Generalizing the optimal axisymmetric point-absorber wave energy converter to irregular waves

Emma C. Edwards[1,2*], Martyn Hann[1], Deborah Greaves[1], Dick K. P. Yue[2]
[1] School of Engineering, Computing and Mathematics, University of Plymouth, Plymouth, UK
[2] Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA, USA
*emma.c.edwards@plymouth.ac.uk

HIGHLIGHTS

We extend analysis which found the optimal shape of an axisymmetric point-absorber wave energy converter for a monochromatic wave, called $\mathscr{S}_O$, to irregular waves. We show that as the incident wave spectrum width increases, the optimal resonant wavenumber for $\mathscr{S}_O$ decreases and the optimal power take-off coefficient increases.

1 INTRODUCTION AND THEORY

In [1], we perform an optimization of geometry of an axisymmetric point-absorber wave energy converter (WEC), in which we assume a monochromatic unidirectional incident wave, all within the context of linearized potential theory. The optimization framework involves ensuring maximum power (derived in [2]), specifying practical motion constraints and then minimizing surface area as a proxy for cost. We carry out a nonlinear optimization for a broad range of geometries for four different constraint regimes, for the problem of a WEC moving and extracting energy in the heave-mode only, as well as the full 3D problem of a WEC moving and extracting energy in heave, surge and pitch. In this paper, we focus on the shape that results from the heave-only problem, where the heave motion is constrained to be no more than three times the incident wave amplitude, and the draft of the body times the wavenumber over the heave motion is required to be larger than 0.1. This shape, which we call $\mathscr{S}_O$, is shown in figure 1.

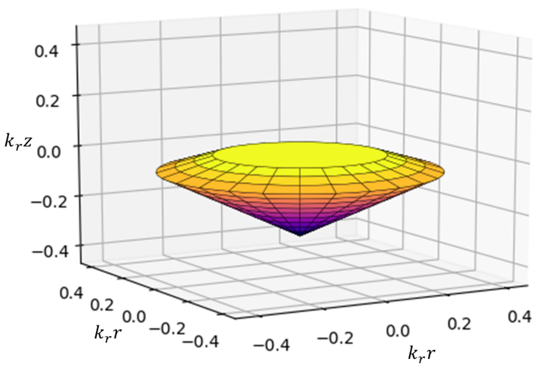

Figure 1: Shape $\mathscr{S}_O$, the optimal shape for an axisymmetric heaving WEC [1]

In this paper we extend the analysis to irregular waves. Instead of performing a full new optimization, we assume the shape to be $\mathscr{S}_O$, and we develop a methodology to determine how the optimal properties of $\mathscr{S}_O$ depend on the incident sea-state, described by a spectrum. As shown in figure 1 and described in [1] and [3], geometric descriptions of $\mathscr{S}_O$ are nondimensionlized by incident resonant wavenumber k_r. In this paper, we will determine what k_r of $\mathscr{S}_O$ should be to maximize power given an incident spectrum, and how this depends on spectrum width and shape. Specifically, this tells us how the optimal size of $\mathscr{S}_O$ depends on the incident sea state.

For regular waves, there is a known expression for the optimal power take-off (PTO) coefficient ([2], [3]), but for irregular waves there is no such expression. Therefore, we also look in this paper how the optimal PTO coefficient depends on spectrum width and shape.

We assume linear potential flow, deep water, heave-only motion, and a PTO modeled as a linear damper with damping coefficient β_{33}. The incident sea state is assumed unidirectional. We consider three types of incident spectrum. The first, a top-hat spectrum, is described by

$$S_{TH}(k) = \begin{cases} 1/(2\delta), & \text{if } k_m - \delta < k < k_m + \delta \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where k_m is the mean wavenumber. The second, a Gaussian spectrum, is described by

$$S_G(k) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-1}{2}\frac{(k-k_m)^2}{\sigma^2}\right), \tag{2}$$

where σ is the standard deviation. Finally, the Bretschneider spectrum is described by

$$S_B(k) = \frac{5}{16}\frac{k_m^2}{\sqrt{k^5 g}} H_S^2 \exp\left(\frac{-5k_m^2}{4k^2}\right), \tag{3}$$

where H_S is the significant wave height, and k_m is the modal wavenumber. We define $[f]^S \equiv \int f(k)S(k)dk$. To quantify the extractable power, given an incident spectrum $S(k)$, we look at the 'spectrum capture width,'

$$W^S = [P(\beta_{33})]^S / [\Pi]^S, \tag{4}$$

where $P(k, \beta_{33})$ is extractable power at wavenumber k with PTO coefficient β_{33} and $\Pi(k)$ is incident power per unit crest-length at wavenumber k. Note that β_{33} cannot change for different wavenumbers. To compare PTO coefficient with the optimal value for a monochromatic wave, we define

$$\beta_{33}^{'} = \frac{\beta_{33}}{B_{33}(k_r)}. \tag{5}$$

For the top-hat spectrum (equation 1), the spectrum width, Δ, is measured by half-width of the spectrum, $\delta/2$, and for the Gaussian spectrum (equation 2), Δ is measured by the half-width at half-max. We nondimesionalize Δ, k_r and W^S by mean or modal wavenumber k_m.

We define $\mathcal{W}$ to be the maximum $k_m W^S$ for a given $\beta_{33}^{'}$ and spectrum, and $\mathcal{K}$ to be the optimal k_r/k_m that gives that $\mathcal{W}$. Furthermore, we define $\beta_{33}^{'*}$ to be the optimal value of $\beta_{33}^{'}$ for a given spectrum, with $\mathcal{W}^*$ being the corresponding maximum $\mathcal{W}$ and $\mathcal{K}^*$ the corresponding optimal $\mathcal{K}$. We determine how $\beta_{33}^{'*}, \mathcal{K}^*$ and $\mathcal{W}^*$ change with different shape (type) and width, Δ/k_m, of spectra.

2 RESULTS AND DISCUSSION

Figures 2 a) and b) show that the optimal resonant wavenumber $\mathcal{K} = k_r/k_m$ decreases as Δ/k_m increases, for both the top-hat and Gaussian spectra. Because $\mathcal{K} < 1$, we see that for wider spectra the optimal WEC is not in resonance at the mean/modal wavenumber of the spectrum. Figure 2 c) and d) show that there is an optimal $\beta_{33}^{'}$ for each spectra, which increases as Δ/k_m increases. That is, the optimal PTO coefficient is larger for wider spectra. Figure 3 shows that, as Δ/k_m increases, $\mathcal{W}^*$ a) increases for top-hat spectra but b) stays approximately constant for Gaussian spectra. Furthermore, figures 3 a) and b) show that $\mathcal{K}^*$ decreases and $\beta_{33}^{'*}$ increases with increasing Δ/k_m for both types of spectra.

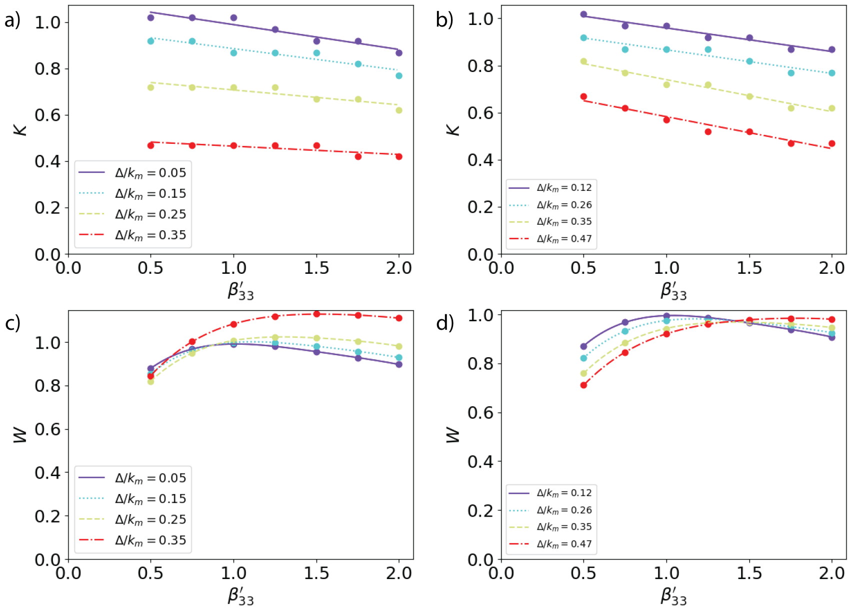

Figure 2: The optimal $\mathcal{K} = k_r/k_m$ as a function of β'_{33}, for different Δ/k_m values for (a) the top-hat spectra, and (b) Gaussian spectra, and the corresponding $\mathcal{W} = k_m W^S$ values, for (c) the top-hat spectra and (d) Gaussian spectra

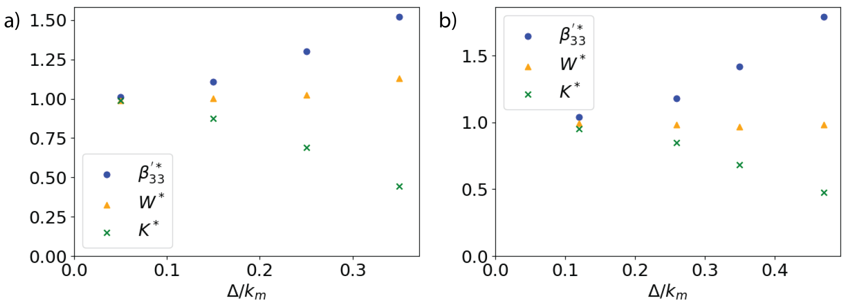

Figure 3: The optimal $\beta_{33}{}'*$, and corresponding maximum $\mathcal{W}^*$ and optimal $\mathcal{K}^*$, as functions of Δ/k_m for (a) the top-hat spectra and (b) the Gaussian spectra

And finally, we consider a more realistic incident sea state by considering the Bretschneider spectrum (equation 3). The Bretschneider spectrum differs from the other two spectra we consider because: (i) it is not symmetric– the spectrum is wider above the peak than below it, and (ii) its width is not changeable. We calculate that the width below k_m at half-max is $\Delta/k_m = 0.37$, and above k_m it is $\Delta/k_m = 0.89$. For the Bretschneider spectrum we calculate $\beta'^*_{33} = 1.5, \mathcal{K}^* = 0.84$, and $\mathcal{W}^* = 0.74$. This extractable power is less and the optimal resonant wavenumber is larger than an equivalently wide symmetric spectrum. We hypothesize that this is because of the asymmetry of the spectrum. Since there is not as much energy at small wavenumbers for the Bretschneider spectrum, having a smaller $\mathcal{K}$ will not be as beneficial, and thus the resonant wavenumber should be close to the modal wavenumber.

When determining $\mathcal{S}_O$ in [1], we apply constraints (following from [4]) on (i) the body heave body motion ξ_3 relative to the incident wave amplitude A: $\alpha_3 < \alpha_0$, where $\alpha_3 = |\xi_3|/A$ and α_0 is the design constraint, and (ii) wavenumber k times draft at the centerline H relative to α_3: $\epsilon_3 > \epsilon_0$, where $\epsilon_3 = kH/\alpha_3$ and ϵ_0 is the design constraint.

For irregular waves, the 'spectrum motion constraint' becomes $\alpha_3^S < \alpha_0^S$ where

$$\alpha_3^S = \left(\left[(|\xi_3|/A)^2 \right]^S / [1]^S \right)^{1/2}, \tag{6}$$

and α_0^S is the design constraint. The 'spectrum steepness constraint' becomes $\epsilon_3^S > \epsilon_0^S$, where

$$\epsilon_3^S = \left([1]^S / \left[\left(\frac{|\xi_3|/A}{kH} \right)^2 \right]^S \right)^{1/2}, \tag{7}$$

and ϵ_0^S is the design constraint. For the Bretschneider spectrum, using the optimized parameters of $\mathcal{K}^* = 0.84$ and $\beta_{33}^{\prime *} = 1.5$, we calculate $\alpha_3^S = 1.62$ and $\epsilon_3^S = 0.19$. If $\alpha_0^S = 3$ and $\epsilon_0^S = 0.1$, the calculated values clearly adhere to the spectrum motion and steepness constraints. If stricter motion constraints were desired, the methodology described in this paper is general and could therefore be applied to a different shape.

3 CONCLUSION

In a recent paper, we perform an optimization of the geometry of an axisymmetric point-absorber WEC. In this paper, we extend analysis on the optimized shape, called $\mathcal{S}_O$ and shown in figure 1, to irregular waves. We consider top-hat and Gaussian spectra to investigate how width and shape of spectrum influences the optimal resonant wavenumber of $\mathcal{S}_O$ and optimal PTO coefficient, and we also consider a Bretschneider spectrum to study a more realistic sea-state. We show that as the spectrum width increases, the optimal resonant wavenumber of $\mathcal{S}_O$ decreases and the optimal PTO coefficient increases. Compared to the idealized spectra, the extractable power for the Bretschneider spectrum is less, and the optimal resonant wavenumber is larger than an equivalently wide symmetric spectrum.

These conclusions give us insights into WEC design. Namely, when designing a WEC in irregular waves, it should be larger than if it was in resonance at the peak wavenumber, and the size increases with wider spectra. The PTO should not be tuned to be equal to the radiation damping at resonance, as was the case for regular waves, and should be larger with wider spectra. When designing a WEC for a particular location the spectrum shape is an important part of the design consideration. While we begin with the optimized shape found from regular waves, our methodology of determining size and PTO of the WEC is general and could be used for any WEC shape and incident spectrum. It would be interesting to perform a full optimization of the shape in irregular waves to compare the result to $\mathcal{S}_O$. Finally, this work assumes linear potential flow, so physical modelling and/or higher-order methods (i.e. [5]) should be used for further analysis.

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