Flexible Ice Floe Melting due to Waves by Smoothed Particle Hydrodynamics

Michael H. Meylan[1] **Thien Tran-Duc**[2] and Ngamta Thamwattana
[1]School of Information and Physical Sciences, University of Newcastle, NSW 2308, Australia
mike.meylan@newcastle.edu.au

HIGHLIGHTS Ice melting under the impacts of water waves is studied numerically via Smoothed Particle Hydrodynamics (SPH) simulations. Effects due to the ice elasticity are also included. The simulations show that water waves' effects on the ice melting are seen via overflow over the top surface and local fluid circulations in the submerged region due to water-ice interactions and wave motions. Those effects result in a melting amount of the ice plate up to 1.78 times higher than the ice in still water.

1 INTRODUCTION

The interaction of waves with sea ice has far-reaching implications in geophysics and marine engineering, and it is the subject of extensive ongoing research. In particular, understanding how waves break up and melt the ice is of great importance in marine safety and accurate climate models. Recently, overwash [1–3], the process by which waves induce over-topping of water on ice floes, has been recognised as a significant effect which is very poorly understood. In this current work, a smooth particle hydrodynamics model for the motion, including overwash, is extended to model the floe melting.

2 SPH MODEL

The fluid is modelled by a set of SPH fluid particles who obey the Navier-Stokes equations,

$$\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_i \cdot \boldsymbol{\nabla} \mathbf{u}_i = -\frac{1}{\rho_i} \boldsymbol{\nabla} P_i + \nu_i \boldsymbol{\nabla}^2 \mathbf{u}_i + \mathbf{g}, \tag{1}$$

in which $\mathbf{u}_i$, P_i, ν_i, and $\mathbf{g}$ are the particle's velocity, pressure, viscosity, and gravity acceleration, respectively. In addition, the particles' pressure satisfies the Poisson equation,

$$\boldsymbol{\nabla} \cdot \left(\frac{1}{\rho_i} \boldsymbol{\nabla} P_i \right) = \frac{1}{\triangle t} \boldsymbol{\nabla} \cdot \mathbf{u}_i. \tag{2}$$

In an incompressible SPH (ISPH) scheme using the prediction-correction algorithm in [4], the intermediate value for the SPH particle's velocity and position is estimated first

$$\mathbf{u}_i^* = \mathbf{u}_i^n + \triangle t \left(\nu_i \boldsymbol{\nabla}^2 \mathbf{u}_i^n + \mathbf{g} \right), \tag{3}$$

$$\mathbf{r}_i^* = \mathbf{r}_i^n + \triangle t \mathbf{u}_i^*, \tag{4}$$

in which $\mathbf{u}_i^n$ and $\mathbf{r}_i^n$ are velocity and position of SPH particles of the previous step. The viscous term in the right-hand side of Equation (3) is discretized as follows

$$\nu_i \boldsymbol{\nabla}^2 \mathbf{u}_i^n = \sum_j V_j \frac{\nu_i + \nu_j}{\left(r_{ij}^n \right)^2 + \epsilon^2} \mathbf{u}_{ij}^n \left(\mathbf{r}_{ij}^n \cdot \boldsymbol{\nabla}_i W_{ij} \right), \tag{5}$$

in which ν_i and ν_j are viscosity of particle i and j, V_j is the volume of particle j, $\mathbf{u}_{ij}^n = \mathbf{u}_i^n - \mathbf{u}_j^n$, $\mathbf{r}_{ij}^n = \mathbf{r}_i^n - \mathbf{r}_j^n$, and $r_{ij}^n = |\mathbf{r}_{ij}^n|$. The parameter $\epsilon = 0.001h$ is used to prevent numerical singularity if two particles come too close to each other. In the prediction step, the fluid particle is assumed compressible, and their dynamic density is estimated from the intermediate velocity and position of the particles,

$$\frac{d\rho_i^*}{dt} = \sum_j m_j \mathbf{u}_{ij}^* \cdot \boldsymbol{\nabla}_i W_{ij}, \tag{6}$$

in which m_j is mass of particle j. The pressure field is then obtained by solving Poisson's equation (2) using the intermediate values ($\mathbf{u}_i^*$, $\mathbf{r}_i^*$, ρ_i^*). In the discretized SPH form, Equation (2) reads

$$\sum_j \frac{8m_j}{\left(\rho_i^* + \rho_j^*\right)^2} \frac{\left(P_i^{n+1} - P_j^{n+1}\right)}{\left(r_{ij}^*\right)^2 + \epsilon^2} \mathbf{r}_{ij}^* \cdot \boldsymbol{\nabla}_i W_{ij} = -\frac{1}{\triangle t} \sum_j V_j \mathbf{u}_{ij}^* \cdot \boldsymbol{\nabla}_i W_{ij}. \tag{7}$$

The ice phase is modelled as an elastic object and represented by a set of solid SPH particles. We are allowing for the elastic motion of the ice as we want a very general model, and we have already developed an elastic model [2]. For each of the ice particles, the deformation gradient is evaluated by

$$\mathbf{F}_a = \sum_b V_b^0 \left(\mathbf{x}_b - \mathbf{x}_a\right) \mathbf{N}_a^0 \boldsymbol{\nabla}_a^0 W_{ab}^0, \tag{8}$$

where $\mathbf{x}_a$ and $\mathbf{x}_b$ are the coordinates of the ice particles a and b in the current deformed object. We note that the subscripts a and b are used for the ice particles to distinguish them from the fluid SPH particles, which have the subscripts i and j. The nonlinear Saint-Venant strain tensor is then written for each ice particle as

$$\boldsymbol{\epsilon}_a^0 = \frac{1}{2}\left(\mathbf{F}_a^T \mathbf{F}_a - \mathbf{I}\right). \tag{9}$$

The particle stress is evaluated from

$$\boldsymbol{\sigma}_a^0 = 2G\left(\boldsymbol{\epsilon}_a^0 - \frac{1}{3}Tr(\boldsymbol{\epsilon}_a^0)\mathbf{I}\right) + KTr(\boldsymbol{\epsilon}_a^0)\mathbf{I}. \tag{10}$$

Elastic forces act on the ice particles inducing a force. The accelerations are evaluated from the divergence of the elastic stress. The temperature difference drives heat flux from the ambient water to the ice. The heat transfer process is governed by

$$\rho c \frac{dT}{dt} = -\boldsymbol{\nabla} \cdot \left(k \boldsymbol{\nabla} T\right), \tag{11}$$

in which c and k are heat capacity and thermal conductivity, respectively. Equation (11) is written for every SPH particle, for either the fluid and ice particles, as

$$\rho_i c_i \frac{dT_i}{dt} = \sum_j V_j \left(k_i + k_j\right) T_{ij} \frac{\partial W_{ij}}{\partial r_{ij}}, \tag{12}$$

in which $T_{ij} = T_i - T_j$ is the temperature difference between the reference i and its neighbouring particles j, k_i and k_j are heat conductivity of particles i and j, and c_i is heat capacity of particle i.

3 TWO PHASE COUPLING

Fluid and ice SPH particles at the interface interact with each other via viscous and pressure forces. For convenience, subscripts (i, j) and (a, b) are used to denote fluid particles and solid particles, respectively. The forces acting on the fluid and solid SPH particles can be written

$$\mathbf{a}_i = \frac{1}{\rho_i} \sum_{k \in \{b,j\}} \left(\mathbf{f}^p_{ik} + \mathbf{f}^v_{ik}\right), \tag{13}$$

$$\mathbf{a}_a = \frac{1}{\rho_a} \left[\sum_b \mathbf{f}^e_{ab} + \sum_{k \in \{b,j\}} \left(\mathbf{f}^p_{ak} + \mathbf{f}^v_{ak}\right) \right]. \tag{14}$$

Here, $\mathbf{f}^p_{ik}$ and $\mathbf{f}^v_{ik}$ are the density of the viscous and pressure forces acting on fluid-particle i from its neighbouring fluid $(k = j)$ and solid $(k = b)$ particles.

4 NUMERICAL RESULTS

Numerical experiments are carried out in a water basin of 16 m in length (L) and 4 m in height (H). Height of the water column is set at $d = 1.2$ m. A moving wall is placed at the left boundary of the basin and oscillated sinusoidally according to the following rule,

$$x_{lw} = S \cos\left(\frac{2\pi}{T} t - \pi\right), \tag{15}$$

to generate water waves. In all the simulations, the period of the oscillation is fixed at $T = 1.5$ s, while the oscillation amplitude is adjusted to generate different water-wave scenarios. In order to absorb the energy of incoming water waves and reduce wave reflection, a wall at a slope angle of 18^o is placed on the right boundary of the water basin. An ice plate with 2 m in length (L_p), 0.5 m in thickness (H_p), Young modulus of 10^8 Pa, and temperature of -1 ^{0}C is used in the simulations. In the SPH approximations the cubic weight function,

$$W(r_{ij}, h) = \alpha_d \begin{cases} -\frac{\eta^3}{6} + \eta^2 - 2\eta + \frac{4}{3} & 0 \le \eta \le 1, \\ -\frac{\left(2-\eta^2\right)^3}{6} & 1 < \eta < 2, \\ 0, & \eta \ge 2, \end{cases} , \tag{16}$$

is used. The notation $\eta = r_{ij}/h$ represents the dimensionless distance between particles i and j. The coefficient $\alpha_d = 0.596831$. The particle size of $\triangle x = 1$ cm is used in the simulations based on our previous study on wave-plate interactions(2). Accordingly, the number of SPH particles for the fluid phase, the wall boundaries, and the ice plate are about $151,191$, $18,863$, and $8,864$, respectively. Each simulation is run over a physical period of 5 minutes. Water temperature is initially assumed 20 ^{0}C. We note that this value is much higher than would occur in the marginal ice zone. We choose a high value to give significant melting in short simulations. Water waves are generated by oscillating the left wall at the amplitude of $S = 10$ cm. The generated waves' measured wave height and wavelength are $H = 0.32$ m and $\lambda = 3.44$ m, respectively. Those values are very close to values obtained from the linear wave theory, which are 0.34 m and 3.43 m(2). Figures 1(b)-1(d) shows snapshots of the system taken at different times of $t = 100$ s, 200 s, and 300 s, respectively. Interaction

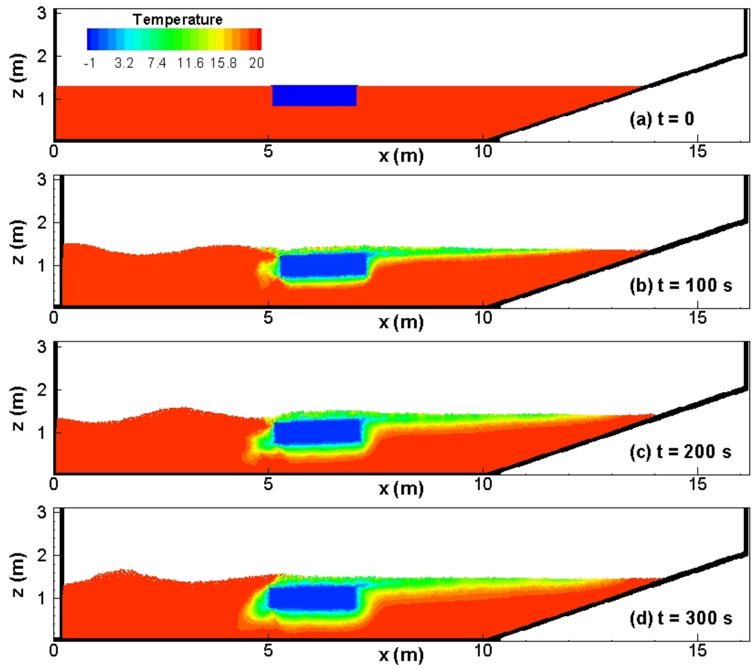

Figure 1: Snapshots for the wave-ice system for wave height $H = 0.32$ m at (a) the equilibrium state ($t = 0$), at (b) $t = 100$ s, (c) $t = 200$ s, and (d) $t = 300$ s.

of the ice plate and incoming water waves results in a fairly strong water flow over the top surface of the ice plate. The highly cooled water is seen not only in close proximity to the ice plate but also in the downstream region.

5 CONCLUSIONS

In this work, melting mechanisms of ice under the impact of water waves are studied numerically using Smoothed Particle Hydrodynamics. The simulations show that, in still water, the ice melting occurs only in the submerged region, while wave-ice interactions in the wave scenarios could create overflows washing over the top surface of the ice plate and local fluid circulations at the contact regions and hence increase melting rate of the ice.

REFERENCES

[1] Tran-Duc, T., M. H. Meylan, M. H., Thamwattana, N., and Lamichhane, B. P. 2020. *Wave interaction and overwash with a flexible plate by smoothed particle hydrodynamics.* Water 12(12), 3354.

[2] Tran-Duc, T., Meylan, M. H., and Thamwattana, N. 2022. *Dynamical Response of a Floating Plate to Water Waves Using a Smoothed Particle Hydrodynamics Algorithm for Nonlinear Elasticity.* Phys. Fluids 34, 053316.

[3] Pitt, J., Bennetts, L., Meylan, M. H., Massom, R., and Toffoli, A. 2022. *Model predictions of wave overwash extent into the marginal ice zone.* J. Geophys. Res. Oceans, e2022JC018707.

[4] Nomeritae, N., Daly, E., Grimaldi, S., and Bui, H. H. 2016. *Explicit Incompressible SPH Algorithm for Free-Surface Flow Modelling: A Comparison with Weakly Compressible Schemes.* Adv. Water Res. 97, 156–167.