A Time-Domain Slender Ship Theory: A Body-Exact Approach Using the Method of Matched Asymptotic Expansions

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1 INTRODUCTION

The objective of this research is to develop a time-domain implementation of the classic slender body theory of Tuck[1] and Newman[2] to support computation of ship motions in time-domain simulations, with intermediate accuracy and execution time between strip theory and 3D methods.

2 FORMULATION

The formulation of the problem assumes inviscid, irrotational, fluid flow. For the purposes of this project we are considering only vertical plane motions in head seas. We adopt the slender body assumption that longitudinal derivatives of potential are an order of magnitude smaller than lateral derivatives. We also adopt the weak scatterer hypothesis of Pawlowski[3], which assumes that body disturbances of the incident wave are higher order and thus the inner problem can be linearized about the incident wave profile, with the usual linearized free surface boundary condition.

The slender body assumption allows the problem to be separated into incomplete inner and outer problems. The inner problem includes the body boundary condition, equating the normal component of body velocity with the normal derivative of velocity potential on the body surface; and with a rigid wall free-surface boundary condition. The outer problem includes the far-field radiation condition with the linearized time-domain free-surface boundary condition. The complete problem is then solved using the method of matched asymptotic expansions, matching the inner expansion of the outer expansion (IEOE) with the outer expansion of the inner expansion (OEIE).

3 SOLUTION

The 3D outer expansion is represented by a line array of transient sources (Green functions per Wehausen & Laitone[4]). The 2D inner expansion is represented by a heaving 2D section with a body-boundary condition based on the instantaneous immersion (the 'body-exact' assumption). The weak scatterer assumption permits the problem in the time domain to be formulated in a transformed physical domain, as illustrated in figure 2. This approach follows Sclavounos[5] and Walree & Turner[6].

3.1 The Inner Solution

Far enough away from the body, the 2D disturbance behaves as if generated by an equivalent 2-D logarithmic source plus an additive function of axial position to render each section solution unique. This general outer expansion of the inner solution (OEIE) is then given as

$$\Phi(X,t) = \lim_{R>>1} \{\Phi_{2D}(X,t) + b(X,t)\} \simeq \sigma(x,t)\ln r_0 + b(X,t) \tag{1}$$

In the time domain, the body boundary condition for the inner problem has two principal parts: a vertical, 'plunging wave-maker' component due to the relative vertical velocity of body-wave radiation/diffraction motions; and a forward speed section-area dilation effect related to the product UA_x, as implied in figure 3, with $\delta x = U\delta t$, and with ship velocity U adjusted by the longitudinal component of wave orbital velocity, which also partially accounts for the diffraction component of the problem. This body boundary condition is nonlinear (or 'body exact'), being applied at the instantaneous section immersion. On the instantaneous position of the wave surface at each section the first order (rigid wall) free surface condition is applied.

3.2 The Outer Solution

This solution is based on the 3D transient boundary Green function leading to a general outer solution, for a line array of such sources on the **free surface**, with general axial source strength distribution $m(\xi,\tau)$, in a space-fixed reference frame,

$$\phi(\mathbf{x},t) = 2\int_L d\xi \int_0^\infty dk\,(gk)^{1/2}\;e^{kz}\int_0^t d\tau\, m(\xi,\tau)\sin\left[(gk)^{1/2}(t-\tau)\right]\mathbf{J}_0(kR(x-\xi(\tau),y)) \tag{2}$$

In order to obtain an inner expansion of the outer expansion (IEOE) with a corresponding functional term in common with the OEIE, we need to obtain a logarithmic singularity. This we do by performing an integration by parts. This yields

$$\begin{aligned}\phi(\mathbf{x},t) \approx\; & -2m(x,t)\ln r + \int_{-\infty}^{\infty} m'(\xi,t)\left[\operatorname{sgn}(x-\xi)\ln 2|x-\xi|\right]d\xi \\ & - \lim_{y,z\to 0} 2\int_L d\xi\, m(\xi,0)\int_0^\infty dk\, e^{kz}\cos\left[(gk)^{1/2}(t)\right]\mathbf{J}_0(kR(x-\xi(0),y)) \\ & - \lim_{y,z\to 0} 2\int_L d\xi \int_0^\infty dk\, e^{kz}\int_0^t d\tau\,\cos\left[(gk)^{1/2}(t-\tau)\right]\Big(\dot m(\xi,\tau)\mathbf{J}_0(kR(x-\xi(\tau),y)) \\ & \qquad + m(\xi,\tau)\dot{\mathbf{J}}_0(kR(x-\xi(\tau),y))\Big)\end{aligned} \tag{3}$$

3.3 The General Matching Solution

Matching the OEIE and IEOE leads to the general matching solution

$$m(\xi,t) = \frac{1}{2}\sigma(x,t) \tag{4}$$

$$\begin{aligned}b(X,t) =\; & \int_L m'(\xi,t)\left[\operatorname{sgn}(x-\xi)\ln 2|x-\xi|\right]d\xi \\ & - 2\int_L d\xi\, m(\xi,0)\int_0^\infty dk\,\cos\left[(gk)^{1/2}(t)\right]\mathbf{J}_0(kR(x-\xi(0),0)) \\ & - 2\int_L d\xi \int_0^\infty dk\int_0^t d\tau\,\cos\left[(gk)^{1/2}(t-\tau)\right]\Big(\dot m(\xi,\tau)\mathbf{J}_0(kR(x-\xi(\tau),0)) \\ & \qquad + m(\xi,\tau)\dot{\mathbf{J}}_0(kR(x-\xi(\tau),0))\Big)\end{aligned} \tag{5}$$

This result is **completely general** for any **spatial-temporal source-strength distribution** $m(\xi,\tau)$, ***without*** the restriction that $R \neq f_n(\tau)$.

1. the first term of $b(X,t)$ accounts for the longitudinal interaction of the non-wave part;
2. the second term of $b(X,t)$ accounts for the initial transient effect; and
3. the third term of $b(X,t)$ accounts for the longitudinal interaction of the wave part, dependent on the dynamics of vertical motion of the body relative to the free surface (represented by $m(\xi,\tau)$ and $\dot m(\xi,\tau)$) and the horizontal plane motions of the body (represented by $\mathbf{J}_0(kR(x-\xi(\tau),0)$ and $\dot{\mathbf{J}}_0(kR(x-\xi(\tau),0)))$.

3.4 Reduction to Computational Form - Use of the Kochin Integral Approach

For application in a step-wise time-domain code, the general time-domain result must be reduced to a computational form in which the temporal variable is the outer integral, and the axial integral is the second-outer integral.

This is necessary to allow progressive integration of the evolving time series of results; that is, in the most general case, at each time-step we have to perform an axial integration to calculate whole-body force and moment to determine the motion of the body at the next time-increment, and the axial integration includes the contributions of all previous disturbances by virtue of the dk integral including the propagation lag from the prior trajectory of motion. This requires that the wavenumber integral be reduced first to get a computational form featuring only temporal and spatial integrations.

Consider the time-dependent part of the general solution for the interaction function $b(X,t)$, the third line of eqn (5). With the transform of coordinates to body-fixed axes (since the matching must occur in the moving reference frame of the slender body), and a change of order of integration, the function $b(X,t)$ becomes

$$\begin{aligned} b_3(X,t) = & -2\int_0^t d\tau \int_L d\xi\, \dot{m}(\xi,\tau) \int_0^\infty dk\, \mathbf{J}_0(k[|x-\xi| + U(t-\tau)]) \cos\left[(gk)^{1/2}(t-\tau)\right] \\ & -2\int_0^t d\tau \int_L d\xi\, m(\xi,\tau) \int_0^\infty dk\, \dot{\mathbf{J}}_0(k[|x-\xi| + U(t-\tau)]) \cos\left[(gk)^{1/2}(t-\tau)\right] \end{aligned} \tag{6}$$

By use of the Kochin integral approach, in [4], § 22, the first dk integral can be reduced to closed form as a difference of products of fractional Bessel functions of the first kind. Thus, we have

$$I_{k_1} = \int_0^\infty \cos\left[(gk)^{1/2}(t-\tau)\right] \mathbf{J}_0(kR)dk = -\frac{\pi}{\sqrt{2}}\frac{\rho}{R}\left[\mathbf{J}_{1/4}(\rho)\,\mathbf{J}_{3/4}(\rho) - \mathbf{J}_{-1/4}(\rho)\,\mathbf{J}_{-3/4}(\rho)\right] \tag{7}$$

$$\rho = \omega^2/2 = g(t-\tau)^2/(8R), \quad R = [|x-\xi| + U(t-\tau)] \tag{8}$$

For the corresponding second dk integral we need to adapt the Kochin formula approach, ending with

$$\begin{aligned} I_{k_2} = U\frac{\pi}{\sqrt{2}}\left\{\frac{g(t-\tau)^2}{R^2}\right\}\Bigg\{ & \left[\mathbf{J}_{-1/4}(\rho)\,\mathbf{J}_{-3/4}(\rho) - \mathbf{J}_{1/4}(\rho)\,\mathbf{J}_{3/4}(\rho)\right]\left(\frac{1}{4R}\right) \\ & + \left[\mathbf{J}_{1/4}\mathbf{J}_{7/4} + \mathbf{J}_{3/4}\mathbf{J}_{5/4} - \mathbf{J}_{-1/4}\left(2\mathbf{J}_{1/4} - \mathbf{J}_{-7/4}\right) - \mathbf{J}_{-3/4}\left(2\mathbf{J}_{3/4} - \mathbf{J}_{-5/4}\right)\right]\left(\frac{g(t-\tau)^2}{128R^2}\right)\Bigg\} \end{aligned} \tag{9}$$

4 VALIDATION

Analytical verification involves the substitution of the motion particulars of two special cases—the calm water forward speed problem and the zero-speed regular-wave radiation problem—into the general solution at eqn (5) and reduction of the inner ($d\tau$) integral. This allows the classic solutions to be recovered. This has been done. It is intended to validate the formulation against forces from the Wigley hull experiments of Journée[7], and the USN 'Force Study' results of Telste & Belknap [8],[9].

5 CONCLUSION

The foregoing has summarized the objectives, approach, and interim results of this attempt to formulate a time-domain slender ship theory for a vessel advancing into large amplitude random head seas.

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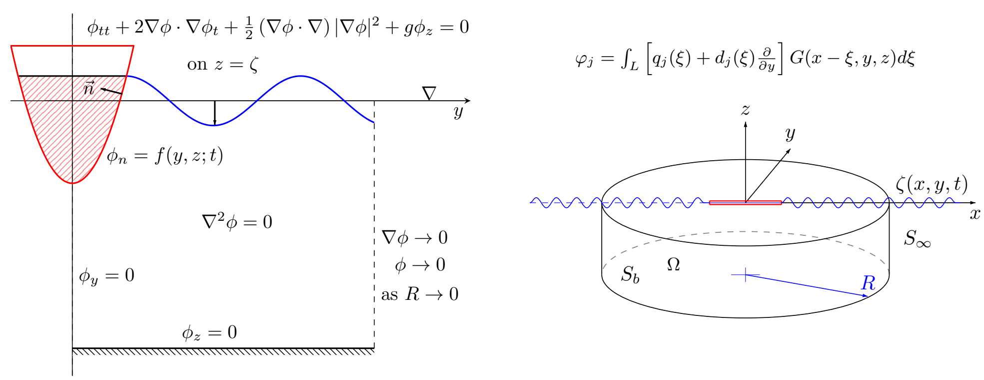
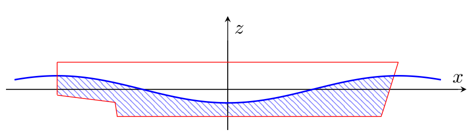
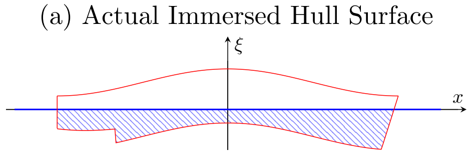


(a) Inner Problem - the Near-field (b) Outer Problem - the Far-field

Figure 1: Two Views: The Essence of Slender Ship Theory (General Case)



(a) Actual Immersed Hull Surface



(b) Virtual Immersed Hull Surface

Figure 2: Computational Domain

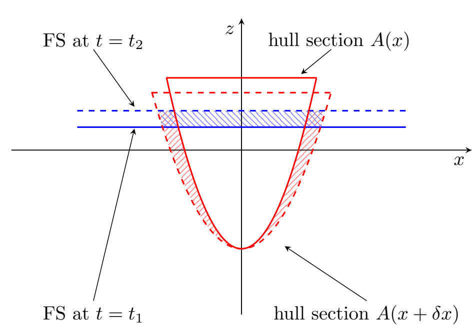


Figure 3: 2D Body Dilation
with forward speed and section heave