High-speed vessel moving along the edge of ice field

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1 Introduction

We are concerned with motion of a high-speed vessel that operates in close proximity of an ice field. The presence of the floating ice sheet changes the hydrodynamic forces acting on the vessel. The waves generated by the ship may break the ice near its edge.

In the present study, the ice is continuous with constant thickness and identical material properties in all directions at every point. The ice is modeled as a thin elastic semi-infinite plate. The rest of the water surface is free of ice. The ship moves in open water of finite depth, see Fig. 1. The flow generated by the ship is nonlinear and potential. Viscous and surface tension effects are not included in the present model. The problem is challenging because the moving vessel causes the ice

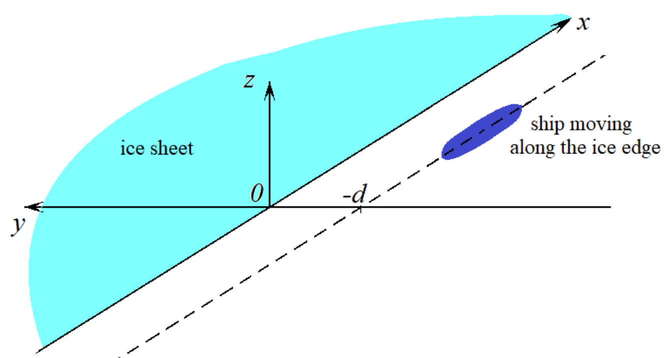

Figure 1: Sketch of the problem.

deflection which in turn modifies the vessel motions. The problem is coupled, generally speaking. The coupling is expected to be weak if the vessel moves far enough from the ice edge. We shall find the conditions of ice and the vessel motion where the presence of the ice field affects the vessel motions.

The forces acting on the ship and the ship motions are computed by the 2D+t approach. Within this approach the three-dimensional flow is approximated by several two-dimensional earth fixed flow problems. As the ship moves forward it causes unsteady flow in each cross-plane, which is solved using a boundary element method based on potential theory. General requirements for the validity of 2D+t theory are that the ship hull is slender and moves fast. Corrections are typically needed at the transom stern as the physically required drop to atmospheric pressure is not captured by the 2D+t approximation. For the ship moving in open water of constant depth, the flow is symmetric with respect to the symmetry plane of the ship. If the ship moves along either a vertical wall or an ice edge, then the flow is not symmetric and there are forces acting on the ship in the transverse direction. Disturbances generated by the ship in the elastic ice propagate both from the ice edge, which are consistent with the 2D+t approach, and along the edge, which leads to interaction between the cross-planes.

In case of open water and constant forward speed of the ship, it is sufficient to use a single cross-plane to approximate the three-dimensional wave field because the temporal development of the flow in a earth fixed cross-plane can be directly translated to the spatial distribution of the wave field along the ship axis. This is also the case if the ice sheet is modeled by a rigid plate of constant thickness, as there is no interaction between the respective cross planes If the ice sheet is modeled as elastic, then all cross-planes need to be modeled simultaneously. This is comparable to previously published problems with the ship moving in incident waves, where interaction between the flows in the cross planes is implied by the unsteady ship motion [1; 2]. However, in our problem, it is not clear how to describe the deflection of the ice sheet within the 2D+t approach, which does not account for the hydroelastic waves propagating in the direction of the ship motion.

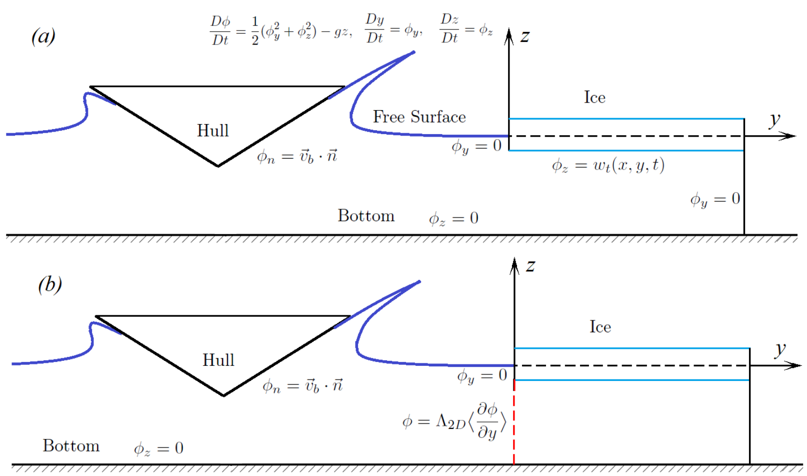

Figure 2: Boundary conditions in a cross plane for the BEM 2D+t (a) and 2D+t with the non-local boundary condition on the fictitious vertical boundary (b).

The idea of the present study is to use the domain decomposition method dividing the flow domain into the domain of open water, $y < 0$, where the ship moves, and the domain covered with ice, $y > 0$. We model the flow in the ice covered region and the ice deflection caused by the ship moving near the ice edge as linear and match the velocity potential in the ice covered region with the potential of nonlinear flow in the open water region at a fictitious boundary shown in Fig. 2 by the vertical dashed line. To this aim, we shall solve the linear problem in the ice covered region, and find a non-local relation between the velocity potential under the ice and its normal derivative at the fictitious vertical wall at the ice edge. This relation is suggested to use as the boundary condition for the non-linear problem in the open water region with the ship. The boundary element method, which is used to solve the flow in each cross-plane for the open-water region $y < 0$, incorporates fully non-linear free surface boundary conditions. This enables it to capture important flow effects associated with high speed ships such as spray evolution and flow separation at chines. Details on the implementation can be found in [1]. Due to the potential flow assumptions made for the flow solver, viscous effects are neglected.

This approach is tested first for the rigid ice model, where the 2D+t approach is well applicable with a single cross plane needed for computations. The rigid-ice plate can be included in the boundary element method without using the 3D or 2D non-local boundary condition at $y = 0$.

In this way we can investigate the applicability of the non-local boundary condition and gain experience of working with this condition. We consider using the boundary element method for both open water and ice-covered regions as too extensive. Note that non-local boundary condition at $y = 0$ does not include the memory effect for the rigid ice model.

2 Non-local boundary condition

We consider here only the ice-covered region, $y > 0$, $-\infty < x < \infty$ and $-H < z < 0$, where H is the water depth and $z = 0$ is the position of the ice/water interface at equilibrium. The ice sheet of constant mass per unit area m_i and rigidity D_i corresponds to the half-plane $y > 0$. The ice deflection $w(x, y, t)$ is governed by the linear Kirchhoff-Love equation of a thin elastic plate,

$$m_i w_{tt} + D_i \nabla^4 w = p(x, y, w(x, y, t), t) \qquad (y > 0,\ t > 0), \tag{1}$$

where $p(x, y, z, t)$ is the hydrodynamic pressure acting on the ice/water interface $z = w(x, y, t)$. This linear model of continuous elastic ice plate is acceptable because ice breaks for relative elongations of ice elements (strains) of order of 10^{-4}. Large and moderate deformations of the ice plate lead to ice breaking before such non-linear deformations are achieved. The plate equation (1) is solved subject to the zero initial conditions, $w(x, y, 0) = 0$ and $w_t(x, y, 0) = 0$, free-free boundary conditions at the edge of the ice plate, and the far-field condition, $w(x, y, t) \to 0$ as $x^2 + y^2 \to \infty$ and $t < +\infty$.

We assume that the equations of the flow under the ice sheet and corresponding boundary conditions can be linearised, even if the flow outside the ice-covered region is fully nonlinear. The linear theory of hydroelasticity provides the following equations describing the flow under the ice:

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \quad (-H < z < 0,\ y > 0,\ |x| < \infty), \qquad \phi \to 0 \quad (x^2 + y^2 \to \infty),$$

$$\phi_z = 0 \quad (z = -H,\ \ y > 0,\ |x| < \infty), \qquad \phi_z = w_t(x, y, t) \quad (z = 0,\ y > 0,\ |x| < \infty). \tag{2}$$

The hydrodynamic pressure on the ice/water interface is given by the linearised Bernoulli equation,

$$p(x, y, w(x, y, t), t) = -\rho\phi_t - \rho g w(x, y, t),$$

where ρ is the liquid density and g is the gravitation acceleration. We assume that the normal velocity of the flow on the fictitious vertical boundary of the ice covered region is given,

$$\phi_y(x, 0, z, t) = G(x, z, t) \qquad (H < z < 0,\ y = 0,\ -\infty < x < \infty,\ t \geq 0). \tag{3}$$

We shall solve the problem (1)-(3) and determine the potential $\phi(x, 0, z, t)$ at the vertical fictitious boundary as an operator acting on $G(x, z, t)$ in the form $\phi(x, 0, z, t) = \Lambda <G>$. The latter relation is suggested to be used as a boundary condition at the fictitious boundary of the open water region with a ship moving there.

To solve the problem (1)-(3), we use the Fourier transform in the x-direction and the normal mode method in the vertical z-direction. The velocity potential $\phi(x, y, z, t)$ is sought in the form $\phi(x, y, z, t) = \phi_0(x, y, z, t) + \Phi(x, y, z, t)$, where $\phi_0(x, y, z, t)$ is the solution of the problem (2)-(3), where $w_t(x, y, t) = 0$. Correspondingly, $\Phi(x, y, z, t)$ is the solution of the problem (1)-(3), where $\Phi_y(x, 0, z, t) = 0$ on the fictitious boundary and the right hand side in (1) reads $-\rho\Phi_t - \rho\phi_{0t} - \rho g w(x, y, t)$. The potential ϕ_0 is the solution of the problem for the rigid ice model. Note that the solution of this problem exists only if the integral of the function $G(x, z, t)$ over the fictitious boundary is zero. We start with the potential $\phi_0(x, y, z, t)$. This potential satisfies Laplace's equation and the conditions $\phi_{0z} = 0$ at $z = -H$ and $z = 0$. The resulting relation on the fictitious boundary reads

$$\phi_0(x, 0, z, t) = \int_{-\infty}^{\infty} \int_{-H}^{0} \frac{\partial \phi_0}{\partial y}(x_0, z_0, 0, t) k_{3D}(x - x_0, z, z_0) \mathrm{d}x_0 \mathrm{d}z_0, \tag{4}$$

$$k_{3D}(x - x_0, z, z_0) = -\frac{1}{2\pi}\Bigg\{ \frac{1}{\sqrt{(x-x_0)^2 + (z-z_0)^2}} + \frac{1}{\sqrt{(x-x_0)^2 + (2H+z+z_0)^2}} +$$

$$\frac{\pi}{H} S_R(|x - x_0|, 2H + z + z_0) + \frac{\pi}{H} S_R(|x - x_0|, |z - z_0|) \Bigg\},$$

$$S_R(x, \zeta) = \sum_{k=1}^{\infty} \left[\frac{1}{\sqrt{(2\pi k - \pi\zeta/H)^2 + (\pi x/H)^2}} + \frac{1}{\sqrt{(2\pi k + \pi\zeta/H)^2 + (\pi x/H)^2}} - \frac{1}{\pi k} \right].$$

The relation (4) written within the 2D+t approach without interaction between cross planes is

$$\phi_0(x, 0, z, t) = \int_{-H}^{0} \frac{\partial \phi_0}{\partial y}(x_0, z_0, 0, t) k_{2D}(z, z_0) \mathrm{d}z_0, \quad k_{2D}(z, z_0) = \frac{1}{\pi} \log \left| \cos\left(\frac{\pi z}{H}\right) - \cos\left(\frac{\pi z_0}{H}\right) \right|. \tag{5}$$

Note that here $z = 0$ corresponds to the ice-water interface but not to the equilibrium level of the liquid, see Fig. 2. The lower part of the ice edge is submerged in water, which makes the derivative $\phi_{0,y}(x, z, 0, t) = O(|z|^{-\frac{1}{3}})$ to be singular at the lower corner of the ice edge.

3 Numerical results

The computer code *impact2d* which was developed at TUHH, has been adjusted to account for rigid level ice by implementing the proposed boundary conditions (4) and (5). Condition (5) is consistent with classical 2D+t theory, where a single cross plane is used to approximate the 3D flow. This problem is also solved directly by using panels on the rigid boundaries of the ice-covered region without considering any conditions on the fictitious boundary. Additionally, condition (4) has been implemented. As it requires integration along the x-axis, all cross planes need to be modelled simultaneously. In that regard it is considered a first step towards implementation of elastic ice sheets. The results obtained with the respective approaches are compared to each other throughout the study. The results of the comparison will be presented at the Workshop.

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