On head-on collision of the depression internal solitary wave and the elevation internal solitary wave

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Highlights

- Head-on collision of the depression internal solitary wave (internal-wave mode) and the elevation internal solitary wave (surface-wave mode) is studied by use of the two-layer high-level Green-Naghdi model.
- During the head-on collision, two wave crests and one wave trough are observed at the interface.
- After the head-on collision, no following wave trains are found, and the two internal solitary-wave profiles come back to their initial wave profiles.

1 Introduction

Internal solitary waves have been observed frequently in the density stratified lakes, coastal areas and continental shelf regions. For experimental and numerical studies on internal solitary waves, the two-layer fluid system is often considered in order to simplify the problems, see e.g. Grue et al. (1999), Choi and Camassa (1999) and Zhao et al. (2020). Prof. Kashiwagi studied a various of problems of the floating bodies in a two-layer fluid system, such as the radiation and diffraction problems, wave-induced motions and wave drift forces, see Ten and Kashiwagi (2004), Kashiwagi et al. (2006) and Kashiwagi (2007).

Collisions between internal solitary waves in a two-layer fluid system attract many attentions. Jo and Choi (2008) studied the head-on collision of two large-amplitude internal solitary waves numerically based on the MCC model. Hsu et al. (2013) conducted a series of laboratory experiments on head-on collision of two internal solitary waves. Zou et al. (2020) studied two internal solitary-wave collisions by use of the boundary element method. Choi et al. (2020) studied the rear-end collisions of two depression internal solitary waves by using a high-order unidirectional model and the MCC model.

In the density stratified oceans, surface solitary waves, such as tsunamis, may induce the elevation internal solitary waves. At the same time, if depression internal solitary waves exist, interaction between the depression internal solitary waves and the elevation internal solitary waves may happen. Thus, we are motivated to study head-on collision of the depression internal wave (internal-wave mode) and the elevation internal solitary wave (surface-wave mode).

This paper is organized as follows. The two-layer HLGN-FS model is described in Section 2. Numerical results of head-on collision of the depression internal wave (internal-wave mode) and the elevation internal solitary wave (surface-wave mode) are presented and discussed in Section 3. Conclusions are reached in Section 4.

2 Two-layer HLGN-FS model

Sketch of the two-layer fluid system is shown in Fig. 1. The two fluids are both incompressible, immiscible and inviscid. The coordinate axis is established at the undisturbed internal-wave interface. The mass densities of the upper and lower layers are ρ_2 and ρ_1. The undisturbed depths of the upper and lower layers are h_2 and h_1. The free surface, interface and bottom are expressed by $z=\eta_2(x,t)$, $z=\eta_1(x,t)$ and $z=-h_1$, respectively.

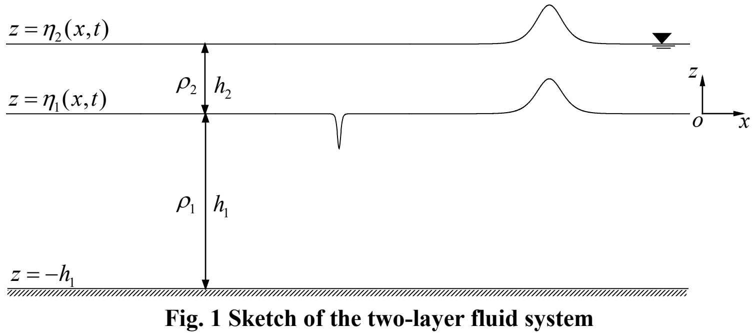


Fig. 1 Sketch of the two-layer fluid system

For each layer of fluid, the conservation equations of mass and momentum can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{2a}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}-g, \tag{2b}$$

where u and w are the horizontal and vertical velocity components, respectively, p is pressure and g is the gravitational acceleration.

The kinematic boundary conditions are written as

$$w^{\mathrm{U}}=\frac{\partial \eta_2}{\partial t}+u^{\mathrm{U}}\frac{\partial \eta_2}{\partial x},\quad z=\eta_2(x,t) \tag{3a}$$

$$w^{\mathrm{U}}=\frac{\partial \eta_1}{\partial t}+u^{\mathrm{U}}\frac{\partial \eta_1}{\partial x},\quad z=\eta_1(x,t) \tag{3b}$$

$$w^{\mathrm{L}}=\frac{\partial \eta_1}{\partial t}+u^{\mathrm{L}}\frac{\partial \eta_1}{\partial x},\quad z=\eta_1(x,t) \tag{3c}$$

$$w^{\mathrm{L}}=0,\qquad z=-h_1 \tag{3d}$$

where the upper corner U and L represent the variables related to the upper and lower layer respectively.

The dynamic boundary conditions are written as

$$\hat{p}^{\mathrm{U}}=p\big|_{z=\eta_2(x,t)}=0, \tag{4a}$$

$$\overline{p}^{\mathrm{U}}=\hat{p}^{\mathrm{L}}=p\big|_{z=\eta_1(x,t)}. \tag{4b}$$

We use the two-layer HLGN-FS model to solve this physical problem, where the horizontal and vertical velocities are expressed in polynomial forms for each fluid layer as

$$u^{\mathrm{U}}(x,z,t)=\sum_{n=0}^{K^{\mathrm{U}}-1}u_n^{\mathrm{U}}(x,t)z^n,\qquad w^{\mathrm{U}}(x,z,t)=\sum_{n=0}^{K^{\mathrm{U}}}w_n^{\mathrm{U}}(x,t)z^n, \tag{5a}$$

$$u^{\mathrm{L}}(x,z,t)=\sum_{n=0}^{K^{\mathrm{L}}-1}u_n^{\mathrm{L}}(x,t)z^n,\qquad w^{\mathrm{L}}(x,z,t)=\sum_{n=0}^{K^{\mathrm{L}}}w_n^{\mathrm{L}}(x,t)z^n, \tag{5b}$$

where K^{U} and K^{L} are the levels of the HLGN model applied to the upper and lower fluids respectively. The HLGN-FS equations for the two-layer fluid system are obtained by substituting Eq. (5) into Eqs. (1)-(4). The detailed derivation process and the numerical algorithm of the two-layer HLGN-FS model are shown in Zhao et al. (2020).

3 Numerical results

Here, we use the two-layer HLGN-FS model to study head-on collision of the depression internal wave and the elevation internal solitary wave. The parameters are as follows. Mass densities of the upper and lower layers are $\rho_2=1000\mathrm{kg/m^3}$ and $\rho_1=1001\mathrm{kg/m^3}$. Undisturbed depths of the upper and lower layers are $h_2=100\mathrm{m}$ and $h_1=900\mathrm{m}$. Amplitude of the depression internal solitary wave is $a_1=50\mathrm{m}$. The elevation internal solitary wave with the amplitude of $a_2=50\mathrm{m}$ is induced by the surface solitary wave with amplitude $a_3=55.8\mathrm{m}$ in this case. Speeds of the depression and elevation internal solitary waves are $c_1=1.1\mathrm{m/s}$ and $c_2=101.8\mathrm{m/s}$. Initially, crest of the depression internal solitary wave that propagating from left to right is located at $x=100000\mathrm{m}$, and crest of the elevation internal solitary wave that propagating from right to left is located at $x=160000\mathrm{m}$. Sketch of the physical problem is shown in Fig. 2.

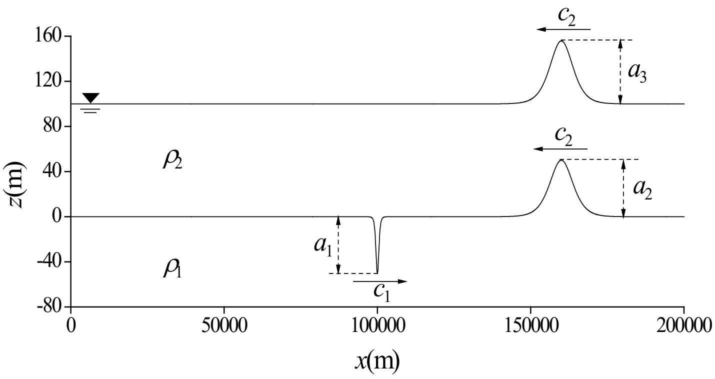
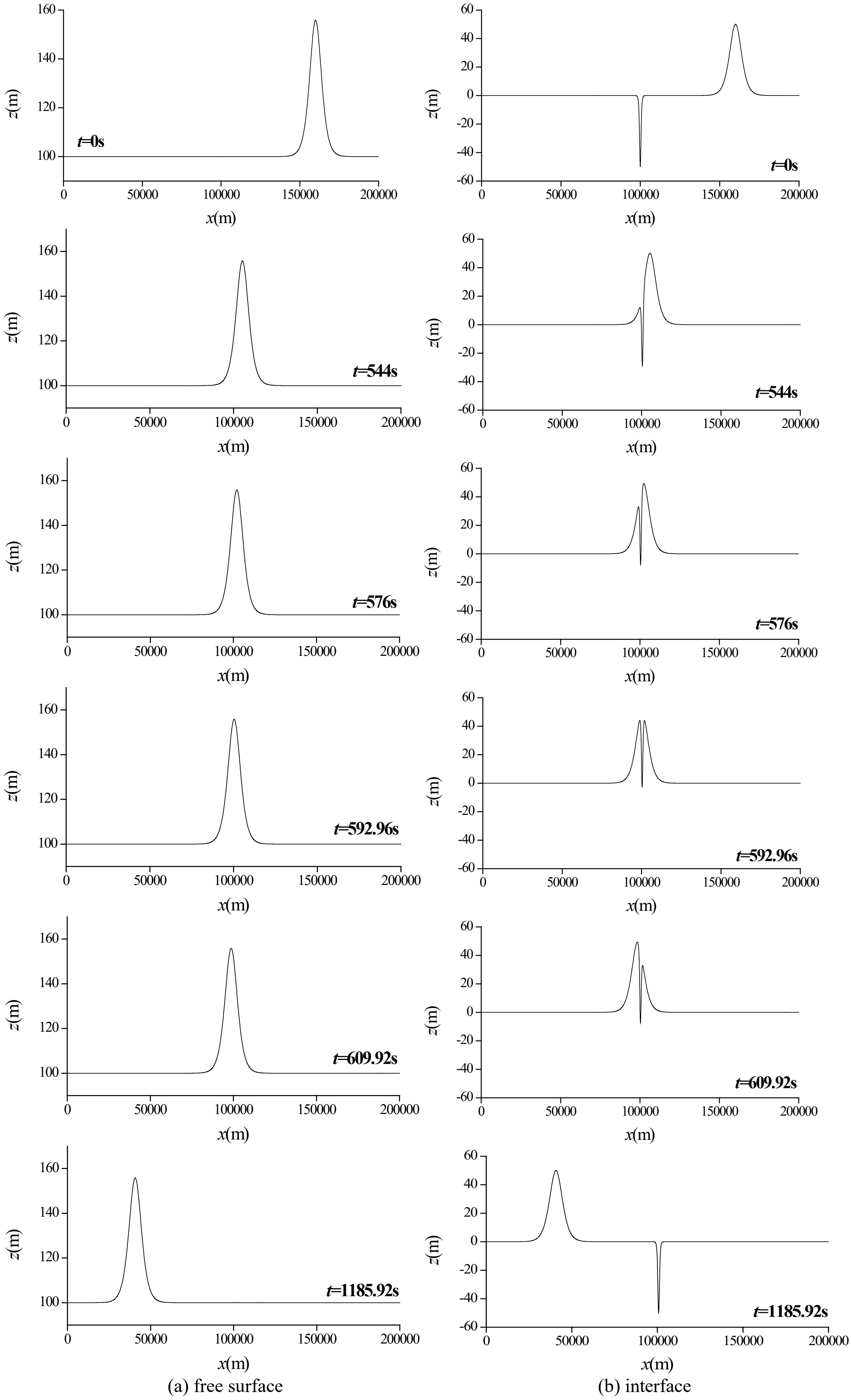


Fig. 2 Sketch of the head-on collision of the depression internal wave and the elevation internal solitary wave (scale of the lower-layer depth is not presented exactly in this figure)

Snapshots of the surface and interface elevation at different moments are shown in Fig. 3.



(a) free surface (b) interface

Fig. 3 Snapshots of the wave profiles at different times (the free surface on the left and the internal interface on the right)

For the free surface elevation shown in Fig. 3(a), we observe that the surface elevation at different moments do not change. On the free surface, amplitude of the surface wave that induced by the depression internal solitary wave (internal-wave mode) is 0.03m, which is much smaller than amplitude of another surface solitary wave (surface-wave mode). Thus, interaction between the two surface waves is very weak.

For the interface elevation shown in Fig. 3(b), at $t = 0\text{s}$, distance between the two internal solitary-wave crests is 60000m. We also observe that the elevation internal solitary-wave profile is much wider than the depression internal solitary-wave profile. During the head-on collision, see e.g. at $t = 544\text{s}$ and $t = 576\text{s}$, two wave crests and one wave trough are found. At $t = 592.96\text{s}$, the interface elevation shows very good symmetry. The two obvious wave crests reach 44.0m and the wave trough reaches 0m. After $t = 592.96\text{s}$, the two internal solitary waves begin to separate with each other. At $t = 1185.92\text{s}$, no following wave trains are found between the two internal solitary waves. Profiles of the depression internal solitary wave and the elevation internal solitary wave come back to their initial profiles.

4 Conclusions

In this paper, head-on collision of the depression internal solitary wave (internal-wave mode) and the elevation internal solitary wave (surface-wave mode) with same amplitude is studied by use of the two-layer HLGN-FS model. During the head-on collision, two wave crests and one wave trough are observed. The interface elevation in good symmetry state is captured. After the head-on collision, no following wave trains are found and the two internal solitary- wave profiles come back to their initial wave profiles respectively.

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