Energy budgets in the interaction between a surface gravity wave and a sumberged cylindrical resonator

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HIGHLIGHTS
In this work, we investigate the energy transfer between a surface propagating wave and a sumberged resonator. We provide evidence that close to resonance conditions, not only is the energy transfer enhanched, but also the viscous dissipation rate, due to increased velocity differences between the wave and the resonator.

1 INTRODUCTION

Metamaterials are engineered materials developed to control standard properties of wave propagation such as dispersion, refraction or diffraction. They were first developed in the field of optics [1] and then extended to phononic crystals and elastic waves [2]. Metamaterials are typically arranged in periodic patterns and their dispersion relation can display band gaps, *i.e.* frequency regions of forbidden propagation. Similar effects have been reported also for surface gravity waves propagating over a periodic structure [3].

In this context, the METAREEF project aims at developing a novel device for coastal protection based on the idea of metamaterials and their enhanced wave control. The device is composed of a lattice of reversed pendula anchored to the seabed. The interplay between the propagating wave and the submerged device can produce wave filtering, depending on the ratio between the wave frequency and the natural frequency of the pendula [4]. Although in the past potential theory has been successfully used to describe wave forcing on submerged structures, recently it has been shown that viscosity can play an important role, especially close to resonant conditions [5]. For this reasons, Direct Numerical Simulations (DNS) of the Navier-Stokes equations can represent a powerful tool to understand the energy exchange between the wave and the submerged structure.

In this work, we present results of DNS simulations of a wave propagating over a sumberged horizontal resonator, oscillating about an equilibrium position, which is an approximation of the reversed pendula of the METAREEF device. The aim is to provide evidence that the resonators absorb energy from the wave, which is dissipated inside the fluid by viscous stresses. Results are presented in terms of energy budget and velocity distributions.

2 Governing Equations

The physical problem is governed by the Navier-Stokes equations for multiphase flows:

$$\rho(\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u}) = -\nabla p + \nabla\cdot(\mu\mathbf{D}) + \rho\mathbf{g} + \mathbf{f} \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0 \tag{2}$$

with $\mathbf{u} = (u, w)$ the velocity field, p the pressure field, $\mathbf{D}$ the deformation tensor defined as $D_{ij} = (\partial_i u_j + \partial_j u_i)/2$, $\mathbf{g}$ the gravity vector and $\mathbf{f}$ the Immersed Boundary Method force which enforces the no-slip boundary condition at the solid boundary. The material properties ρ and μ of the fluid are related to the volume fraction field $\mathcal{F}(\mathbf{x}, t)$ as

$$\rho(\mathcal{F}) = \mathcal{F}\rho_1 + (1 - \mathcal{F})\rho_2 \tag{3}$$

$$\mu(\mathcal{F}) = \mathcal{F}\mu_1 + (1 - \mathcal{F})\mu_2, \tag{4}$$

where ρ_1, ρ_2, μ_1 and μ_2 are the density and viscosity of the two fluids; the volume fraction field (defined as the volumetric ratio of the two fluids in each computational cell) is advected by the flow with the following equation

$$\partial_t \mathcal{F} + \nabla \cdot (\mathcal{F}\mathbf{u}) = 0. \tag{5}$$

The dynamic of the resonator is modelled by the Newton's law

$$m\frac{d^2X}{dt^2} + \kappa(X - X_0) = F \tag{6}$$

where X is the position of the centre of mass of the resonator, m its mass, κ is the elastic constant, X_0 the equilibrium position and F the horizontal component of the integral of the hydrodynamic forces acting on it. This force is computed by integrating the surface forces tensor, pressure (p) and the viscous stress tensor ($\boldsymbol{\tau}$) over the surface of the solid body as follows:

$$\mathbf{F} = \int_S (\boldsymbol{\tau} - p\mathbf{I}) \cdot \mathbf{n} dS. \tag{7}$$

Details on the implementation and validation of the solver can be found in [6].

3 Results

We simulate a monochromatic sinusoidal wave propagating inside a unit periodic 2D squared domain of size λ, the wavelenght of the wave. The initial surface elevation and velocity field are taken from linear theory. Since the water depth is equal to $h = \lambda/2$ the wave angular frequency ω_w can be estimated using the dispersion relation in deep-water condition $\omega_r = \sqrt{gk}$, with $k = 2\pi/\lambda$ the wavenumber. The resonator is located at the center of the domain in the horizontal direction ($X_0 = \lambda/2$) and the submergence is $d = -r - 3a$, r being the radius of the resonator and a the initial wave amplitude. The natural frequency of the resonator can be estimated by considering the added mass term $\omega_r = \sqrt{\kappa/(m + \rho_w V)}$, where V is the volume of the resonator and ρ_w the density of the surrounding fluid. The simulation parameters are the following: Reynolds number $Re = \rho_w g^{1/2}\lambda^{3/2}/\mu_w = 10^5$, density ratio $\rho_a/\rho_w = 1/850$, viscosity ratio $\mu_a\mu_w = 1.92 \cdot 10^{-2}$, resonator radius $r/\lambda = 0.1$ and density $\rho_r = \rho_w/2$.

In [4] it has been shows that based on the ratio between the wave frequency and the resonator frequency $\Omega = \omega_r/\omega_w$, wave energy dissipation can change significantly. The wave mechanical energy E^w is given by the sum of the kinetic ($\mathcal{K}$) and potential ($\mathcal{U}$) contribution:

$$E^w(t) = \mathcal{K}^w(t) + \mathcal{U}^w(t) = \frac{1}{2}\int_0^\lambda \int_{-h}^\eta \left(\rho_w|\mathbf{u}|^2 + \rho_w g z\right) dz dx - \bar{\mathcal{U}^w} \tag{8}$$

with $\bar{\mathcal{U}^w}$ the potential energy of the still water level. Similarly, the resonator energy E^r can be computed as

$$E^r(t) = \frac{1}{2}\left[mU^2 + \kappa(X - X_0)^2\right] \tag{9}$$

with U the velocity of the center of mass of the resonator.

In [4], the dissipation was estimated by fitting the time evolution of the wave mechanical energy with an exponential expression, similar to the linear theory decay $E^w(t) = E^w(t = 0)e^{-4\nu k^2 t}$. Here, instead, we directly compute the dissipation rate by viscous stresses as

$$\mathcal{P}^w(t) = \int_0^\lambda \int_{-h}^\eta 2\mu \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] dxdz \tag{10}$$

at every timestep of the simulation. The amount of energy dissipated at time t is then computed as $\mathcal{D}^w(t) = \int_0^t \mathcal{P}^w dt$.

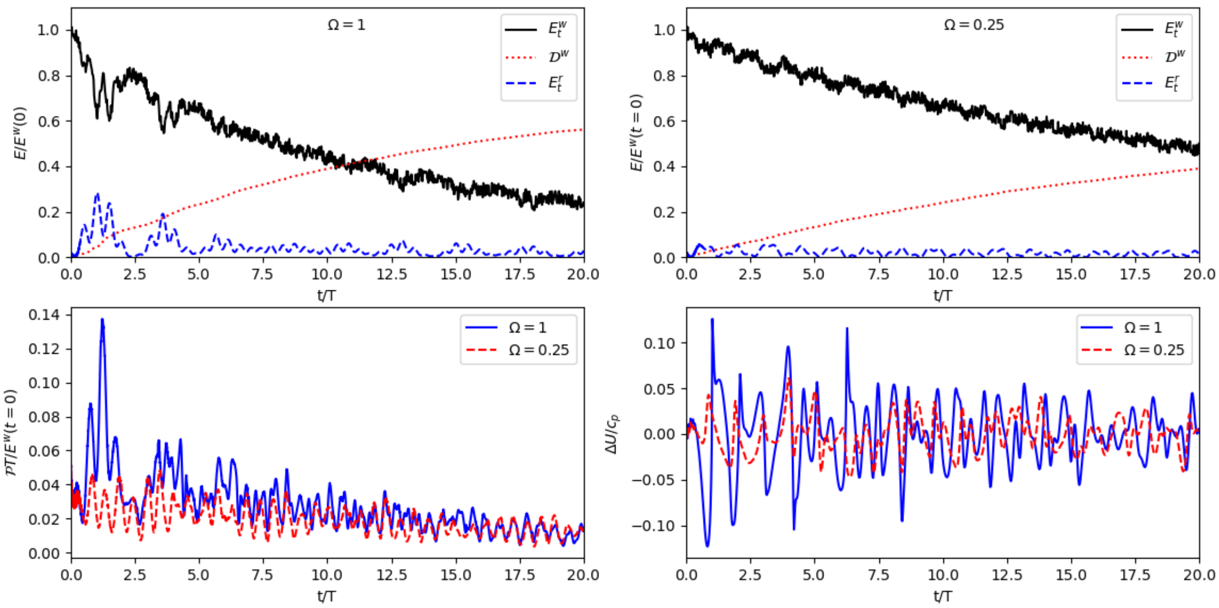

Figure 1: Time history of wave and resonator mechanical energy and dissipation for the case $\Omega = 1$ (top left panel) and $\Omega = 0.25$ (top right panel); instantaneous dissipation rate for the two cases (bottom left panel); instantaneous velocity difference between the wave and the resonator (bottom right panel).

The time history of the wave and resonator mechanical energy are shown in fig.1 (top panels) alongside the dissipated energy. Energy is normalized with the initial wave mechanical energy $E^w(t = 0)$ and time is made non-dimensional using the wave period $T = 2\pi/\omega_w$. For the case $\Omega = 1$, close to resonance conditions, the energy absorbed by the resonator reaches a maximum of about 30% of the initial wave mechanical energy; instead, when $\Omega = 0.25$ the maximum energy of the resonator is about 10%. This energy then is partially returned to the wave and partially dissipated by viscous stresses. Looking at the instantaneous dissipation rate (fig.1 bottom left panel), it can be seen that in resonance conditions not only the energy transfer, but also the dissipation rate increases. This implies that the increased absorbed energy by the resonator leads to an increase in viscous dissipation. Since this term is proportional to the velocity gradient (see eq.(10)) we compute the difference between the resonator velocity U and the interface velocity at the same horizontal coordinate $\Delta U(t) = u(x = X, z = \eta, t) - U(t)$. Results, reported in the bottom right panel of fig. 1, clearly show that in the case $\Omega = 1$ the velocity difference can reach values about 4 times greather than for $\Omega = 0.25$ and exhibits a maximum about the maximum dissipation rate (compare with the bottom left panel). The differences in the velocity field are highlighted in figure 2 where the color map of the horizontal velocity is reported, after one wave period. The overall mechanism can be explained as follows: when the frequency of the wave and the resonator are comparable, the elastic force acts on a timescale similar to the hydrodynamic forcing on the resonator and its motion does not follow the wave. On the other hand, when the timescale of the elastic force is larger, the resonator follows the wave motion more easily, with a reduced dissipation.

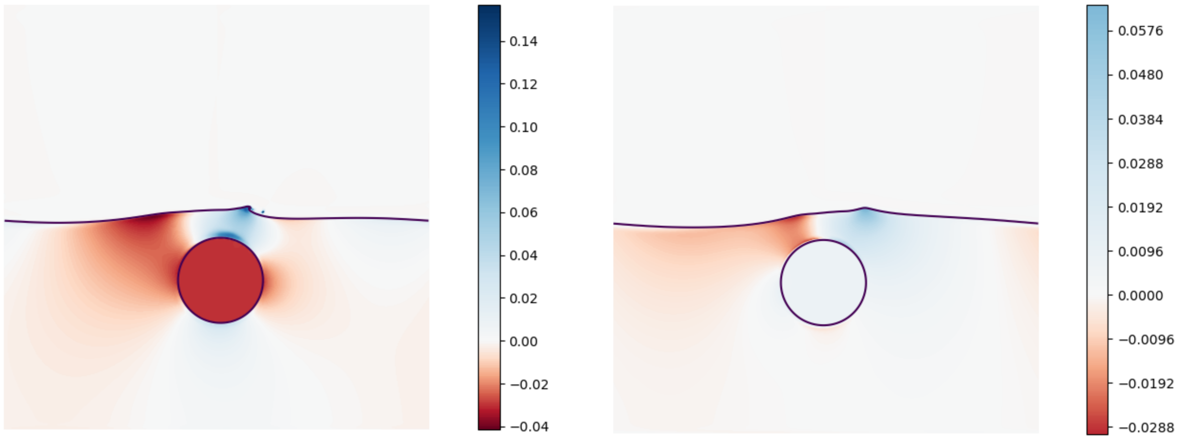

Figure 2: Horizontal velocity colormap after one wave period: $\Omega = 1$ (left panel), $\Omega = 0.25$ (right panel). Velocity is made non-dimensional using the phase velocity.

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