On the radiation problem for two floating solar panels separated by a small gap in the long-crested case

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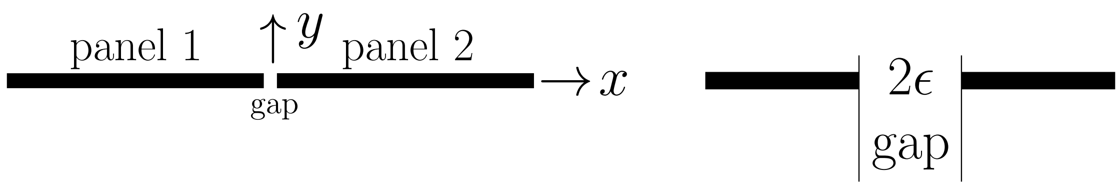

Figure 1: Model of two floating solar panels (left) separated by a small gap (right) in the heave mode of motion.

1 Introduction

The world wide push for renewable energy solutions has led to the development of Floating PhotoVoltaic systems (FPV) on the water surface. Systems are tested in sheltered lakes or man-made reservoirs where the FPVs benefit from a cooling from the water which increases the performance. The FPVs are constructed by connected solar panels or thin films floating directly on the water. Panels may alternatively be mounted on pontoons (e.g., Dai et al., 2020; Kaymak and Sahin, 2021). FPV systems intended for localisation in the ocean area are exposed to wave (and wind) effects. Intended horizontal extension of the FPV systems are a few hundred meters squared. FPV systems may be composed by several connected components, each as a rigid member responding in six degrees of freedom. This background provides motivation for the analysis that follows.

In this note, a two-dimensional analysis is carried out for two solar panels located next to each other and separated by a small gap (Figure 1). The length of each panel is $2a$. They are separated by a small opening of width of 2ϵ. In reality, the length $2a \simeq 2$ m and the gap distance $2\epsilon \simeq 5$ cm. The radiation problem of the vertical modes is considered where the effects of added mass and damping are evaluated. The geometry is oscillating with frequency ω. The corresponding non-dimensional wavenumber $(\omega^2 a/g)$ is assumed to be small, where g denotes the acceleration of gravity. The analysis is carried out in the long wave regime. At the scale of the panel, the double body approximation may be justified. The two-dimensional analysis is relevant to the case of long-crested waves and to experiments in narrow wave flumes.

2 Mathematical formulation

A frame of reference is introduced with the x-axis at the mean free surface. The two panels each of length $2a$ are located symmetrically at each side of $x = 0$. The y-axis is vertical, where gravity acts along the negative vertical. Panel 1 is given a vertical velocity V_1, and panel 2 a vertical velocity V_2. The fluid velocity in the gap may be characterised by a vertical velocity V_g. This is interpreted by an average across and along the gap.

The motion of the fluid is modelled by the potential Φ where incompressibility and irrotationality are assumed. Note that in the gap, flow separation effects are taking place which are modelled by a quadratic Darcy formulation. The potential Φ satisfies the Laplace equation in the fluid. Green's theorem applied to the potential and the double body Green function in two dimensions is used to connect Φ and the vertical velocity V along the bottom of the panels and at the gap, where V is the union of V_1, V_g and V_2, and where $V = 0$ for $|x| > 2a + \epsilon$. The double body Green function reads $G = \ln(rr_1) - \ln a^2$ where $r^2 = (x - \bar{x})^2 + (y - \bar{y})^2$ and $r_1^2 = (x - \bar{x})^2 + (y + \bar{y})^2$. The term $-\ln a^2$ provides a non-dimensional form of the argument of G. This added constant is discussed further in Section 3 below. From Green's theorem we find (e.g., Newman, 2017)

$$\pi\Phi + \int_{-2a-\epsilon}^{-\epsilon} (-G_n\Phi + G\Phi_n)d\bar{x} + \int_{-\epsilon}^{\epsilon} (-G_n\Phi + G\Phi_n)d\bar{x} + \int_{\epsilon}^{2a+\epsilon} (-G_n\Phi + G\Phi_n)d\bar{x} = 0, \quad (1)$$

where the point of evaluation is at the underside of the plates and the gap. The index n indicates derivative along the plate normal, pointing out of the fluid. The $\bar{x}$ is integration variable. The Green function satisfies $\partial G/\partial \bar{y} = 0$ on $\bar{y} = 0$. Since the plates localised on the water surface are thin, the Green function is evaluated at $y = \bar{y} = 0$ obtaining

$$\pi\Phi(x) + \int V \ln\Big((x - \bar{x})/a\Big)^2 d\bar{x} = 0, \quad |x| < 2a + \epsilon, \quad (2)$$

and the integration is from $-2a - \epsilon$ to $2a + \epsilon$.

Harmonic motion of time-dependency $e^{\mathbf{i}\omega t}$ is assumed. Consider the case when panel 1 is moving with excursion ξ_1 in heave and panel 2 is fixed. Then, $\Phi = Re(\mathbf{i}\omega\xi_1\phi_1 e^{\mathbf{i}\omega t})$, $V_1 = Re(\mathbf{i}\omega\xi_1 e^{\mathbf{i}\omega t})$, $V_g = Re(\mathbf{i}\omega\xi_1 v_1 e^{\mathbf{i}\omega t})$, $V_2 = 0$, where Re means real part and $\mathbf{i}$ imaginary unit.

2.1 Quadratic Darcy law of the gap motion

A quadratic Darcy law formulation is used to connect the vertical pressure jump and the vertical velocity at the gap at $x = 0$ by $-\Delta p/\rho = -\partial\Phi/\partial t = (1/2)K_0|V_g|V_g$ where K_0 is a dimensionless constant – a drag coefficient – that characterises the flow of the gap, ρ is density and t time (e.g., Molin and Remy, 2013, Dokken et al., 2017). Applying equivalent linearisation we obtain at the gap:

$$Re\Big(-(\mathbf{i}\omega)^2\xi_1\phi_1(x=0)e^{\mathbf{i}\omega t}\Big) = A_0\omega|\xi_1||v_1|Re\Big(\mathbf{i}\omega\xi_1 v_1 e^{\mathbf{i}\omega t}\Big), \quad (3)$$

where $A_0 = (1/2)(8/3\pi)K_0 = (4/3\pi)K_0$, giving $-\mathbf{i}\phi_1(x = 0) = A_0|\xi_1||v_1|\, v_1$.

Use of (2) obtains the potential at the gap by $\pi\phi_1(x=0) \simeq -\int_{-2a-\epsilon}^{-\epsilon} \ln\Big(\bar{x}/a\Big)^2 d\bar{x} = 4a(1-\ln 2)$ where terms $\epsilon/a << 1$ are omitted. This obtains

$$v_1 = -\mathbf{i}\,[\phi_1(0)/(A_0|\xi_1|)]^{1/2} = -\mathbf{i}\,[3(1-\ln 2)/K_0)]^{1/2}\,(a/|\xi_1|)^{1/2} \tag{4}$$

2.2 Force on panel 1

The force on panel 1 is obtained by integrating the time derivative of (2) over the panel. The pressure is given by $p = -\rho\Phi_t = -\rho Re((\mathbf{i}\omega)^2\xi_1\phi_1 e^{\mathbf{i}\omega t})$, and the unit normal by 1. The potential $\phi_1(x)$ is integrated from $-2a-\epsilon$ to $-\epsilon$:

$$\int_{-2a-\epsilon}^{-\epsilon} \phi_1(x)dx$$
$$= -\frac{1}{\pi}\int_{-2a-\epsilon}^{-\epsilon}\int_{-2a-\epsilon}^{-\epsilon} \ln\Big((x-\bar{x})/a\Big)^2 dx d\bar{x} - \frac{v_1}{\pi}\int_{-2a-\epsilon}^{-\epsilon}\int_{-\epsilon}^{\epsilon} \ln\Big((x-\bar{x})/a\Big)^2 dx d\bar{x}. \tag{5}$$

Evaluation of the integrals gives

$$F(t)_{pan1} = Re\Big(e^{\mathbf{i}\omega t}\xi_1[\omega^2\tilde{A}_{pan1} - \mathbf{i}\omega\tilde{B}_{pan1}]\Big), \tag{6}$$

where

$$\frac{\tilde{A}_{pan1}}{\rho a^2} = \frac{4}{\pi}(3-2\ln 2), \qquad \frac{\tilde{B}_{pan1}}{\omega\rho a^2} = \frac{2\epsilon}{\sqrt{K_0 a|\xi_1|}}\frac{4}{\pi}[3(1-\ln 2)^3]^{1/2}. \tag{7}$$

2.3 Added mass vs. mass force

The mass of panel 1 equals $m = d_0\,2a\,\rho$, where d_0 is the submergence. The resonance frequency of the panel becomes (where no restoring forces other than the buoyancy applies):

$$\frac{\omega_n^2 a}{g} = \frac{1}{d_0/a + (2/\pi)(3-2\ln 2)}. \tag{8}$$

The first term of the denominator is approximately 0.01 while the latter is 1.03. The expression suggests that the effect of added mass is very important to the vertical response where resonance occurs for waves that are three times the length of the panel. Estimate of the factor K_0 is needed to properly determine the damping coefficient.

2.4 Force on panel 2

The potential $\phi_1(x)$ integrated from ϵ to $2a+\epsilon$ obtains:

$$\int_{\epsilon}^{2a+\epsilon} \phi_1(x)dx$$
$$= -\frac{1}{\pi}\int_{\epsilon}^{2a+\epsilon}\int_{-2a-\epsilon}^{-\epsilon} \ln\Big((x-\bar{x})/a\Big)^2 dx d\bar{x} - \frac{v_1}{\pi}\int_{\epsilon}^{2a+\epsilon}\int_{-\epsilon}^{\epsilon} \ln\Big((x-\bar{x})/a\Big)^2 dx d\bar{x}, \tag{9}$$

giving

$$F(t)_{pan2} = Re\Big(e^{\mathbf{i}\omega t}\xi_1[\omega^2\tilde{A}_{pan2} - \mathbf{i}\omega\tilde{B}_{pan2}]\Big), \tag{10}$$

where

$$\frac{\tilde{A}_{pan2}}{\rho a^2} = \frac{12}{\pi}\,(1-2\ln 2), \qquad \frac{\tilde{B}_{pan2}}{\omega\rho a^2} = \frac{2\epsilon}{\sqrt{K_0 a|\xi_1|}}\,\frac{4}{\pi}\,[3(1-\ln 2)^3]^{1/2}. \tag{11}$$

3 Comments on the Green function

Use of $G = \ln(rr_1)$ implies that the added mass forces become:
$\tilde{A}_{pan1}/(\rho a^2) = (4/\pi)\,(3-2\ln 2) - (8/\pi)\ln a$,
$\tilde{A}_{pan2}/(\rho a^2) = (12/\pi)\,(1-2\ln 2) - (8/\pi)\ln a$.
The potential ϕ_1 at $x=0$, with implications for the damping coefficient, becomes
$\phi_1(x=0) = (4/\pi)a(1-\ln 2) - (4/\pi)a\ln a$.
Adding the constant $-\ln a^2$ contributes to the added mass forces by $(2a)^2\ln a^2$ which should be divided by the prefactor π, and cancels exactly the log-terms in $\tilde{A}_{pan1}/(\rho a^2)$ and $\tilde{A}_{pan2}/(\rho a^2)$. This shows that the added mass coefficients are proportional to the panel length squared. The similar conclusion regards the damping coefficient, where the correction term in ϕ_1 becomes $2a\ln a^2/\pi$. The potential $\phi_1(x=0)$ and ϕ_1 as such is proportional to the panel length.

4 Generalisation

The forces due to oscillations in heave of panel 2 are readily obtained from the forces on panel 1, because of symmetry. This is true also for the force on panel 1 due to the oscillatory panel 2. If both panels are oscillating, the nonlinear relation (3) includes the coupled effects of the oscillation amplitudes of panels one and two. The moment on the panels are easily evaluated. The effect of a yaw (or roll) motion is modelled similarly as for the heave problem. The vertical velocity V in (2) is then generalised including the yaw velocity of the panels. The force on several panels is obtained by extending the procedures for the two panels.

References

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