$), and the free surface node velocity is assumed to be equal to the fluid particle velocity($\vec{v}_1 = \nabla \phi^{(1)}$) for material node approach(Full Lagrangian approach).

$$\frac{\delta \phi^{(1)}}{\delta t} = g\eta_1 + \vec{v}_1 \cdot \nabla \phi^{(1)} - \frac{1}{2}\left|\nabla \phi^{(1)}\right|^2 \qquad \text{on } z=0 \tag{4}$$

$$\frac{\delta \vec{x}_1}{\delta t} = \nabla \phi^{(1)} \qquad \text{on } z=0 \tag{5}$$

where g is a gravity acceleration and η_1 is a free surface elevation. Nonlinear interface boundary conditions are defined as follow Eq. (6), (7). As with the free surface boundary conditions, interface nodes are rearranged using total derivative($\frac{\delta}{\delta t} = \frac{\partial}{\partial t} + \vec{v}_2 \cdot \nabla$), and the interface node velocity is assumed to be equal to the particle velocity on the interface($\vec{v}_2 = \nabla \phi^{(2)}$).

$$\rho_1\left(\frac{\delta \phi^{(1)}}{\delta t} + g\eta_2 - \vec{v}_2 \cdot \nabla \phi^{(1)} + \frac{1}{2}\left|\nabla \phi^{(1)}\right|\right) = \rho_2\left(\frac{\delta \phi^{(2)}}{\delta t} + g\eta_2 - \vec{v}_2 \cdot \nabla \phi^{(2)} + \frac{1}{2}\left|\nabla \phi^{(2)}\right|\right) \qquad \text{on } z=-h_1 \tag{6}$$

$$\frac{\partial \phi^{(1)}}{\partial n} = -\frac{\partial \phi^{(2)}}{\partial n} \;, \qquad \frac{\delta \vec{x}_2}{\delta t} = \nabla \phi^{(2)} \qquad \text{on } z=-h_1 \tag{7}$$

Finally, a rigid and impervious boundary condition is applied on the side wall and bottom boundaries as follow equation.

$$\frac{\partial \phi^{(m)}}{\partial n} = 0 \tag{8}$$

3 RESULTS AND DISCUSSION

In this study, numerical calculation is conducted under follow conditions: density ratio(γ=0.5), forced oscillation amplitude(Y=0.05m), water depth(h_1=0.15m, h_2=0.25m), body draft(d=0.08m) and body width(B=0.2m) as shown in Fig. 1. The dispersion relationship[11] of propagating waves in two-layer fluids can be expressed as follow equation. Based on this equation, wave numbers according to oscillating frequencies are shown in Table 1.

$$\omega^2 = \frac{gk}{2(1+\gamma th1 \cdot th2)}\left[th1 + th2 \pm \sqrt{(th1+th2)^2 - 4th1 \cdot th2(1-\gamma)(1+\gamma th1 \cdot th2)}\right] \tag{9}$$

in which γ is density ratio of upper and lower fluid domain(=ρ_1/ρ_2); $th1$=tanh(kh_1); $th2$=tanh(kh_2); k is wave number(=$2\pi/\lambda$). Positive(+) and negative(-) signs in RHS in Eq.(9) correspond to barotropic and baroclinic modes, respectively. Therefore, there are two different wave numbers(k_1: Barotropic and k_2: Baroclinic mode) according to one frequency(ω). In general, free surface wave is predominant in barotropic mode and internal wave is predominant in baroclinic mode. Detailed characteristics of each wave mode were presented in [10, 11].

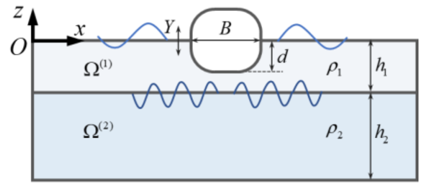
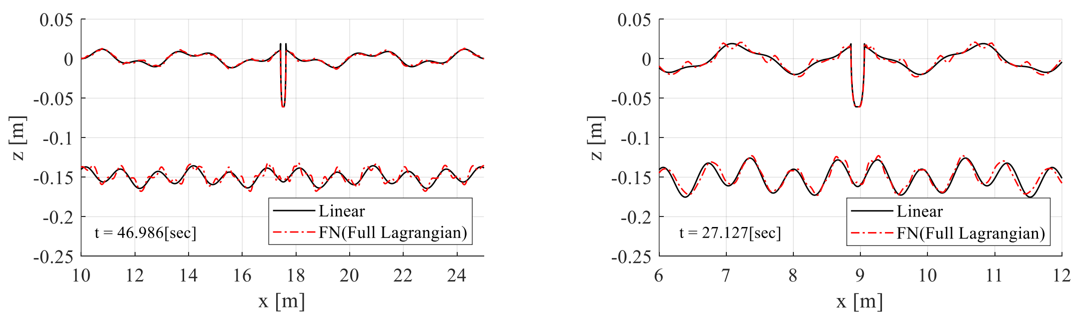


Figure 1: Sketch of two-layer fluids system with an oscillating body

Table 2: Wave number in two-layer fluids system at each wave mode

	***KB*/2**	***ω*[rad/s]**	***k*₁(*ω*)[rad/m]**	***k*₂(*ω*)[rad/m]**	***k*₁(2*ω*)[rad/m]**	***k*₂(2*ω*)[rad/m]**
Case I	0.1	3.13	1.803	4.665	4.383	12.431
Case II	0.3	5.43	3.547	9.768	12.009	36.019

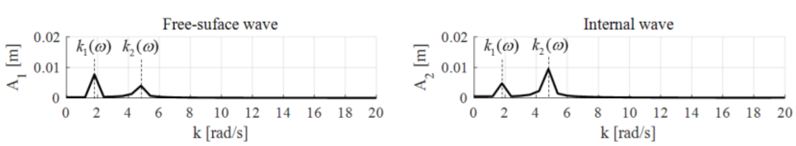
In Fig. 2, snapshots of free surface and internal wave elevations are compared for linear and fully nonlinear calculation. When the oscillation frequency is low, there is the difference in internal waves between linear and fully nonlinear calculations (in Fig. 2(a)). However, when the oscillation frequency increases, the nonlinearity of free surface waves becomes predominant (in Fig. 2(b)).



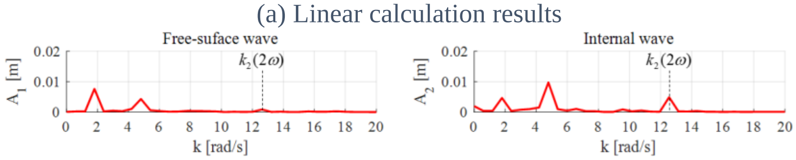
(a) Case I ($KB/2$=0.1) (b) Case II ($KB/2$=0.3)

Figure 2: Snapshots of free surface and internal wave elevation

Also, using these snapshots, wave amplitudes(A_1:Surface wave, A_2:Internal wave) corresponding to each wave number are extracted by Fast Fourier Transform(FFT) technique in Fig. 3 to 4. In the linear calculation, only primary frequency components in each wave mode($k_1(\omega)$, $k_2(\omega)$) are extracted, but in the fully nonlinear calculation, higher order frequency components of each wave mode are also detected.

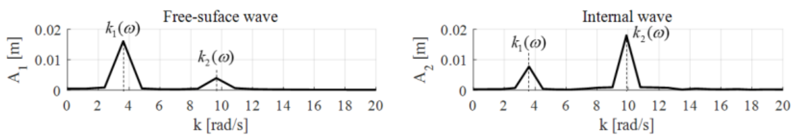


(a) Linear calculation results

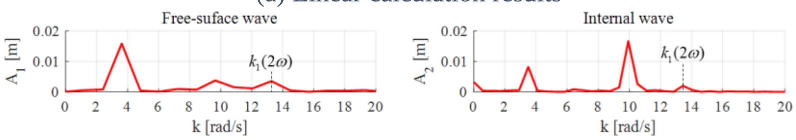


(b) Fully nonlinear calculation results

Figure 3: Fast Fourier Transform(FFT) results of wave amplitudes corresponding to wave numbers(k) [$KB/2$=0.1]



(a) Linear calculation results



(b) Fully nonlinear calculation results

Figure 4: Fast Fourier Transform(FFT) results of wave amplitudes corresponding to wave numbers(k) [$KB/2$=0.3]

In the low frequency($KB/2$=0.1), FFT results of fully nonlinear calculation can extract the wave number component corresponding to 2nd order frequency($k_2(2\omega)$) in baroclinic mode not only internal wave but also surface wave (in Fig. 3(b)). On the other hand, in case of $KB/2$=0.3, the wave number component corresponding to 2nd order frequency($k_1(2\omega)$) in barotropic mode is detected from surface and internal waves (in Fig. 4(b)). Also, the baroclinic mode effect(k_2) on free surface waves decreases when the oscillation frequency increases.

4 CONCLUSIONS

In this study, fully nonlinear interaction between free surface and internal waves was studied by using FN-NWT in two-layer fluids. The FN-NWT was modeled by dual-domain BEM, and the free-surface and interface boundaries were rearranged by MEL method with Full Lagrangian approach. The waves were decomposed into respective-order wave frequency components by using FFT technique. Higher-order frequency components can be detected in not only barotropic mode but also baroclinic mode as nonlinearity increases in the fully nonlinear system. When the oscillation frequency was low, the nonlinearity in the baroclinic mode became stronger, and as the frequency increased, the nonlinearity in the barotropic mode became dominant.

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