Shielding effects of a floating ring-shaped poroelastic plate on a cylinder

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Abstract

At the 18th IWWWFB, Malenica & Korobkin [1] presented water wave scattering by a cylinder in partially frozen sea. In the present study, their work is generalised to the wave interactions with a circular cylinder surrounded by a floating ring-shaped poroelastic plate. Despite the analogy to [1] in mathematical aspects, here the physical focus is placed on the shielding effects due to the floating poroelastic ring-shaped plate on the cylinder.

Boundary value problem

Consider a rigid vertical cylinder of a radius R standing through the water depth h surrounded coaxially by a ring-shaped poroelastic plate, with the inner and outer radii a and b, lying on the free surface under water wave actions. It is assumed that the fluid is incompressible and inviscid, and the flow is irrotational so that a velocity potential is existent. Three-dimensional Cartesian and cylindrical coordinate systems are defined with the $z = 0$ plane on the undisturbed free surface and Oz axis coinciding with the cylinder's axis and orienting positively upward, as in Fig. 1. The horizontal coordinates are $(x, y) = r(\cos\theta, \sin\theta)$.

In a time-harmonic steady state, a time oscillator $\mathrm{e}^{-\mathrm{i}\omega t}$ can be extracted and suppressed here, where ω denotes the angular frequency of oscillation. The complex velocity potential ϕ is subjected to the following boundary value problem

$$\nabla^2\phi = 0 \qquad \text{in the fluid domain} \tag{1}$$

$$-K\phi + \phi_z = 0 \qquad \text{on } z = 0 \text{ and } r \in [R, a] \cup [b, \infty) \tag{2}$$

$$(\chi\bar{\nabla}^4 - K\gamma + 1)\left(\frac{\partial}{\partial z} - \mathrm{i}\sigma\right)\phi - K\phi = 0 \qquad \text{on } z = 0 \text{ and } r \in [a, b] \tag{3}$$

$$\phi_z = 0 \qquad \text{on } z = -h \tag{4}$$

$$\phi_r = 0 \qquad \text{on } r = R \tag{5}$$

$$\lim_{r\to\infty} \sqrt{r}\left(\frac{\partial}{\partial r} - \mathrm{i}k_0\right)(\phi - \phi_I) = 0 \qquad \text{when } r \to \infty \tag{6}$$

where $K \equiv \omega^2/g$ denotes the wavenumber in deep water, k_0 is the wavenumber in finite water depth satisfying the gravity wave dispersion relation $k\tanh(kh) = K$, g is the acceleration of gravity, $\bar{\nabla}$ is the gradient with respect to horizontal coordinates, $\sigma = k_0 b/2\pi$ with b a nondimensional parameter associated with solidity ratio, and χ and γ are defined as $\chi = D/(\rho g)$ and $\gamma = m_u/\rho$, respectively, with m_u denoting the mass of unit area of the plate, D the flexural rigidity of the plate and ρ is the density of water. In Eq. (6) ϕ_I is the incident wave potential written as [2]

$$\phi_I = -\frac{\mathrm{i}gA}{\omega}\frac{\cosh[k_0(z+h)]}{\cosh(k_0 h)}\mathrm{e}^{\mathrm{i}k_0 x} = -\frac{\mathrm{i}gA}{\omega}\frac{\cosh[k_0(z+h)]}{\cosh(k_0 h)}\sum_{m=0}^{\infty}\varepsilon_m \mathrm{i}^m J_m(k_0 r)\cos m\theta, \tag{7}$$

where ε_m equals to 1 when $m = 0$ and 2 otherwise, A is the wave amplitude, and $J_m(\cdot)$ means the mth-order Bessel function of the first kind.

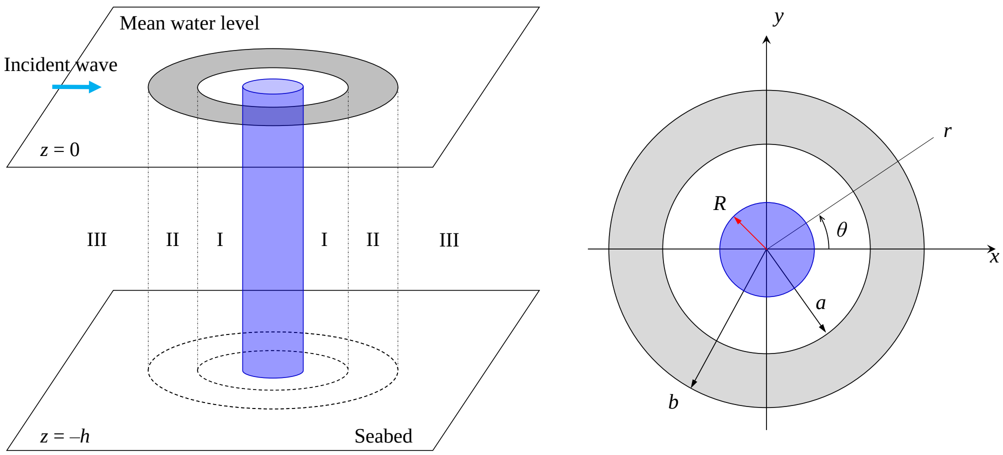

Figure 1: Schematic of wave interactions with a cylinder surrounded by a ring-shaped plate.

At the edges of the annular ring, the edge conditions free of shear force and bending moment are imposed

$$\left[\bar{\nabla}^2 - \frac{1-\nu}{r}\left(\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^2}{\partial \theta^2}\right)\right] w = 0 \quad \text{at } r = a \text{ and } r = b, \tag{8a}$$

$$\left[\frac{\partial}{\partial r}\bar{\nabla}^2 - \frac{1-\nu}{r^2}\left(-\frac{\partial}{\partial r} + \frac{1}{r}\right)\frac{\partial^2}{\partial \theta^2}\right] w = 0 \quad \text{at } r = a \text{ and } r = b \tag{8b}$$

where w denotes the hydroelastic deflection which can be obtained based on the kinematic condition on the plate

$$w = \left(\frac{\mathrm{i}}{\omega}\frac{\partial \phi}{\partial z} + \frac{\sigma}{\omega}\phi\right)_{z=0} \tag{9}$$

Eigenfunction expansion method

The boundary value problem is solved by an eigenfunction expansion method. The fluid domain is divided by two cylindrical surfaces of radii $r = a$ and $r = b$ into three subdomains, as illustrated in Fig. 1. The velocity potentials in each subdomain are expressed as

$$\phi^{\mathrm{i}} = -\frac{\mathrm{i}gA}{\omega}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\varepsilon_m Z_n(z) A_{mn}\left[J_m(k_n r) - \frac{J'_m(k_n R)}{H'_m(k_n R)} H_m(k_n r)\right]\cos m\theta, \tag{10}$$

$$\phi^{\mathrm{ii}} = -\frac{\mathrm{i}gA}{\omega}\sum_{m=0}^{\infty}\sum_{n=-2}^{\infty}\varepsilon_m \mathcal{Z}_n(z)[B_{mn} J_m(\mu_n r) + C_{mn} Y_m(\mu_n r)]\cos m\theta, \tag{11}$$

$$\phi^{\mathrm{iii}} = -\frac{\mathrm{i}gA}{\omega}\sum_{m=0}^{\infty}\varepsilon_m\left[\mathrm{i}^m Z_0(z) J_m(k_0 r) + \sum_{n=0}^{\infty} D_{mn} Z_n(z) H_m(k_n r)\right]\cos m\theta, \tag{12}$$

where superscripts ‘i’, ‘ii’, and ‘iii’ correspond to three subdomains. A_{mn}, B_{mn}, C_{mn}, and D_{mn} are coefficients to be determined, $H_m(\cdot) = J_m(\cdot) + \mathrm{i}Y_m(\cdot)$ is the mth-order Hankel function of the first kind, and vertical mode functions $Z_n(z)$ and $\mathcal{Z}_n(z)$ for gravity waves and flexural-gravity waves are expressed as

$$Z_n(z) = \frac{\cosh[k_n(z+h)]}{\cosh(k_n h)} \quad \text{with} \quad n \geq 0, \quad \text{and} \quad \mathcal{Z}_n(z) = \frac{\cosh[\mu_n(z+h)]}{\cosh(\mu_n h)} \quad \text{with} \quad n \geq -2. \tag{13}$$

In Eq. (13), k_n are roots of the gravity wave dispersion relation $k \tanh(kh) = K$, and μ_n are roots of the flexural-gravity wave dispersion relation $K = (1 - K\gamma + \chi\mu^4)[\mu \tanh(\mu h) - \mathrm{i}\sigma]$ [3]. From Eq. (11), the hydroelastic deflection of the ring-shaped plate is written as

$$w = \frac{A}{K} \sum_{m=0}^{\infty} \sum_{n=-2}^{\infty} \varepsilon_m S_n [C_{mn} J_m(\mu_n r) + D_{mn} Y_m(\mu_n r)] \cos m\theta, \quad \text{with } S_n = \mu_n \tanh(\mu_n h) - \mathrm{i}\sigma. \tag{14}$$

To setup an equation system, the matching conditions, requiring the continuity in potential and its radial derivative at juncture boundaries $r = a$ and $r = b$, are imposed in a Galerkin manner via integrating a test function $Z_l(z)\mathrm{e}^{-\mathrm{i}k\theta}$, with $k \geq 0$ and $l \geq 0$, over $z \in [-h, 0] \cup \theta \in [-\pi, \pi]$. To obtain a determined system, the free edge conditions (8a) and (8b) for the flexible ring at $r = a$ and $r = b$ are supplemented. For the sake of limited spaced, detailed equations are not presented here.

Results and discussions

As illustrative examples, we consider a setup $R/h = 0.5$, $a/h = 1.0$, and $b/h = 2.0$. The mass density of the floating plate is $\gamma/h = 0.1$.

Figure 2 depicts the wave exciting force and mean drift force, which are nondimensionalised with respect to $\pi\rho g R^2 A$ and $\rho g A^2 R$, respectively, as a function of wavenumber $k_0 h$ for different parameters of flexural rigidity $\bar{\chi} = \chi/h^4$. Both impervious $b = 0.0$ and perforated $b = 5.0$ ring plates are considered. Comparison is made with the results determined by the MacCamy-Fuchs formula for an isolated cylinder. For impervious plate $b = 0$, the linear wave exciting force is much greater than that by an isolated cylinder near $k_0 h = 3.0$ and $k_0 h = 7.7$, indicating the resonance of waves near the cylinder may occur. Moreover, the mean drift force experiences oscillations near $k_0 h = 3.0$, and becomes spiky when $k_0 h > 7.0$. When the ring-shaped plate is perforated $b = 5.0$, however, both linear wave exciting force and mean drift force are significantly lower than that experienced by an isolated cylinder. At relatively high frequencies $k_0 h > 4.0$, the reduction of the linear wave exciting force due to the shielding effects can be 90%, whereas that of the mean drift force can be up to 99%. Moreover, in contrast to the impervious scenario, there is no resonance of waves observed when the plate is perforated. The reason is that energy will be dissipated when there is flow past a perforated plate. Therefore, the deployment of a poroelastic ring-shaped plate can help to reduce both linear wave exciting force and mean drift force.

To delve into the resonance of waves, the coloured contour plots of the free surface elevation and hydroelastic deflection at $\bar{\chi} = 1.0$ are exhibited in Fig. 3 for two resonant frequencies $k_0 h = 2.92$ and $k_0 h = 7.76$ in the left and right subplots. The real and imaginary parts are displayed on the upper and lower panels, respectively. White circles correspond to the edges of the ring-shaped plate. In both cases, the amplitude of free surface waves in the annular region between the cylinder and inner ring edge is very large, and the amplification can be up to 3.3 and 5.6. In the lower panels, wave trough and wave crest appear at the

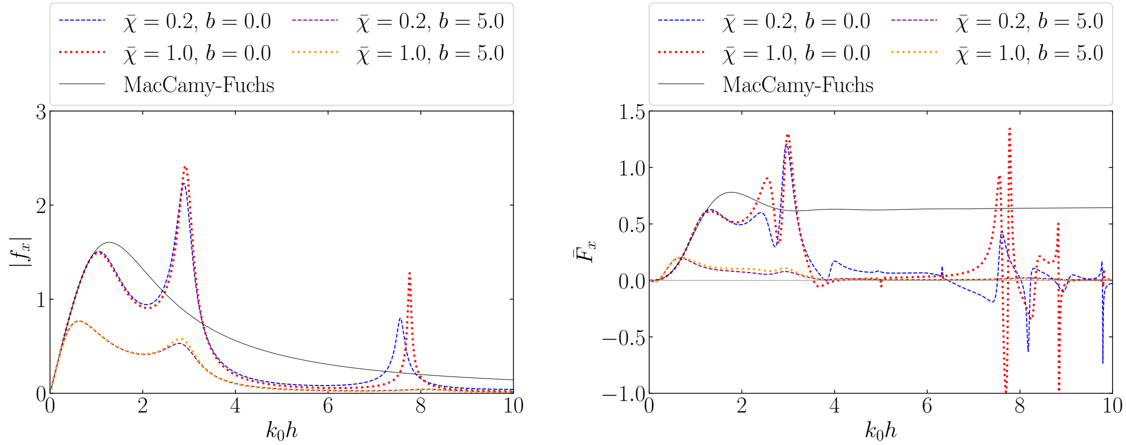

Figure 2: Wave exciting force and mean drift force on the cylinder for different parameters of $\bar{\chi}$ and b. Comparison is made with the MacCamy-Fuchs formula for an isolated cylinder.

front part and rear part of the cylinder, resulting in large wave force. At the workshop, more details and results will be presented.

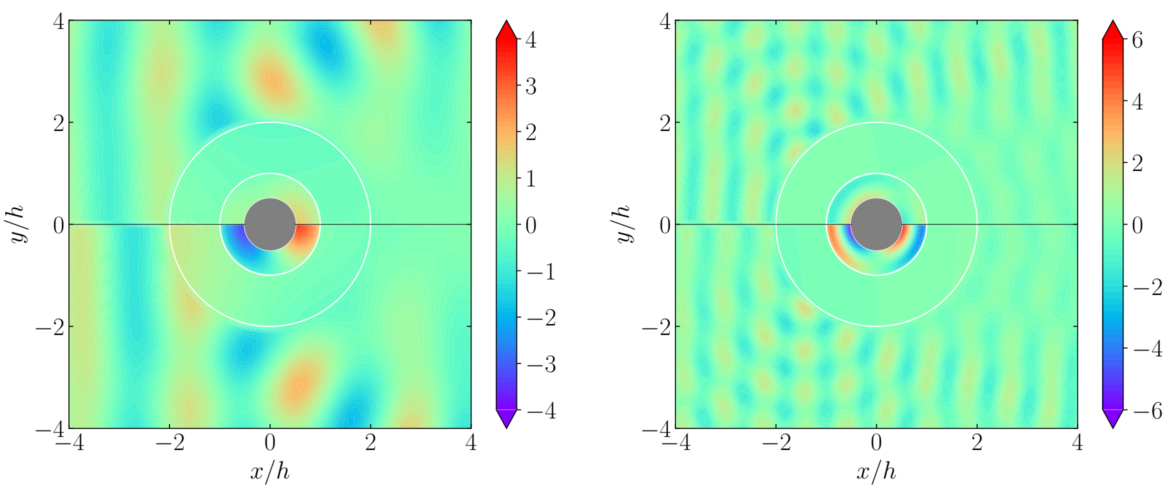

Figure 3: Free surface elevation and hydroelastic deflection for $k_0h = 2.92$ (left) and $k_0h = 7.76$ (right) at $\bar{\chi} = 1.0$. The real and imaginary parts are exhibited on the upper and lower panels, respectively.

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