Physical properties of the ship wake and its detection

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Abstract

At the 37th IWWWFB, Liang, et al. [1] presented the numerical simulation of the ship wake as well as its time-frequency spectrograms. As a sequel to [1], this study considers the fundamental physical properties of the ship wake at a fixed location. The insight into the physics of the ship wake leads to the development of a method requiring two-probe measurements to determine the speed and sailing direction of a ship.

Physical properties

Consider a ship steadily advancing in calm water. We define two coordinate systems: one is the space-fixed coordinate system $OXYZ$ with the OX axis pointing to ship's bow and OZ axis pointing positively upward, and the other one is the coordinate system $oxyz$ moving with the ship along OX axis. At $t = 0$, two frames of reference coincide.

By using the coordinate transform $X = x + Ut$, the free surface elevations for transverse and divergent waves in the fixed coordinate system $OXYZ$ are written in the form of

$$\pi E_\pm(X, Y, t) = \kappa \| F_\pm^{KH} \| \cos[P_\pm(X, Y, t)], \tag{1}$$

where $\kappa = g/U^2$, and the phase functions $P_\pm(X, Y, t)$ are

$$P_\pm(X, Y, t) = \kappa\sqrt{1 + q_\pm^2}(X - Ut + q_\pm Y) \mp \pi/4 + \mathrm{Arg}(F_\pm^{KH}), \tag{2}$$

where $F_\pm^{FH}$ are defined in [1], with subscripts $-$ and $+$ corresponding to transverse and divergent waves, respectively, and $q_\pm$ are stationary phase points. Suppose that the censor location is at $(X, Y) = (0, Y_s)$ in the fixed coordinate system, and the coordinates in the moving frame of reference are $(x, y) = (-Ut, Y_s)$. Then we can define a nondimensional time

$$\tau \equiv Ut/Y_s = -x/y, \quad \text{and} \quad q_\pm = (\tau \pm \sqrt{\tau^2 - 8})/4. \tag{3}$$

According to the geometrical relation $\tan\gamma_c = 1/\sqrt{8}$, where γ_c is the Kelvin angle, the observation point locates inside the Kelvin wake when $\tau > \sqrt{8}$, and the cusp line of the Kelvin wake meets the censor at $\tau = \sqrt{8}$. Based on the phase function (2), we obtain the frequencies of transverse waves ω_- and divergent waves ω_+ measured at the censor location

$$U\omega_\pm/g = \sqrt{1 + q_\pm^2} = \frac{\sqrt{2}}{4}\sqrt{\tau^2 + 4 \pm \tau\sqrt{\tau^2 - 8}}. \tag{4}$$

The long time asymptotic expressions for frequencies are

$$\omega_- = g/U + O(\tau^{-2}), \quad \text{and} \quad \omega_+ = g\tau/(2U) + O(\tau^{-3}) = gt/(2Y_s) + O(\tau^{-3}). \tag{5}$$

Figure 1 depicts normalised frequencies of transverse and divergent waves at a censor location as a function of nondimensional time τ determined by Eq. (4). At $\tau = \sqrt{8}$, frequencies of transverse and divergent waves are identical, and equal to $\omega U/g = \sqrt{3/2}$. With the time marching, the frequency of transverse waves becomes nearly constant, whereas that of divergent waves keeps increasing. When τ is large, as in Eq. (5), we have $\omega_- = g/U$, indicating that the frequency of transverse waves is independent of time t, and inversely proportional to ship's speed U. For divergent waves, however, the frequency is $\omega_+ = gt/(2Y_s)$, linearly proportional to the time t but independent of ship's speed U.

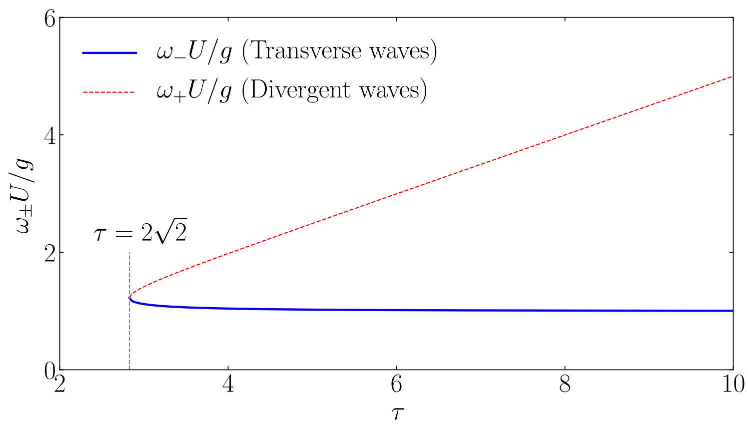

Figure 1: Normalised frequencies of transverse and divergent waves versus τ.

According to the phase function (2), wavenumbers in X- and Y-directions are obtained

$$k_\pm^X = \kappa\sqrt{1+q_\pm^2}, \quad \text{and} \quad k_\pm^Y = \kappa q_\pm \sqrt{1+q_\pm^2}. \tag{6}$$

Furthermore, we can rewrite the frequencies (4) as

$$\omega_\pm = \sqrt{g k_\pm} \quad \text{with} \quad k_\pm = \sqrt{(k_\pm^X)^2 + (k_\pm^Y)^2} \tag{7}$$

called the wavenumber modulus. Equation (7) indicates that both transverses and divergent waves satisfy the deep water dispersion relation $\omega^2 = gk$ [2].

According to Eq. (6), the heading angles of transverse and divergent waves are determined

$$\beta_\pm = \arctan(k_\pm^Y/k_\pm^X) = \arctan\left(\frac{\tau \pm \sqrt{\tau^2-8}}{4}\right). \tag{8}$$

Given the dispersion relation (7), i.e. the relationship between frequency and wavenumbers, the phase velocity vector is obtained

$$\mathbf{c}_\pm = \frac{\omega_\pm}{\|\mathbf{k}_\pm\|^2}\frac{(k_\pm^X, k_\pm^Y)}{k_\pm} = \frac{U}{1+q_\pm^2}(1, q_\pm) = \frac{8U}{\tau^2+4\pm\tau\sqrt{\tau^2-8}}\left(1, \frac{\tau\pm\sqrt{\tau^2-8}}{4}\right), \tag{9}$$

and the group velocity vector according to [3] is written as

$$\mathbf{v}_\pm = \left(\frac{\partial\omega_\pm}{\partial k_\pm^X}, \frac{\partial\omega_\pm}{\partial k_\pm^Y}\right) = \frac{U}{2(1+q_\pm^2)}(1, q_\pm) \equiv \frac{\mathbf{c}_\pm}{2}. \tag{10}$$

Consistent with the deep water wave theory, Eqs. (9) and (10) indicate that, in the fixed coordinate system, the phase velocity of both transverse and divergent waves is in alignment with the group velocity, and is twice the group velocity in magnitude.

Ship wake detection

Given the physical properties, we aim at developing a two-probe method to determine the speed and direction of the sailing ship. As in Fig. 2, there are two probes A and B at a distance λ, and we can plot a deployment line going through A and B. Suppose that the angle between the ship's sailing line and deployment line is θ, which is to be determined. We define t_A and t_B as the time instants when the cusp meets probes A and B, respectively, and their difference is $t_B - t_A = T$. At t_A and t_B, the mid-ship locates at P and Q, respectively.

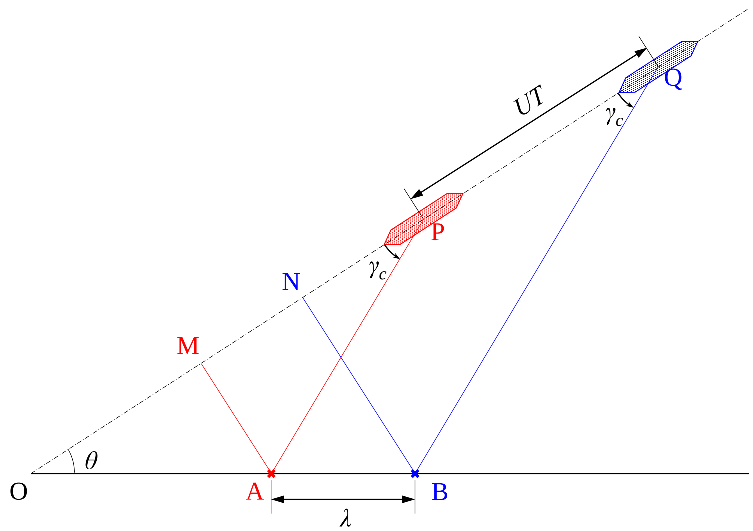

Figure 2: Sketch of a two-probe method to determine ship's speed and sailing direction.

By applying the short-time Fourier transform, the components of different frequencies can be separated, and one can obtain an amplitude heat map. Therefore, it is feasible to obtain the time difference T as well as the time-dependent wave frequencies $\omega_{\pm}^{A}$ and $\omega_{\pm}^{B}$ measured at A and B by processing the time series. Then, the task is to determine ship's speed U and sailing angle with respect to the deployment line θ.

Based on Eq. (5), the ship's speed can be obtained in a straightforward manner

$$U = g/\omega_{-}^{A,B} \quad \text{when transverse waves are in a steady state,} \tag{11}$$

because the frequency of transverse waves is nearly independent of time. According to the geometrical relation, the angle between the deployment line and sailing line θ is

$$\theta = \arcsin[UT/(3\lambda)] - \gamma_c. \tag{12}$$

To showcase the two-probe method, we devise a problem with a ship sailing at a speed $U = 1.6$ m/s. Two wave probes A and B are at a distance $\lambda = 2$ m, and the sailing direction with respect to the deployment line is $\theta = 30°$.

Figure 3 exhibits the time histories and the corresponding time-frequency spectrograms obtained from the measurements at probes A (left) and B (right), respectively. By measuring the frequency of the lower branch of heat map, the frequency of transverse waves is $\omega_- \approx$ 6.1 rad/s, and thus the corresponding velocity determined by Eq. (11) is $U_{\text{cal}} \approx 1.61$ m/s, which is close to the true value 1.6 m/s.

Based on Eq. (5), the frequency for divergent waves is linearly increasing with time at a slope $S = g/(2Y_s)$. The measurement of the slope of the upper branch associated with divergent waves gives $S_A = g/(2Y_s^A) \approx 1.6$ and $S_B = g/(2Y_s^B) \approx 1.2$, and then the lateral distances from the sailing line Y_s^A and Y_s^B can be obtained. Suppose that the cusp line meets the probes A and B at time instants t_A and t_B, respectively. Then, we have

$$U(t_A - t_0)/Y_s^A = 2\sqrt{2} = [U(t_B - t_0) - \lambda\cos\theta]/Y_s^B. \tag{13}$$

Combining with Eq. (12), we obtain an equation with respect to θ

$$\theta = \arcsin\left[\frac{2\sqrt{2}(Y_s^B - Y_s^A) + \lambda\cos\theta}{3\lambda}\right] - \gamma_c. \tag{14}$$

As a result, the computed sailing angle is $\theta_{\text{cal}} \approx 30.7°$, which is in good agreement with the true value $\theta_{\text{true}} = 30°$. Therefore, the two-probe method developed here can be used to estimate the ship's speed and sailing angle via measuring the time-frequency spectrograms.

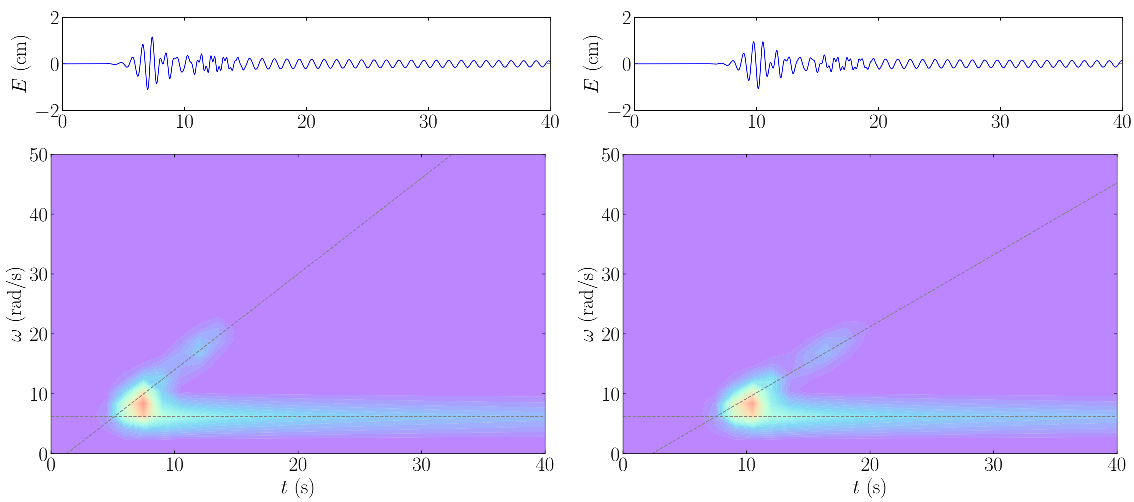

Figure 3: Free surface elevations (top), and the corresponding time-frequency spectrograms (bottom). The measurements at A and B are displayed in the left and right, respectively.

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