38th IWWWFB, University of Michigan, Ann Arbor, Michigan, USA, May 7-10, 2023

On a New Formulation for Wave Added Resistance

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1 Introduction

Our objective is to present preliminary calculations using a new formulation for the wave added resistance recently derived by Kashiwagi [1]. This method is based on far-field momentum conservation with the far-field integrals moved to the body using Green's second identity. An *incomplete* form of this formulation with body-surface integrals has existed for many years, but the *complete* version was first given by [1], where it is also shown to be equivalent to the time average of the Lagally theorem. We compare calculations using the new method with near-field pressure integration and Maruo's far-field method using both strip theory and three-dimensional (3D) potential-flow models.

2 Background, Theory, and Results

Traditionally, in the far-field method, the Transport Theorem is used to write an equation for the average rate of change of fluid momentum inside a closed volume. Then the mean drift force is expressed in terms of an integral over a far-field control surface S_∞. One way of treating this far-field integral is to employ the Kochin function to be evaluated by an integral over the body surface, and to compute the drift force using Maruo's formulation. At the previous workshop, we presented some numerical challenges related to this approach of computing added resistance for floating bodies. Another way of treating this far-field integral is to apply Green's second identity to transfer the drift force integrals to the wetted-body surface S_b. This approach was employed by Newman for the vertical mean drift force in [2]. Based on his derivation we can derive a formulation for the horizontal drift force as

$$R_w = \overline{F_x} = -\frac{\rho}{2}\Re\left\{\int_{S_b}\left[\phi_B\phi^*_{0nx} - \phi_{Bn}\,\phi^*_{0x}\right]\,ds\right\} - \frac{\rho}{4}\Re\left\{\int_{S_b}\left[\phi_B\phi^*_{Bnx} - \phi_{Bn}\,\phi^*_{Bx}\right]\,ds\right\}. \quad (2.1)$$

Here ρ is the fluid density, ϕ_B is the combined radiation and scattering velocity potential and ϕ_0 is the incident wave potential, all in the frequency domain. The subscript n denotes a derivative normal to the body surface, and subscript x denotes the x-derivative. The asterisks indicate the complex conjugate and $\Re$ takes the real part. In (2.1) we denote the first integral by F_{ϕ_0} and the second integral by F_{ϕ_B}. In his well-known paper [3], Salvesen applied (2.1) to derive a formulation for the added resistance inside Salvesen, Tuck and Faltinsen (STF) strip theory. Based on the *weak-scatterer* assumption, he neglected the second integral in (2.1). Invoking the *long-wave* assumption, he then converted the first 3D body integral in (2.1), F_{ϕ_0}, to 2D sectional and line integrals along the ship length. In a recent paper [4] we have shown that the prediction of added resistance using strip theory and equation (2.1) can be remarkably improved if none of the above-mentioned assumptions are employed in computing the 2D sectional integrals. We have reproduced part of the results of that research in Fig. 1. The results are for the Modified Wigley hull ($Fr = 0$, $Fr = 0.1$), and the RIOS bulk carrier ($Fr = 0.18$). The results are compared to measurements and Enhanced Unified Theory (EUT) [5]. As can be seen, a complete evaluation of (2.1) inside STF strip theory leads to considerably more accurate results in comparison with the classical formulation by Salvesen.

Motivated by the improved results using the strip theory, we have implemented equation (2.1) inside our in-house 3D finite-difference potential-flow solver (OW3D-Seakeeping), and calculated the mean drift force for several closed form and ship geometries. The computations are for freely-floating bodies at zero speed and for fixed submerged bodies at both zero and non-zero forward speed, see Fig. 2. The mean drift force calculations are also compared with the classical far-field and the near-field methods. Note also that two of the computations show the vertical mean drift force $\overline{F_z}$ on the submerged spheroid both with and without forward speed. From these plots, an impressive agreement can be observed between the results from equation (2.1) and the results based on the classical near-field or far-field methods. To illustrate the errors associated with the weak-scatterer assumption, we have added two examples in which only F_{ϕ_0} is considered for the computation. Similar findings in 3D were reported in (2009) by Tsubogo [6], though only for closed-form geometries at zero-speed.

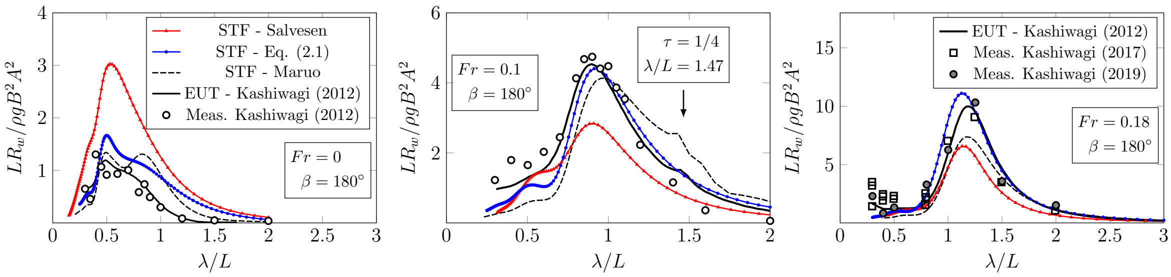

Figure 1: Added resistance computed using equation (2.1) using STF strip theory.

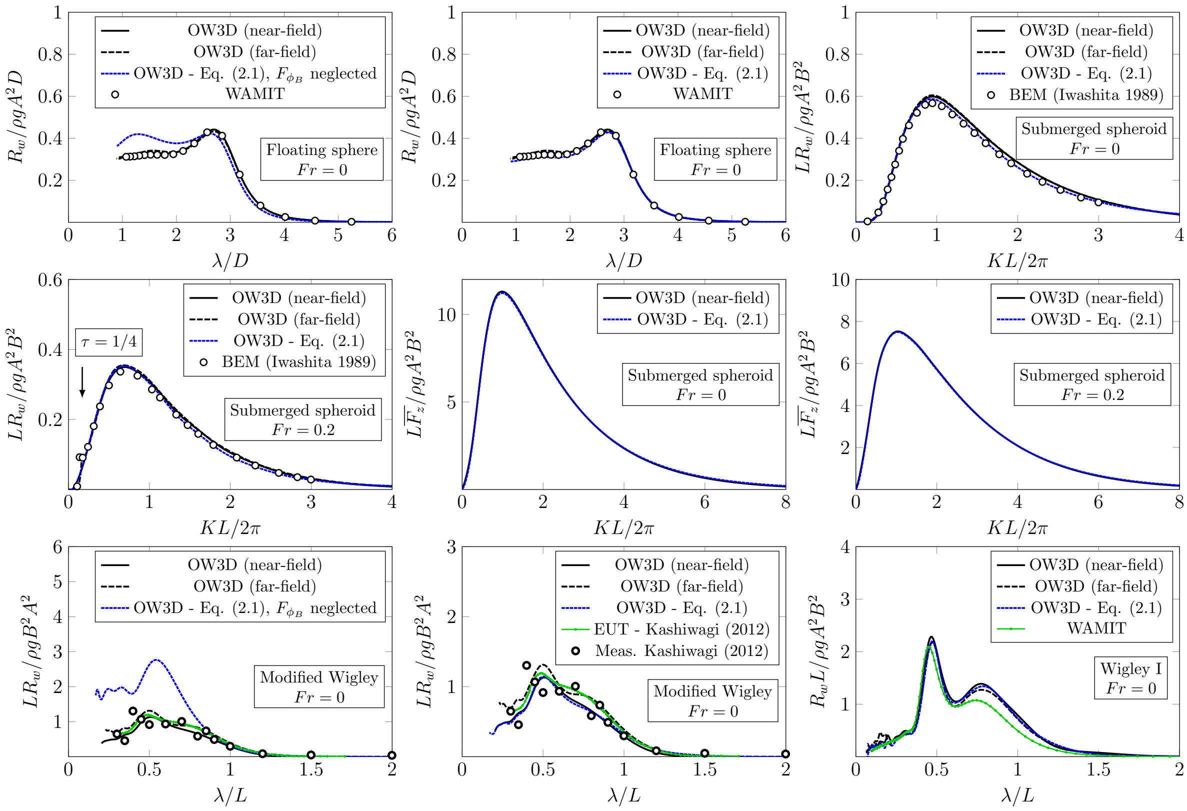

Figure 2: Mean drift force computed using equation (2.1) using a 3D potential-flow solver.

Eq. (2.1) is however, incomplete for floating bodies with non-zero forward speed, which can be seen through the following sketch of the complete derivation from [1]. As noted above, in the far-field method the mean drift force is expressed in terms of far-field integrals over S_∞ as

$$\overline{F_x} = \rho \overline{\int_{S_\infty} \left(\Phi_t + \frac{1}{2}\nabla\Phi \cdot \nabla\Phi + gz\right) n_x ds} - \rho \overline{\int_{S_\infty} \Phi_x \Phi_n ds}. \tag{2.2}$$

where Φ is the total velocity potential (including incident, radiation and scattering waves) in the time domain. Differentiation with respect to time is denoted by a subscript t and the over-bar indicates the average over one wave period. Since the mean drift force is second order, equation (2.2) can be collected into two integrals: an integral of the zeroth- and the first-order pressure terms over $z \in [0\ \ \zeta]$, and an integral of the second-order pressure terms over $z \in [-\infty\ \ 0]$ as

$$\overline{F_x} = \rho \overline{\int_{S_\infty} \left(\frac{1}{2}\nabla\Phi \cdot \nabla\Phi\, n_x - \Phi_x \Phi_n\right) ds} + \rho \overline{\int_0^\zeta \int_{C_\infty} (\Phi_t + gz)\, n_x\, dz\, dl} = I_1 + I_2. \tag{2.3}$$

Here $\zeta(t) = -\left(\Phi_t - U\Phi_x\right)/g = -\Re\left\{\left(\mathrm{i}\omega_e - U\Phi_x\right)\phi\, e^{\mathrm{i}\omega_e t}\right\}/g$ is the instantaneous free-surface elevation with ω_e the encounter frequency, and U the forward speed. C_∞ is the waterline at the far-field surface. For brevity, the integral over $[-\infty\ \ 0]$ is denoted by S_∞. Note that for example $\overline{\Phi_1\Phi_2} = \Re\left(\phi_1\phi_2^*\right)/2 = \Re\left(\phi_1^*\phi_2\right)/2$. Defining $\boldsymbol{A} = \left(0, -\phi\phi_z^*, \phi\phi_y^*\right)$ it can be shown that

$$\int_{S_\infty} \mathbf{n} \cdot (\nabla \times \boldsymbol{A})\, ds = -\int_{C_\infty} \boldsymbol{A} \cdot d\mathbf{r} = \int_{C_\infty} \phi\, \phi_z^* dy. \tag{2.4}$$

$$\frac{1}{2}\nabla\phi \cdot \nabla\phi^*\, n_x - \phi_x\phi_n^* = \frac{1}{2}\mathbf{n} \cdot (\nabla \times \boldsymbol{A}) + \frac{1}{2}\left[\phi\, \phi_{nx}^* - \phi_n^*\phi_x\right]. \tag{2.5}$$

$$I_1 = \frac{\rho}{4}\Re\left\{\int_{S_\infty} \left[\phi\, \phi_{nx}^* - \phi_n^*\phi_x\right] ds\right\} + \frac{\rho}{4}\Re\left\{\int_{S_\infty} \mathbf{n} \cdot (\nabla \times \boldsymbol{A})\, ds\right\}. \tag{2.6}$$

For the Neumann-Kelvin linearization, $\phi_z = \nu\phi + 2\mathrm{i}\tau\phi_x - \phi_{xx}/K_0$ and invoking Green's second identity, the first far-field integral in (2.6) can be converted to a body integral over S_b, and two waterline integrals. One of them over the body C_b and the other over the far-field C_∞ as follows.

$$\frac{\rho}{4}\Re\left\{\int_{S_\infty} \left[\phi\, \phi_{nx}^* - \phi_n\, \phi_x^*\right] ds\right\} = -\frac{\rho}{4}\Re\left\{\int_{S_f} + \int_{S_b} \left[\phi\, \phi_{nx}^* - \phi_n\, \phi_x^*\right] ds\right\}. \tag{2.7}$$

$$-\frac{\rho}{4}\Re\left\{\int_{S_f} \left[\phi\, \phi_{zx}^* - \phi_z\, \phi_x^*\right] dx\, dy\right\} = -\frac{\rho}{4}\Re\left\{\int_{C_\infty} \left[-2\mathrm{i}\tau\phi\, \phi_x^* + \left(\phi_x\, \phi_x^* - \phi\, \phi_{xx}^*\right)/K_0\right] n_x\, dl\right\}$$
$$-\frac{\rho}{4}\Re\left\{\int_{C_b} \left[-2\mathrm{i}\tau\phi\, \phi_x^* + \left(\phi_x\, \phi_x^* - \phi\, \phi_{xx}^*\right)/K_0\right] n_x\, dl\right\}. \tag{2.8}$$

Here $\nu = \omega_e^2/g, \tau = U\omega_e/g, K_0 = g/U^2$. Note that for $U = 0$, no integral over the free-surface S_f appears after invoking Green's second identity in the first integral of (2.6). This integral over S_f was in fact missing from all previous derivations of (2.1). The second integral in (2.3) can be evaluated as

$$I_2 = -\frac{\rho g}{2}\overline{\int_{C_\infty} \zeta^2\, n_x\, dl} + \rho U \overline{\int_{C_\infty} \zeta\Phi_x n_x\, dl} = -\frac{\rho}{4}\Re\left\{\int_{C_\infty} \left(\nu\phi\, \phi^* - \phi_x\, \phi_x^*/K_0\right) n_x\, dl\right\}. \tag{2.9}$$

Using the identity in (2.4), it can be shown that the combination of the second integral in (2.6) and the line integral in (2.9) cancels the C_∞ waterline integral in equation (2.8). Therefore the mean drift force can be computed using only near-field integrals as

$$\overline{F_x} = -\frac{\rho}{4}\Re\left\{\int_{S_b} \left[\phi\, \phi_{nx}^* - \phi_n\, \phi_x^*\right] ds\right\} + \frac{\rho}{4}\Re\left\{\int_{C_b} \left[2\mathrm{i}\tau\phi\, \phi_x^* - \left(\phi_x\, \phi_x^* - \phi\, \phi_{xx}^*\right)/K_0\right] n_x\, dl\right\}. \tag{2.10}$$

If we decompose the total velocity potential ϕ into its components, then it is possible to verify that the first integral in (2.10) is in reality identical to (2.1). Now it should be clear why (2.1) cannot be applied to calculate the mean drift force for floating bodies with forward speed.

At this point, it has not been possible for us to show convergence of the computation for the waterline integral in (2.10), and thus we have not been able to show agreement with the near-field method for floating bodies with forward speed. It is also important to mention that, as shown in [1], a similar formulation for the added resistance also exists for the double-body flow linearization. However, in this case instead of the waterline integral in (2.10), an integral of the steady and unsteady potentials should be evaluated over the free surface S_f. Due to the rapidly decaying behavior of the double-body flow potential away from the body, this surface integral appears to be tractable but we have not yet attempted to evaluate it.

3 Conclusions

We have introduced calculations using a new formulation for wave added resistance. The efficacy of this formulation has been demonstrated by comparison with classical near-field and far-field methods. For floating bodies with forward speed, further computations are required to show convergence of the waterline integral in (2.10), or its counterpart free-surface integral in case of a double-body linearization.

Acknowledgment

Financial support provided by Den Danske Maritime Fond and Orient's Fond are highly appreciated.

References

[1] M. Kashiwagi, "A New Computation Method for Added Resistance and Connection with Lagally's Theorem," *The Japan Society of Naval Architects and Ocean Engineers*, no. 2022A-OS4-5, pp. 253–259, November 2022.

[2] J. N. Newman, "The Second-order Time-average Vertical Force and Moment on a Submerged Slender Body Moving Beneath a Regular Wave System," Tech. Rep. 170, Naval Ship Research and Development Center, Washington, D.C. 20034, May 1970.

[3] N. Salvesen, "Second-order Steady State Forces and Moments on Surface Ships in Oblique Regular Waves," in *International Symposium on Dynamics of Marine Vehicles and Structures in Waves, Paper*, vol. 22, 1974.

[4] M. Amini-Afshar, "Salvesen's Method for Added Resistance Revisited," *Journal of Offshore Mechanics and Arctic Engineering*, vol. 143, no. 5, p. 051902, 2021.

[5] M. Kashiwagi, "Added Resistance, Wave-induced Steady Sway Force and Yaw Moment on an Advancing Ship," *Ship Technology Research (Schiffstechnik)*, vol. 39, no. 1, pp. 3–16, 1992.

[6] T. Tsubogo, "The Reciprocal Form of Mean Wave Drift Force and Yaw Moment," in *Proceedings of 9th International Offshore and Polar Engineering Conference. Osaka, Japan*, 2009.