Local resonances within an array of C-shaped cylinders in water waves

J. Xu[1], D.Z. Ning[1], L. Chen[1, 2], C.W. Zhang[1], B. Teng[1]

[1]State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian, 116024, China.

[2]Oceans Graduate School, Faculty of Engineering and Mathematical Sciences, The University of Western Australia, M053, Perth WA 6009, Australia.

1. Introduction

Recently, the interaction between waves and periodic structures has attracted considerable attention, noting various resonance behaviors, e.g. the so-called Bragg resonance for uniform periodic ripples [1], and rainbow trapping for graded or chirped arrays in which the properties of the array spacing or elements vary spatially [2]. The concepts built on these resonance mechanisms have been proposed for coastal protection by blocking certain waves [3] or for amplifying wave-energy harvesting by focusing wave energy of different frequencies at different spatial locations [2].

It is well recognized that these resonances of periodic solid structures (e.g. bottom mounted vertical cylinders) occur when the wavelength is comparable to the periodic spacing of the structures [4]. Thus, the proposed concepts/systems normally fail to function in long-wave regime [5]. This is crucial as tsunamis and storm swells are in this category.

Metamaterials, consisting of e.g. split-ring/tube or C-shaped cylinder arrays, in acoustics are known to be able to incur a new type of resonance at frequencies far below those of conventional Bragg resonances [4]. This provides attractive alternatives in the water-wave context. Hu et al. [6] and Dupont et al. [7] have demonstrated that the propagation of low-frequency water waves through a line of C-shaped cylinders is indeed prohibited, and Bennetts et al. [8] extended the concept by grading the cylinder properties. Although Chalmers et al. [9] argued that the formation of this low-frequency-type resonance may be associated with the resonance of the air column within each individual resonator, the underlying mechanics applied to water waves, including the effects of wave nonlinearity and water viscosity, remain unclear.

This work investigates nonlinear water wave propagation through a line array of C-shaped cylinders with an emphasis on identifying and characterizing this low-frequency-type resonance. The so-called band diagram derived using the Bloch theorem is used for identifying the frequency bands/gaps within which the wave transmission/propagation is blocked. The band diagram is initially proposed for solid-state physics of semiconductors, and is extended to water waves by e.g. Evans and Porter [10], and McIver [11]. The local wave fields are detailed for investigating the captured resonant behaviors using first linear potential flow theory and latter more sophisticated CFD-based simulations. The effects of various wave parameters and dimensions of the C-shaped cylinder are also analyzed.

2. Results and Discussion

We consider a water domain of infinite horizontal extent (defined as along x-axis), and constant finite depth h (defined as along z-axis). The vertical coordinate z points upwards, with its origin coinciding with the mean water level, and $z = - h$ denoting the bottom. The domain is bounded by a flat bottom and by a free surface, and a line array of M vertical C-shaped cylinders are placed along the centerline of the domain; details referred to Figs. 3-4. The spacings between adjacent C-shaped cylinders in x and y directions are labelled as a and b, respectively, and the individual C-shaped cylinder is characterized by its outer radius R_1, inner radius R_2, and opening l_n - see Fig. 1 left in which a periodic unit is shown. This unit is used to obtain the band diagram using Bloch theorem; details to be followed.

As mentioned above, the problem of wave-periodic structure interaction is first considered within the framework of linear potential flow theory; the flow motion can then be described by the velocity potential,

$$\Phi(x,y,z,t) = \mathrm{Re}\left\{\phi(x,y)\frac{\cosh(k(z+h))}{\cosh(kh)}e^{i\omega t}\right\} \quad (1)$$

where t is the time, and ϕ is depth independent, hence, satisfies the Helmholtz equation in the fluid domain,

$$\left(\nabla^2 + k^2\right)\phi = 0 \quad (2)$$

where k is the wavenumber, satisfying the dispersion relation $\omega^2 = gk\tanh(kh)$ with ω being the angular wave frequency, and g the acceleration due to gravity.

The no-flow boundary conditions are applied on the surfaces of the C-shaped cylinders and the flat bottom,

$$\nabla\phi \cdot \mathbf{n} = 0 \quad (3)$$

where $\boldsymbol{n}$ is the outward unit normal vector.

In Bloch theorem, the solutions for infinite periodic structures, Eqs. (1-3), are resolved within a periodic unit as shown in Fig. 1 (left); the boundaries of which should then satisfy [11],

$$\phi(a,y) = e^{iq_1 a}\phi(0,y) \text{ and } \partial\phi(a,y)/\partial x = e^{iq_1 a}\partial\phi(0,y)/\partial x \text{ in the } x \text{ direction} \quad (4)$$

$$\phi(x,b) = e^{iq_2 b}\phi(x,0) \text{ and } \partial\phi(x,b)/\partial y = e^{iq_2 b}\partial\phi(x,0)/\partial y \text{ in the } y \text{ direction} \quad (5)$$

where q_1 and q_2 are the x- and y-components of Bloch wave vector defined in the reciprocal space, respectively, and can be complex numbers in water waves [11]. For given $q_1 a$ ($\in[0, \pi]$) and q_2 (= 0; as only wave propagation along x-axis is considered), the solutions for the eigenvalues k are obtained by solving Eq. (2) using the finite element method; the first three eigenvalues are considered in this work, see e.g. Fig. 1 (right; solid lines).

The numerical solutions are first used to reproduce the experiments in Dupont et al. [8] for validation. As in Dupont et al., $a = b = 0.65$ m, $R_1 = 0.15$ m, $R_2 = 0.145$ m, and $l_n = 0.16$ m. Firstly, the calculated band diagram considering infinite arrays is shown in Fig. 1 right. It can be seen that there are two frequency ranges (characterized by ka; and shaded by blue) within which no real values of q_1 can be found. These are so-called stopping bands (or band gaps). As a result, the incident wave of the frequency in these stopping bands is blocked by the periodic structure; the wave amplitude is attenuated and eventually decreases to zero as infinite number of cylinders is considered. The frequencies outside the stopping bands are otherwise formed the so-called passing bands within which the waves can propagate freely through the array with changes only in phase. It is found that the band gap $ka\in[3.08, 3.88]$ corresponds to the well-known Bragg resonances associated with the periodic nature of the array [9], and the band gap $ka\in[1.96, 2.80]$ falls in the category of low-frequency-type resonances.

Then the array of three C-shaped cylinders is considered, facilitating a direct comparison with the experiments (only finite number of cylinders can be considered in laboratory). The absorbing boundary conditions (the method of numerical beach is applied in this work) are applied at the left and right ends of the numerical flumes for absorbing reflected waves from the domain. The calculated transmission coefficients are compared with the experimental data in Fig. 2. It can be seen that the transmission coefficients are nearly zero or very small in two separated regions, which are overlapped with the band gaps identified in Fig. 1. Main trend is captured by the present numerical solutions, although differences are observed. This could be due to the energy dissipation and wave breaking at the slits in the experiments; details will be presented at the workshop.

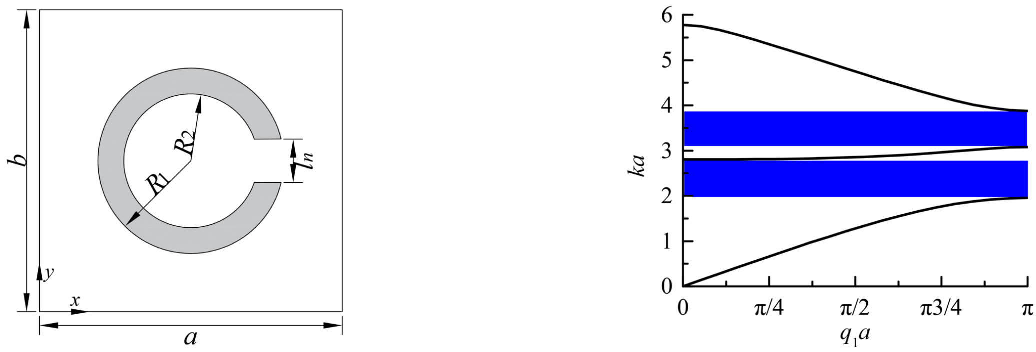

Fig. 1 Layout of a periodic unit of a line array of infinite C-shaped cylinders (left) and the band diagram calculated using Bloch therom (right). The band gaps are shaded by blue. $a = b = 0.65$ m, $R_1 = 0.15$ m, $R_2 = 0.145$ m, and $l_n = 0.16$ m.

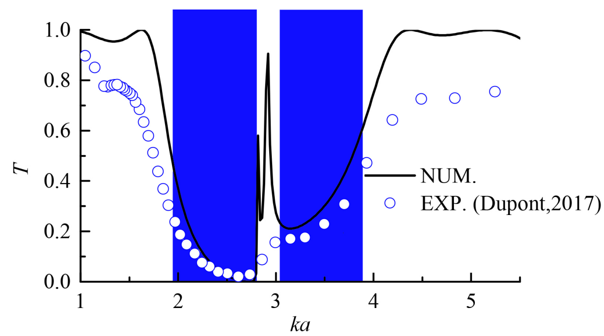

Fig. 2 Comparison of numerical results of the transmission coefficient with experiment data. (M = 3; $a = b = 0.65$ m, $R_1 = 0.15$ m, $R_2 = 0.145$ m, and $l_n = 0.16$ m)

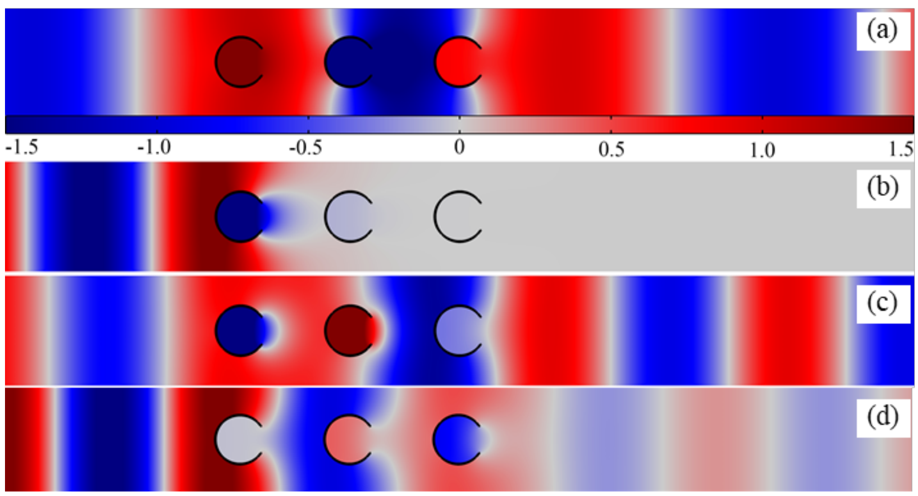

Fig. 3 Contour of non-dimensional free surface elevations at $ka = 1.63$ (a), 2.74 (b), 2.92 (c) and 3.27 (d).

Figs. 3(a) ~ (d) show the contour of non-dimensional free surface elevations for four typical frequencies, $ka = 1.63$, 2.74, 2.92 and 3.27, respectively. As expected, no resonance occurs and the waves of frequencies outside the band gaps are free to propagate through the array, see Figs. 4 (a) and (c). While the wave energy of frequency $ka = 2.74$ in the second band gap and $ka = 3.17$ in the first band gap are blocked by the array; the differences is that for the former, only the first cylinder plays the role (Fig. 3 (b)), and the latter the amplitude attenuation starts at the first cylinder and evolves along the array (Fig. 3(d)). This suggests that the resonance mechanism associated with the second band maybe a locally behavior incurred by the individual periodic unit, and the resonance in the first band should arise due to the periodic nature of the structure, consistent with the Bragg-type resonances. We note that similar resonance phenomenon in the second lower frequency band is observed by Elford et al. [4] in acoustic waves.

This is then further investigated by running numerical experiments with one and five C-shaped cylinders and results are shown in Fig. 4. It can be seen that the resonance still occurs even if there is only one C-shaped cylinder. Meanwhile, comparisons between Fig. 3 (b) and Fig. 4 show that the modes of resonance are the same around the first cylinder for the three cases considered. It indicates that this low-frequency-type resonance is indeed a locally resonance, and may be caused by the wave resonance in the cavity. More results regarding the effect of nonlinearity etc. will be presented at the workshop.

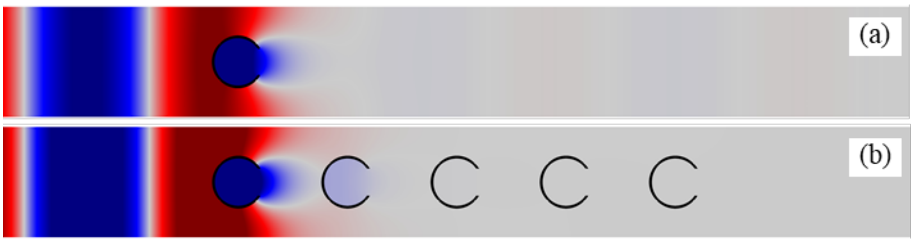

Fig. 4 Contour of non-dimensional free surface elevations at ka = 2.74 for cases with one (a) and five (b) C-shaped cylinders. For color maps see Fig. 3.

Acknowledgements

This work is supported by the LiaoNing Revitalization Talents Program (Grant No. XLYC2002033), Liaoning BaiQianWan Talents Program (Grant No. 2020921007) and Fundamental Research Funds for the Central Universities (Grant No. DUT21LAB116).

References

[1] Zhang, S. B., Chen, L., Ning, D. Z., & Teng, B. *Multi-Bragg reflections over a periodic submerged structure*. The 36h International Workshop on Water Waves and Floating Bodies, Seoul, Korea, 25-28 April, 2021

[2] Archer, A. J., Wolgamot, H. A., Orszaghova, J., Bennetts, L. G., Peter, M. A., & Craster, R. V. 2020. *Experimental realization of broadband control of water-wave-energy amplification in chirped arrays*. Physical Review Fluids, 5(6), 062801.

[3] Liu, H. W., Liu, Y., & Lin, P. 2019. *Bloch band gap of shallow-water waves over infinite arrays of parabolic bars and rectified cosinoidal bars and Bragg resonance over finite arrays of bars*. Ocean Engineering, 188, 106235.

[4] Elford, D. P., Chalmers, L., Kusmartsev, F. V., & Swallowe, G. M. 2011. *Matryoshka locally resonant sonic crystal*. The Journal of the Acoustical Society of America, 130(5), 2746-2755.

[5] Hu, X., Chan, C. T., Ho, K. M., & Zi, J. 2011. *Negative effective gravity in water waves by periodic resonator arrays*. Physical review letters, 106(17), 174501.

[6] Hu, X., Yang, J., Zi, J., Chan, C. T., & Ho, K. M. 2013. *Experimental observation of negative effective gravity in water waves. Scientific Reports*, 3(1), 1-4.

[7] Dupont, G., Remy, F., Kimmoun, O., Molin, B., Guenneau, S., & Enoch, S. 2017. *Type of dike using C-shaped vertical cylinders*. Physical Review B, 96(18), 180302.

[8] Bennetts, L. G., Peter, M. A., & Craster, R. V. 2018. *Graded resonator arrays for spatial frequency separation and amplification of water waves*. Journal of Fluid Mechanics, 854.

[9] Chalmers, L., Elford, D. P., Kusmartsev, F. V., & Swallowe, G. M. 2009. *Acoustic band gap formation in two-dimensional locally resonant sonic crystals comprised of Helmholtz resonators*. International Journal of Modern Physics B, 23(20n21), 4234-4243.

[10] Evans, D. V., & Porter, R. 1999. *Trapping and near-trapping by arrays of cylinders in waves*. Journal of Engineering Mathematics, 35(1), 149-179.

[11] McIver, P. 2000. *Water-wave propagation through an infinite array of cylindrical structures*. Journal of Fluid Mechanics, 424, 101-125.