The 37th International Workshop on Water Waves and Floating Bodies, April 10–13, 2022, Giardini Naxos, Italy.

Waves generated by moving disturbances on the seafloor

Yongbo Chen[a], Masoud Hayatdavoodi[a,b], BinBin Zhao[a] and R. Cengiz Ertekin[a,c]

a. College of Shipbuilding Engineering, Harbin Engineering University, Harbin, China
b. School of Science and Engineering, University of Dundee, Dundee DD1 4HN, UK
c. Department of Ocean & Resources Engineering, University of Hawaii, HI 96822, USA

Email: mhayatdavoodi@dundee.ac.uk

Introduction

Moving disturbances on the seafloor have been used by e.g., Tuck & Hwang (1972) to generate long waves to study tsunami run-up. Bottom moving disturbances can also be used as wave-makers to generate long waves in laboratory e.g., by Jamin et al. (2015). In principle, waves may be generated by a bottom disturbance, moving or oscillating in the vertical or horizontal direction. The bottom disturbance may be a continuous surface or discrete (in the case of a piston located on the seafloor). In this study, waves generated by moving disturbances on the sea floor are studied by use of the Level I Green-Naghdi (GN) equations. Attention is confined to waves generated by continuous bottom surface which is allowed to move in either vertical or horizontal direction. Results are compared with existing laboratory experiments and available numerical data. The model is then used to study waves generated by oscillatory bottom disturbances.

The level I Green-Naghdi Equations

Green et al. (1974) showed that the exact three-dimensional equations of motion of an inviscid and incompressible fluid (Euler's equations) can be simplified by making a single assumption about the distribution of the vertical velocity. The resultant equations are known as the GN equations, and they satisfy the nonlinear boundary conditions and the conservation of mass exactly, and postulate the integrated form of the conservation of momentum and energy laws. The GN equations are classified based on the assumption made about the distribution of the vertical velocity over the fluid column. In the Level I GN equations, used in this study, the vertical velocity varies linearly from the seafloor to the free surface. Consequently, (due to the continuity equation) the horizontal velocity is invariant over the water column in the Level I GN equations. These conditions are mostly applicable to the unsteady and nonlinear fluid motions in shallow waters, the subject of this study.

A rectangular Cartesian coordinate system (x_1, x_2) is used, where the origin is at the still-water level (SWL), with x_1 pointing to the right, and x_2 pointing upward against gravity. Ertekin (1984) provided a compact form of the Level I GN equations. In two dimensions the equations are given by

$$\eta_{,t} + \{(h+\eta-\alpha)u_1\}_{,x_1} = \alpha_{,t}, \tag{1}$$

$$\dot{u}_1 + g\eta_{,x_1} + \frac{\hat{p}_{,x_1}}{\rho} = -\frac{1}{6}\{[2\eta+\alpha]_{,x_1}\ddot{\alpha} + [4\eta-\alpha]_{,x_1}\ddot{\eta} + (h+\eta-\alpha)[\ddot{\alpha}+2\ddot{\eta}]_{,x_1}\}, \tag{2}$$

$$u_2(x_1,x_2,t) = \dot{\alpha} + \frac{x_2+h-\alpha}{h+\eta-\alpha}(\dot{\eta}-\dot{\alpha}), \tag{3}$$

$$\bar{p}(x_1,t) = \frac{\rho}{2}(h+\eta-\alpha)(\ddot{\alpha}+\ddot{\eta}+2g) + \hat{p}, \tag{4}$$

where $\alpha(x_1,t)$ is the bottom profile of the fluid sheet, $\eta(x_1,t)$ is the free surface, measured from the SWL, h is the water depth, g is the gravitational acceleration and ρ is the fluid mass density. The subscripts after commas denote partial derivatives with respect to the variables, while the superposed dots mean first and second material derivatives. $\bar{p}$ is the pressure at the bottom profile and $\hat{p}$ is the pressure on the free surface and $\hat{p} = 0$ in this study without loss in generality. All the variables in this manuscript are dimensionless using ρ, g, and h as a dimensionally independent set, hence

$$x_1^{'} = \frac{x_1}{h}, \quad \eta^{'} = \frac{\eta}{h}, \quad \alpha^{'} = \frac{\alpha}{h}, \quad u_1^{'} = \frac{u_1}{\sqrt{gh}}, \quad t^{'} = t\sqrt{\frac{g}{h}}. \tag{5}$$

For simplicity, the superscript ($'$) are removed from all variables in the foregoing.

Continuous bottom disturbance

The system of equations (1) to (4) is solved to determine the impact of moving bottom disturbances on fluid. In this study, attention is confined to cases where the bottom disturbance is described by a continuous function, i.e. $\alpha(x_1,t)$ is continuous at all times. Cases where the bottom is discontinuous shall be considered in the future. The solution approach of the problem is described below.

At any given time, Eqs. (1) and (2) are solved for two unknowns, η and u_1, with $\alpha(x_1,t)$ given as a known function. For a given $\alpha(x_1,t)$, Eq. (1) contains only time derivatives of function η. Hence, at a given time step, surface elevation is determined explicitly by use of Eq. (1). The first and second material derivatives of η are given by $\dot{\eta} = \eta_{,t} + u_1\eta_{,x_1}$ and $\ddot{\eta} = \dot{\eta}_{,t} + u_1\dot{\eta}_{,x_1}$. Substituting $\eta_{,t}$ and $\eta_{,tt}$ (obtained from Eq. (1)) into $\ddot{\eta}$, gives

$$\ddot{\eta} = \ddot{\alpha} + (h+\eta-\alpha)(u_{1,x_1}^2 - u_{1,x_1t} - u_1u_{1,x_1x_1}). \tag{6}$$

Substituting Eq. (6) and material derivatives, $\dot{\alpha}$, $\ddot{\alpha}$ and $\ddot{\alpha}_{,x_1}$ into Eq. (2), gives

$$\begin{aligned}
&u_{1,t} + u_1u_{1,x_1} + g\eta_{,x} = \\
&-\eta_{,x_1}[\alpha_{,tt} + u_{1,t}\alpha_{,x_1} + 2u_1\alpha_{,x_1t} + u_1u_{1,x_1}\alpha_{,x_1} + u_1^2\alpha_{,x_1x_1}] \\
&-\frac{1}{2}(h+\eta-\alpha)[\alpha_{,x_1tt} + u_{1,x_1t}\alpha_{,x_1} + u_{1,t}\alpha_{,x_1x_1} + 2u_{1,x_1}\alpha_{,x_1t} \\
&+ 2u_1\alpha_{,x_1x_1t} + u_{1,x_1}^2\alpha_{,x_1} + u_1u_{1,x_1x_1}\alpha_{,x_1} + 3u_1u_{,x_1}\alpha_{,x_1x_1} + u_1^2\alpha_{,x_1x_1x_1}] \\
&-\frac{1}{2}(h+\eta-\alpha)(u_{1,x_1}^2 - u_{1,x_1t} - u_1u_{1,x_1x_1})[(2\eta-\alpha)_{,x_1}] \\
&-\frac{1}{3}(h+\eta-\alpha)^2(u_{1,x_1}u_{1,x_1x_1} - u_{1,x_1x_1t} - u_1u_{1,x_1x_1x_1}).
\end{aligned} \tag{7}$$

Equation (7) is similar to that given by Ertekin (1984) for a flat and stationary seafloor, involving the following additional terms due to the moving bottom disturbance: $\alpha_{,x_1tt}$, $2u_{1,x_1}\alpha_{,x_1t}$, $2u_1\alpha_{,x_1x_1t}$, $\alpha_{,tt}$ and $2u_1\alpha_{,x_1t}$. Equations (1) and (7), η and u_1 are solved numerically by use of the finite different method. Equation (7) forms a tridiagonal matrix of u_1, which is solved by use of a Gaussian Elimination method. Then u_2 and $\bar{p}$ are obtained explicitly by Eqs. (3) and (4). Discussion on the applicability of high level GN equations to generation of long waves by a moving bottom disturbance can be found in Zhao et al. (2011).

Results and Discussion

Results include comparisons with available data, followed by results of the GN model for waves generated by a moving bottom. A sketch of an arbitrary-shaped bottom disturbance on an otherwise flat bottom is shown in Fig. 1.

In a study by Tinti & Bortolucci (2000), waves generated by a disturbance moving horizontally on the seafloor are investigated by use of shallow water wave approximation method. The bottom disturbance is given by

$$\alpha(x_1,t) = \begin{cases} \frac{H_s}{2}[1 - cos(\frac{2\pi}{L_s}(x_1 - x_s))], & x_s \le x_1 \le x_e, \\ 0, & x_1 < x_s, x_1 > x_e, \end{cases} \tag{8}$$

where $L_s = 10$ and $H_s = 0.01$. The bottom disturbance moves along x_1 with constant velocity V_s. Shown in Fig. 2, the snapshots of wave profiles calculated by the GN equations are compared with those of Tinti & Bortolucci (2000). Figure 2 shows that two waves are generated: one wave with distinguishable peak and trough propagates in the same direction as the bottom disturbance while the other wave with almost only one trough propagates in the opposite direction. Overall, very good agreement is observed between the results.

Next, the laboratory experiments conducted by Hammack (1973), in which the bottom disturbance oscillates vertically, is considered. The bottom disturbance on the seafloor is given by

$$\alpha(x_1,t) = H_s(t) * H((\frac{L_s}{2})^2 - (x_1 - x_0)^2), \quad x_1 \ge 0, \tag{9}$$

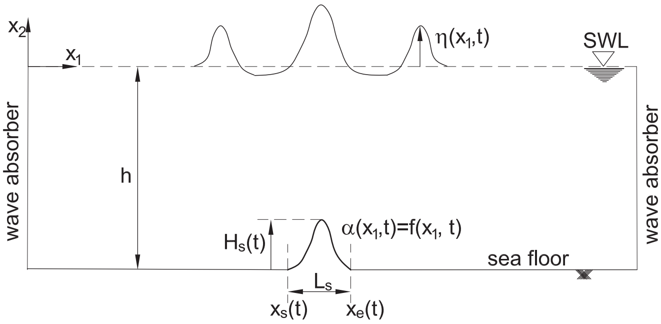

Figure 1: A sketch of the numerical wave tank with an arbitrary-shaped bottom disturbance, defined by a continuous function $f(x_1, t)$. x_s and x_e are the starting and ending position of the bottom disturbance, and $L_s = x_e - x_s$ is the disturbance length.

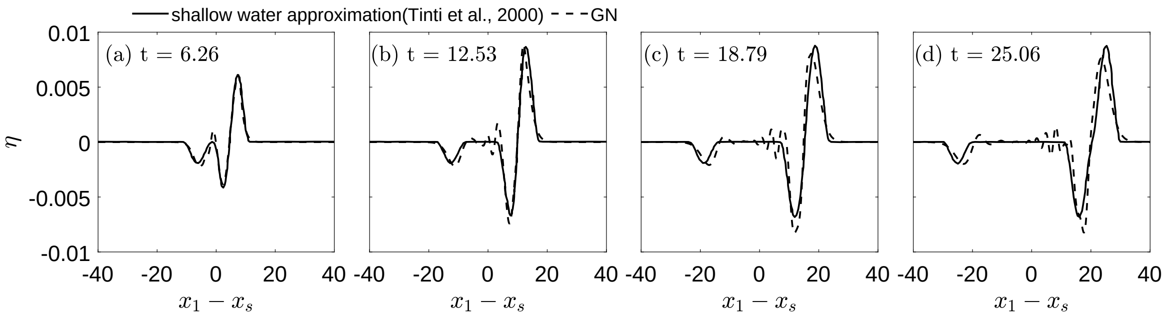

Figure 2: Snapshots of surface elevation at (a) $t = 6.26$, (b) $t = 12.53$, (c) $t = 18.79$ and (d) $t = 25.06$, obtained by the GN model and shallow water approximation of Tinti & Bortolucci (2000). The bottom disturbance moves with constant speed of $V_s = 0.639$ and stops at $t = 26.06$.

where $L_s = 24.4$, x_0 is the coordinate of the middle point of the moving disturbance, $x_0 = \frac{x_s+x_e}{2}$ and $H()$ is the unit step function (whose value is zero for negative arguments and one for positive arguments). $H_s(t)$ is given by

$$H_s = H_0(1 - e^{-qt}), \quad t \geq 0, \tag{10}$$

where H_0 is the oscillating amplitude of the bottom disturbance and q is constant. For the purpose of this comparison, we make sure that the bottom surface is continuous (i.e., the edges of the bottom disturbance are made smooth).

Time series of water surface elevation downstream of the obstacle are shown in Fig. 3, where results of the GN model are compared with the laboratory measurements of Hammack (1973) and the Boussinesq model of Fuhrman & Madsen (2009). At $x_1 = x_e$ and $x_1 = x_e + 20$, GN results show better agreement with the laboratory experiments. Both GN and Boussinesq models (Fuhrman & Madsen (2009)) overestimate the surface elevation at $x_1 = x_e + 180$ and $x_1 = x_e + 400$. Overall, there is close agreement between results of the two models and the laboratory measurements.

In this part, we study waves generated by a continuous moving bottom disturbance whose shape is described by the following hyperbolic function:

$$\alpha(x_1, t) = H_s(t) * R(x_1) * sech^2(x_1 - x_0). \tag{11}$$

The bottom disturbance oscillates vertically, defined by $H_s(t) = H_0 sin(\omega t)$ where ω is the oscillation frequency of the bottom surface. The ramp function is given as $R(x_1) = e^{\frac{-(x_1-x_0)^2}{\sigma^2}}$, where σ is a constant, controlling the effective width of $R(x_1)$.

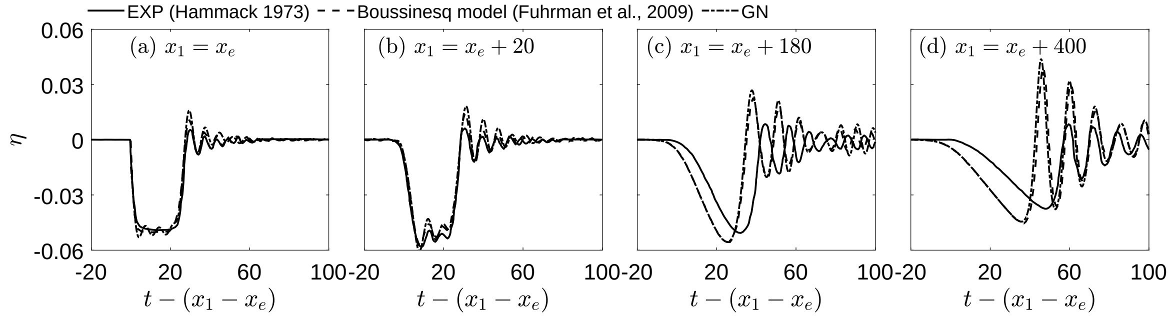

Figure 3: Time series of wave surface elevation recorded at (a) $x_1 = x_e$, (b) $x_1 = x_e + 20$, (c) $x_1 = x_e + 180$ and (d) $x_1 = x_e + 400$, downstream of the disturbance. $H_0 = -0.1$ and $q = 0.978$.

Figure 4 shows snapshots of wave surface elevation and bottom disturbances with $H_0 = 0.05$, 0.1 and 0.15. Wave profiles are very close to each other at $x_1 \geq x_0 + \frac{L_s}{2}$ with $H_0 = 0.1$ and 0.15. This can also be seen in Fig. 5 where time series of the surface elevation are recorded at four gauges.

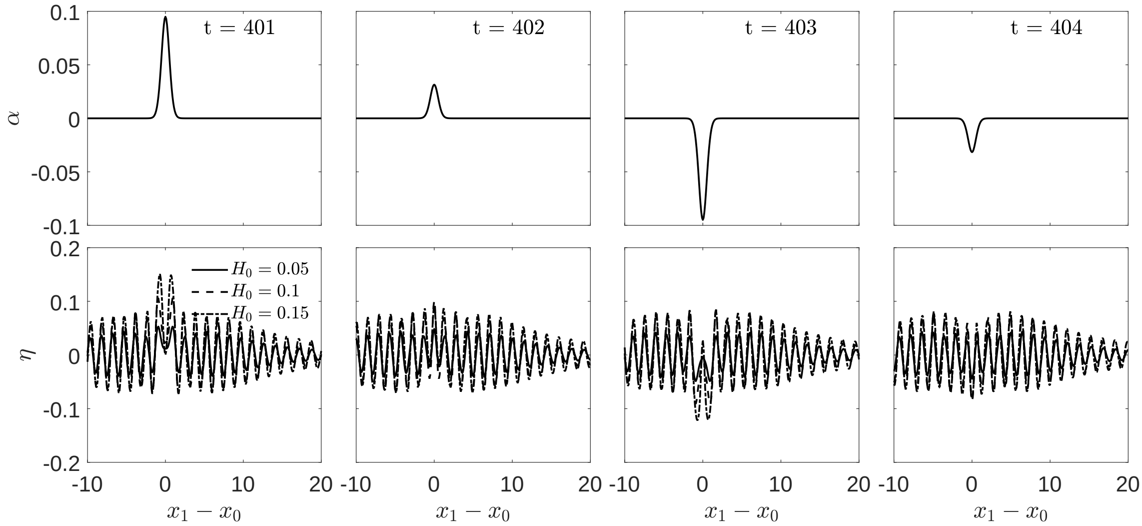

Figure 4: Snapshots of waves generated by the oscillating bottom disturbance with $H_0 = 0.05$, 0.1 and 0.15, $L_s = 4$, $\omega = 1.57$ and $\sigma = 1$.

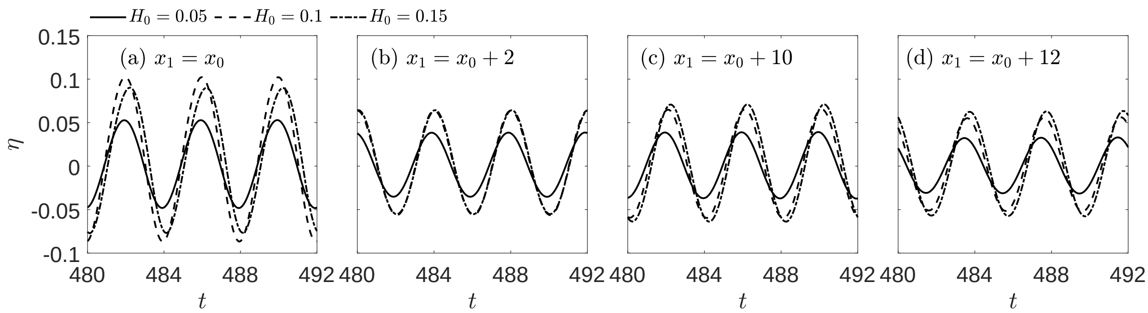

Figure 5: Time series of the wave surface elevation generated by the oscillating bottom disturbance with $H_0 = 0.05$, 0.1 and 0.15, $L_s = 4$, $\omega = 1.57$ and $\sigma = 1$.

Shown in Fig. 5, the oscillatory waves generated by a vertically oscillating disturbance have the same frequency as the moving disturbance and propagate with no change in their frequency. The wave height seems to increase as the oscillation height of the bottom disturbance increases, but the relationship appears to be nonlinear. Overall, the GN model developed in this study shows promising accuracy to study nonlinear waves generated by bottom disturbances.

References

Ertekin, R. C. (1984), *Soliton Generation by moving disturbances in shallow water: theory, computation and experiment*, Ph.D. Dissertation, University of California, Berkeley.

Fuhrman, D. R. & Madsen, P. A. (2009), 'Tsunami generation, propagation, and run-up with a high-order Boussinesq model', *Coastal Engineering* **56**(7), 747–758.

Green, A. E., Laws, N. & Naghdi, P. (1974), 'On the theory of water waves', *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences* **338**(1612), 43–55.

Hammack, J. L. (1973), 'A note on tsunamis: their generation and propagation in an ocean of uniform depth', *Journal of Fluid Mechanics* **60**(4), 769–799.

Jamin, T., Gordillo, L. et al. (2015), 'Experiments on generation of surface waves by an underwater moving bottom', *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences* **471**(2178), 20150069.

Tinti, S. & Bortolucci, E. (2000), 'Energy of water waves induced by submarine landslides', *Pure and Applied Geophysics* **157**(3), 281–318.

Tuck, E. O. & Hwang, L.-S. (1972), 'Long wave generation on a sloping beach', *Journal of Fluid Mechanics* **51**(3), 449–461.

Zhao, B. B., Duan, W. Y. & Webster, W. C. (2011), 'Tsunami simulation with Green–Naghdi theory', *Ocean Engineering* **38**(2), 389–396.