Tertiary interactions due to wave run-up in spread seas

C. L. Hansen[a,*], **P. H. Taylor**[a], **H. Wolgamot**[a], **W. Zhao**[a], **J. Orszaghova**[a], **H. Bredmose**[b]
[a] Oceans Graduate School, The University of Western Australia,
35 Stirling Highway, Crawley, WA, 6009, Australia
[b] DTU Wind and Energy Systems, Technical University of Denmark,
Kgs. Lyngby, DK-2800, Denmark
[*] E-mail: christine.lynggardhansen@research.uwa.edu.au

HIGHLIGHTS

- Wave-by-wave analysis of the run-up on a fixed box is carried out, following initial presentation of experimental results in Zhao *et al.* [1].
- We apply NewWave analysis to explore the nonlinear amplification and phase shifts of the reflected field in front of the box.
- We analyse experiments conducted in both unidirectional and moderately spread irregular waves of normal incidence, and show that realistic spreading reduces the nonlinear amplification down to the linear reflection level but that the phase shifts remain.

1 INTRODUCTION

Tertiary wave-wave interactions are first described by Longuet-Higgins and Phillips [2], who showed that the interactions cause changes to the phase speed of two waves passing each other. Based on experiments in regular waves and fully nonlinear numerical modelling, Molin *et al.* [3, 4], established that tertiary wave-wave interactions between a wave and its reflection off a perpendicular vertical wall cause large amplifications of the run-up on the wall as well as a phase lag of the run-up relative to the incident wave. Results from a Boussinesq-type numerical model have shown a decrease in amplification with rotation of the wall relative to the incident wave direction for a monochromatic wave (Jamois *et al.* [5]).

For experiments in unidirectional irregular waves, Zhao *et al.* [6] were able to extend this result using both spectral and wave-by-wave analysis to document the amplified run-up and phase lag. For irregular waves with oblique heading and spread waves with constant spreading factor $s = 30$ respectively, Zhao *et al.* [1] show that the amplification of the run-up on a box due to tertiary interactions drop down to close to linear diffraction levels. Ouled Houssine *et al.* [7] model the the setup from [6] in the 3D-BEM software Hydrostar using a parabolic model by Molin [3], and show a large drop in amplification with rotation of the box with only 8°.

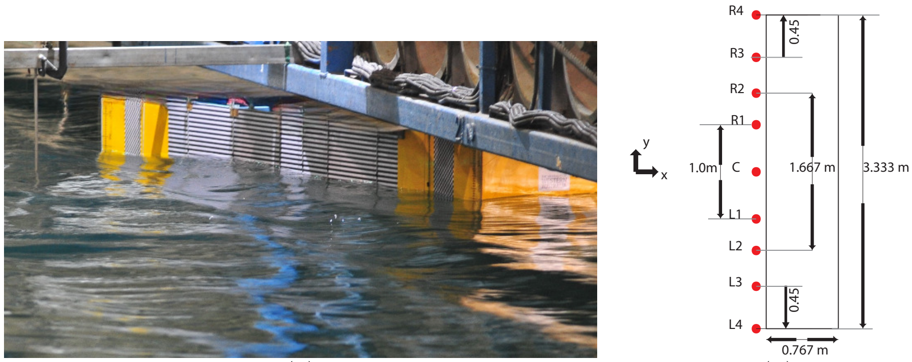
Since the tertiary wave-wave interactions only affect phase speed, similar to a shoal, the run-up on the wall can be analysed locally as a linear response, hence we can apply linearised NewWave analysis. This allows us to simultaneously analyse the consequences of the tertiary interactions on both amplitude and phase. Here, we provide further analysis and estimate the importance of tertiary wave-wave interactions in front of a fixed box in realistic sea states.

2 EXPERIMENTAL SET-UP

Experiments have been performed in the deepwater basin at Shanghai Jiao Tong University and were previously reported by Zhao *et al.*[1]. The basin is 50 m long, 40 m wide, and the depth is 10 m. The basin has 2 neighbouring sides equipped with flap-hinge wavemakers and 2 with absorbing beach.

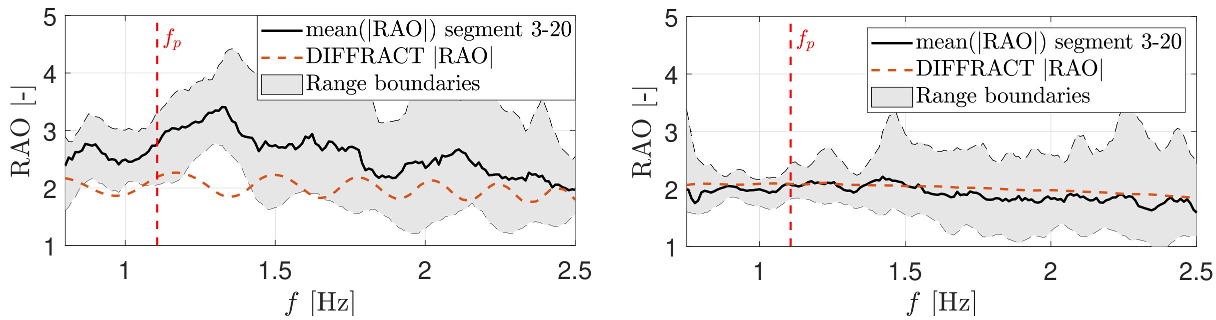
A box was fixed in the basin as shown in fig. 1a. The horizontal dimensions are given in fig. 1b. The box has height 0.425 m, submerged draft 0.185 m and rounded edges along the 2 submerged long sides. The box was fitted with 9 wave gauges placed as shown in fig. 1b.

Tests were run for unidirectional and spread waves, propagating in the x-direction (see fig. 1b). Unidirectional waves were generated by feeding the same paddle signal to all wavemakers along 1



(a) (b)

Figure 1: Experimental setup: (a) photo of the box (yellow colour) rigidly connected to the gantry with wave gauges placed at the front face (b) schematic of the box dimensions and location of wave gauges along the front face.



(a) (b)

Figure 2: Response Amplitude Operator (RAO) for the unidirectional (a) and spread sea state (b).

side of the basin. Spread waves were generated using all wavemakers, resulting in the mean wave direction at 45° relative to both basin sides with wavemakers. The box was rotated around point C to achieve normal incidence along the mean wave direction. An irregular sea state based on a JONSWAP spectrum with significant wave height $H_s = 52.5$ mm, peak period $T_p = 0.9$ s and shape factor $\gamma = 1.5$ at lab scale was run for a duration of $t_{dur} = 1800$ s. We applied a constant spreading factor $s = 13.5$ in the commonly used spreading function $D(\theta) = \cos^{2s}\left(\frac{\theta}{2}\right)$, corresponding to a root mean square spreading angle of 21.3°, and comparable to that often observed in severe storms in the North Sea and elsewhere. All sea states were run without the box in place to calibrate the sea state and measure the undisturbed surface elevation. Then, the box was installed and the sea state re-run with the same paddle signal as previously. The surface elevation at C without the box in place is denoted by η and that with the box as the response, ζ.

3 NEWWAVE ANALYSIS OF THE RUN-UP IN IRREGULAR WAVES

Response Amplitude Operators (RAOs) are obtained using the method of Zhao *et al.* [6] and shown for the unidirectional and spread spread states in figure 2 with a linear diffraction model (red dashed), showing large amplification of the unidirectional response and linear response levels for the spread response. Further RAO-analysis is presented in Zhao *et al.*[1]. The NewWave, presented in Tromans *et al.*[8] describes the most probable linear extreme wave within a sea state. A NewWave in response, with a crest at $x = 0$ and $t = 0$ can be written as the inverse Fourier transform of the response spectrum:

$$\zeta^{NW}(t) = \alpha_\zeta \frac{\mathrm{Re}\left(\sum_n S_\zeta(\omega_n)\ \Delta\omega\ \exp(-\mathrm{i}\omega_n t)\right)}{\sum_n S_\zeta(\omega_n)\Delta\omega}, \tag{1}$$

with the amplitude of the extreme crest response at the 1 in M level given by $\alpha_\zeta = \sqrt{2\sigma_\zeta^2 \ln(M)}$, S_ζ the response spectrum, ω_n the angular frequency and $\Delta\omega$ the frequency discretization. The NewWave in response at a 1 in M level ζ^{NW} is constructed from the measured time series by averaging the N largest linearised crests in the response time series ζ. We first identify the N largest independent crest values in ζ and define a short time sequence around each with the crest at $t = 0$. ζ^{NW} is then found by averaging the N sequences. We take $N = 30$.

To obtain phase shifts between the surface elevation without the box η and with the box ζ, we use the time stamp of the NewWave signal to condition the other signal. We thus construct the conditioned signal 'Wave|NewResponse' ($\eta|\zeta^{NW}$) and 'Response|NewWave' ($\zeta|\eta^{NW}$). These are found as the average of the N time segments from η and ζ occurring simultaneously with the N largest crests in ζ and η respectively. It has been shown (e.g. in Zhao *et al.*[6]) that reciprocity holds for any linear system between the conditioned incident wave $\eta|\zeta^{NW}$ corresponding to a NewWave in response ζ^{NW} and the conditioned response $\zeta|\eta^{NW}$ to an incident NewWave η^{NW}, given a linear response mechanism:

$$\eta(t)|\zeta^{NW} = \frac{\alpha_\zeta}{\alpha_\eta} \frac{\sum_n S_\eta(\omega_n)}{\sum_n S_\zeta(\omega_n)} \cdot \left(\zeta(-t)|\eta^{NW}\right) \tag{2}$$

where the fraction represents the scale factor ξ_a and $\zeta|\eta^{NW}$ is reversed in time.

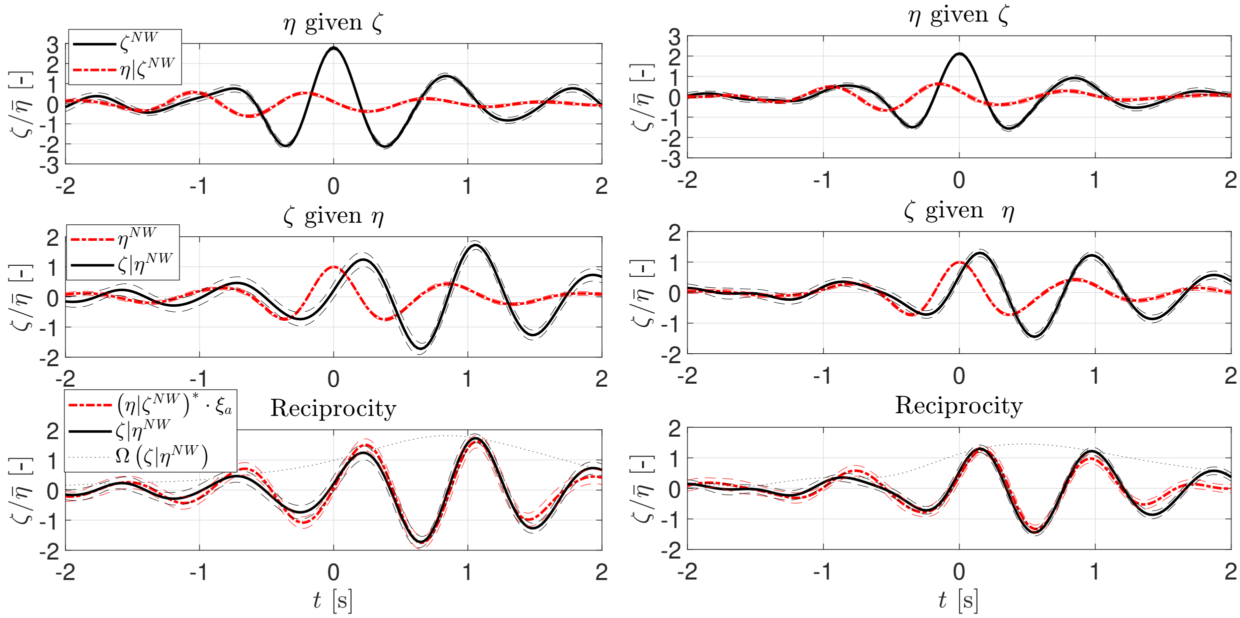
A conditioning analysis of the response at the front centre position of the box is shown in figure 3 for the unidirectional sea state (fig. 3a) and the spread sea state (fig. 3b). All curves are normalised by the largest incident crest amplitude $\max(|\eta^{NW}|)$. The black solid lines represent response signals and the red dash-dotted lines are incident wave curves. The first row shows the NewWave in response ζ^{NW} and the conditioned incident wave $\eta|\zeta^{NW}$. The second row shows the free-field incident NewWave η^{NW} and the corresponding condition response given this NewWave, $\zeta|\eta^{NW}$. The third row shows the reciprocity of the two conditioned signals with the linear wave envelope $\Omega(\zeta|\eta^{NW})$ showing the phase shift. Error margins of width $\pm 2\sigma/\sqrt{N}$ are marked with dashed lines.

Figure 3a shows that the maximum amplitude of ζ^{NW} reaches almost $3\times \max(\eta^{NW})$ in the top row, which clearly exceeds that expected from linear diffraction coefficients. The third row shows clear reciprocity as there is good similarity between $\zeta|\eta^{NW}$ and $\eta|\zeta^{NW}$, the latter scaled with $\xi_a = 2.91$ and mirrored about $t = 0$. A delay of $dt = 0.725$ s of the peak of the linear wave conditioned response envelope $\Omega\left(\zeta|\eta^{NW}\right)$ relative to the occurrence of η^{NW} confirms phase speed reductions caused by tertiary wave-wave interactions. This is in agreement with results from Zhao *et al.* [6].

For the spread sea state shown in figure 3b, the NewWave in response (solid black, first row) reaches $2\times$ the max amplitude of η^{NW}, which corresponds to the expected level of linear diffraction off a wall, and that for reflection from an infinite wall of $2\times$. This result is consistent with the initial results in Zhao *et al.* [1]. However, the persistent delay between the conditioning and conditioned signals and the clear reciprocity observed in the bottom row indicate the presence of tertiary interactions upstream of the box. With the delay of the linear response envelope $dt = 0.45$ s, however, the strength of the interactions have weakened compared to the unidirectional sea state.

The difference in phase shift between the unidirectional and spread tests is assumed to arise from the interaction function $\mathcal{F}(\omega_1, \omega_2, \beta)$, defined by Molin *et al.* [4], which is highly dependent on the interaction angle β between pairs of wave components. We attribute the reduction in amplification to the spatial variation of wave energy which for unidirectional waves of normal incidence is concentrated at the front centre of the box. For spread waves, there is no unique position to concentrate the energy for all frequency and directional interactions and the effect is such that the amplitude remains at a linear level. We consider it striking that the amplification effect beyond any linear diffraction is lost with a realistic level of spreading in an open ocean context.

Tests with unidirectional oblique waves show similar results to the spread sea tests, except that the left-right symmetry of the off-centre wave gauges is lost for the oblique sea states. A full analysis



(a) (b)

Figure 3: Response conditioning (a) normal incidence unidirectional sea state. (b) spread sea state

is presented in a full journal paper currently under preparation.

4 CONCLUDING REMARKS

- We compare the amplification of surface elevations and their phase delay in a unidirectional and a spread normal incidence sea state.
- We demonstrate the presence of phase shifts caused by tertiary interactions in both cases.
- Even with a fairly narrow spreading at a realistic level for open sea conditions, we see that the amplification effect reduces to a linear level.
- Our results suggest that tertiary interactions between the incident and reflected waves in front of a large wall-like structure in the open sea are unlikely to be of practical significance.

AKNOWLEDGEMENTS

C. L. Hansen and H. Wolgamot are supported by the Australian Research Council Early Career Fellowship DE200101478

REFERENCES

[1] Zhao, W., Taylor, P. H., Wolgamot, H. A., and Houssine, C. O. *Heading and spreading effects on wave run-up due to tertiary interactions.* In *35th Int. Workshop on Water Waves and Floating Bodies* (Seoul, Korea, 4 2020).

[2] Longuet-Higgins, M. S., and Phillips, O. M. 1962. *Phase velocity effects in tertiary wave interactions.* Journal of Fluid Mechanics 12(3), 333–336.

[3] Molin, B., Remy, F., Kimmoun, O., and Jamois, E. 2005. *The role of tertiary wave interactions in wave-body problems.* Journal of Fluid Mechanics 528, 323–354.

[4] Molin, B., Kimmoun, O., Remy, F., and Chatjigeorgiou, I. K. 9 2014. *Third-order effects in wave-body interaction.* European Journal of Mechanics - B/Fluids 47, 132–144.

[5] Jamois, E., Fuhrman, D. R., Bingham, H. B., and Molin, B. 11 2006. *A numerical study of nonlinear wave run-up on a vertical plate.* Coastal Engineering 53(11), 929–945.

[6] Zhao, W., Taylor, P. H., Wolgamot, H. A., and Eatock Taylor, R. 6 2019. *Amplification of random wave run-up on the front face of a box driven by tertiary wave interactions.* Journal of Fluid Mechanics 869, 706–725.

[7] Ouled Housseine, C., Molin, B., and Zhao, W. *Third-order wave run-up in diffraction-radiation problems.* In *35th Int. Workshop on Water Waves and Floating Bodies* (4 2020).

[8] Tromans, P. S., Anaturk, A. R., and Hagemeijer, P. *A new model for the kinematics of large ocean waves - application as a design wave.* In *Proc. 1st Int. Offshore and Polar Engineering Conference* (1991).