Effects of water depth and nonlinearity on the wavefront

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Highlights:

- . The weakly nonlinear wavefront results of HLIGN equations are compared with linear analytical solutions and good agreement is achieved.
- . The strongly nonlinear wavefront results of HLIGN equations in different water depths are presented, the water depth effects and the nonlinearity effects on the water wavefront are investigated.

1 Introduction

The research on regular water waves is mainly concentrated in the region of periodic variation. Dai & He (1993) give the transient solutions of plane progressive linear waves. Taylor et al. (1994) investigate the linear analytical solutions of the transient waves generated by wave maker. Chen et al. (2018, 2019) study the exact behavior of wavefront in infinite water depth and define the wavefront function. They further formulate the amplitude of the largest wave amplitude behind the wavefront. The nonlinearity effects on the wavefront are studied by the High Level Irrotational Green-Naghdi (HLIGN) equations in infinite water depth (Zhang et al. 2020).

The main motivation of this work is to investigate the water depth effects and the nonlinearity effects on the water wavefront in finite water depth. As a comparison, the linear analytical solutions of the transient waves in finite water depth are provided.

The theoretic framework of HLIGN equations for finite water depth is briefly described in Section 2. The linear analytical solutions of the transient waves are introduced in Section 3. The numerical cases of the transient regular waves are presented in Section 4 to reveal the effects of water depth and nonlinearity. Summaries and conclusions of this study are provided in Section 5.

2 The HLIGN equations for finite water depth

The fluid is assumed to be incompressible and inviscid and the flow is irrotational. The HLIGN equations satisfy the mass conservation equation and the momentum conservation equation. In two dimensions, the velocity field (u, w) can be expressed by the stream function $\psi(x, z; t)$ as follows (Kim et al., 2001; Zhao et al., 2015),

$$u(x,z,t)=\frac{\partial\psi(x,z;t)}{\partial z},\quad w(x,z,t)=-\frac{\partial\psi(x,z;t)}{\partial x},\tag{1}$$

$$\psi(x,z;t)=\sum_{n=1}^{K}\psi_n(x;t)f_n(\gamma),\tag{2}$$

where K and ψ_n are the level of the HLIGN equations and stream function coefficients, respectively. A higher level wave model usually possesses a stronger nonlinearity and better accuracy (Duan et al., 2017). The $f_n(\gamma)$ is named as the shape function which is a function of z describing the vertical structure of the velocity field. In finite water depth, the shape function is a series of polynomials,

$$f_n(\gamma)=\gamma^{2n-1},\quad \gamma=(z+d)/(\eta+d),\tag{3}$$

where η and d are the surface elevation and the water depth, respectively. Since the fully nonlinear free surface boundary condition is satisfied, the shape function is the only assumption of HLIGN equations. More details on the HLIGN can be referred to Zhao et al. (2015).

3 The linear analytical solutions of transient waves in finite water depth

Within the framework of potential flow theory, the transient solutions of plane progressive waves are provided in Dai & He (1993). Supposing a flexible plate at x=0 to horizontally oscillate in the following form,

$$\left.\frac{\partial \Phi}{\partial x}\right|_{x=0}=\frac{Agk_0}{\omega}\frac{\cosh\left[k_0(z+d)\right]}{\cosh(k_0 d)}\sin(\omega t)\cdot \tag{4}$$

The corresponding linear transient wave elevation on the free surface is analytically expressed as:

$$\frac{\eta(x,t)}{A}=\frac{2k_0}{\pi}\int_0^{\infty}\cos(kx)\frac{\cos(\omega t)-\cos(\beta t)}{k^2-k_0{}^2}dk\,, \tag{5}$$

where $\beta=\sqrt{gk\tanh(kd)}$, and A, ω and k_0 represent the wave amplitude, wave frequency and wave number of the steady-state regular wave.

4 Numerical simulations

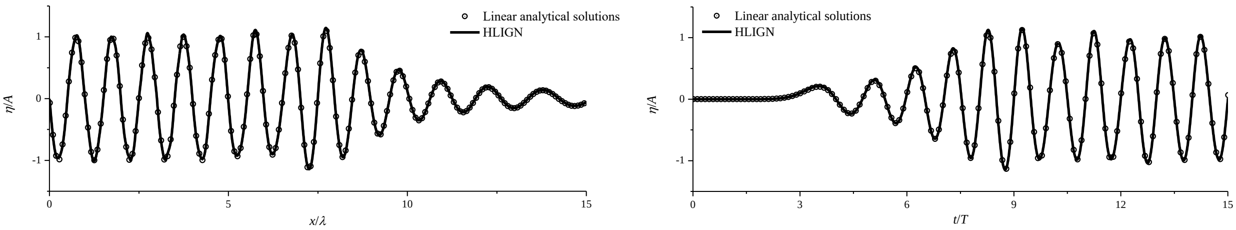
4.1 Weakly nonlinear wavefront in finite water depth

In this section, the weakly nonlinear wavefront in finite water depth is simulated by HLIGN equations and compared with the linear analytical solutions derived in finite water depth. The parameters are listed in Tab. 1, where T, λ and H(H=2A) represent the wave period, the wave length and the wave height of the steady-state regular waves.

Tab. 1 Wave parameters

A (m)	T (s)	d/λ	H/λ
0.02	1.64	0.25	0.01

The convergence study, including the grid size, the time step and the levels, is performed in each simulation. The converged HLIGN results are compared with the linear analytical solutions and displayed in Fig. 1. The snapshot at t/T=15 and the time history at x/λ=4 are given with dimensionless forms.



(a) Snapshot at t/T=15 (b) Time history at x/λ=4

Fig. 1 The comparison between the HLIGN results and the linear analytical solutions (*d/λ*=0.25; *H/λ*=0.01)

It can be observed that the HLIGN results agree well with linear analytical solutions in finite water depth for the regular waves with the small wave steepness (H/λ=0.01). Thus, the HLIGN equations for finite wave depth can be utilized to investigate the effects of water depth and nonlinearity in the following subsection.

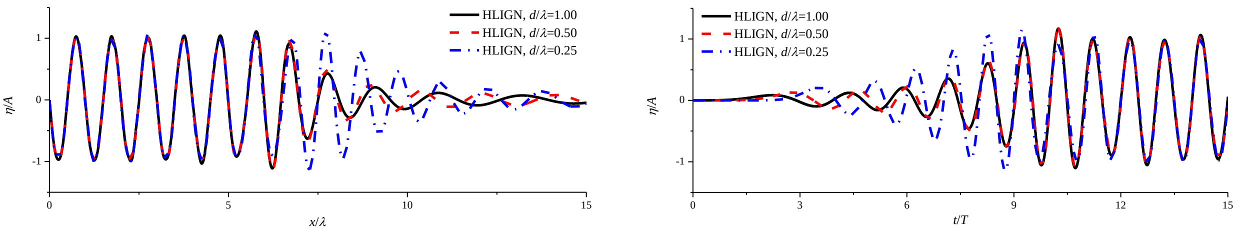
4.2 The effects of water depth and nonlinearity

Keeping the wave period consistent with the above case, the effects of water depth and nonlinearity are investigated by setting the water depth as d/λ=1.00, 0.50, 0.25 and the wave steepness as H/λ=0.01, 0.03, 0.05. The wave parameters are listed in Tab.2.

Tab. 2 Wave parameters

A (m)	T (s)	d/λ	H/λ
0.02	1.64	1.00	0.01
		0.50	
		0.25	
0.06		1.00	0.03
		0.50	
		0.25	
0.10		1.00	0.05
		0.50	
		0.25	

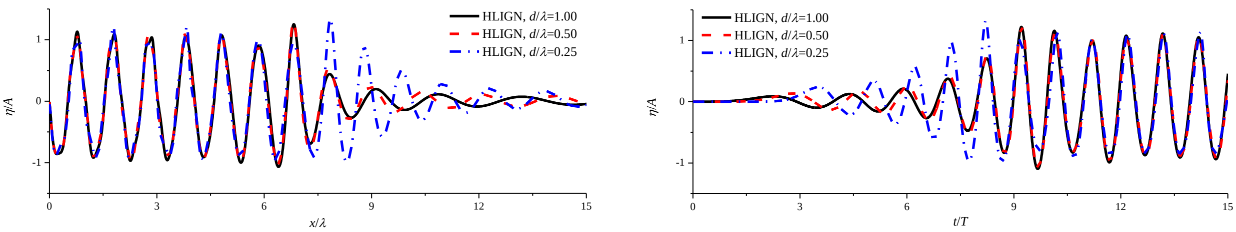
The water depth effects on the transient water waves are studied in Figs. 2 and 3. The snapshot of the transient water wave at t/T=15 and the time history at x/λ=4 are illustrated. The solid, dashed and dot dashed lines represent the HLIGN results obtained in different water depths (d/λ=1.00, 0.50, 0.25).



(a) Snapshot at *t/T*=15 (b) Time history at *x/λ*=4

Fig. 2 The HLIGN results in different water depths (*d/λ*=1.00, 0.50, 0.25; *H/λ*=0.01)

Fig.2 displays the water depth effects on the waves with the small wave steepness (*H/λ*=0.01). It is shown that the water depth affects the profile of wavefront, significantly. As the decreasing of the water depth, the length of waves in wavefront is getting shorter. As an example, from *x/λ*=7.5 to *x/λ*=15 in Fig. 2(a), there are 4, 5, 6 wave crests for *d/λ*=1.00, 0.50, 0.25, respectively. And the corresponding wave periods in wavefront are also getting smaller as the decreasing of water depth, as shown in Fig. 2(b). However, for the water depths considered in this study, the results in the steady-state are almost the same.



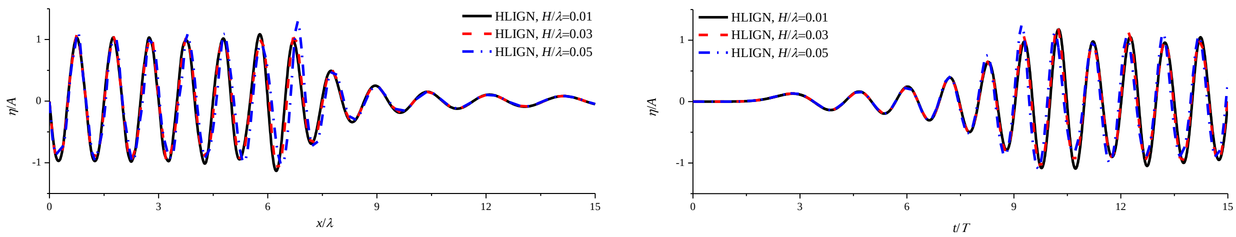
(a) Snapshot at *t/T*=15 (b) Time history at *x/λ*=4

Fig. 3 The HLIGN results in different water depths (*d/λ*=1.00, 0.50, 0.25; *H/λ*=0.05)

Fig.3 displays the water depth effects on the waves with a stronger nonlinearity (*H/λ*=0.05), which can not be accurately studied by the linear analytical solutions. It is shown that the water depth also affects the strong nonlinear waves, significantly. As shown in Fig.3(a), the decreasing of water depth makes the position of the maximum wave crest shifting from *x/λ*=6.83 to *x/λ*=7.85 and the surface elevation increasing from *η/A*=1.25 to 1.34 for *d/λ*=1.00 and 0.25, respectively. Consistent with the observation in Fig.2, the wavelengths in wavefront is also getting shorter as the decreasing of water depth.

The wavelengths in wavefront are longer than the ones in steady-state. The waves in wavefront are thus more sensitive with the water depth, which explains the different behaviors on the wavefront and the steady-state waves for the variation of the water depth.

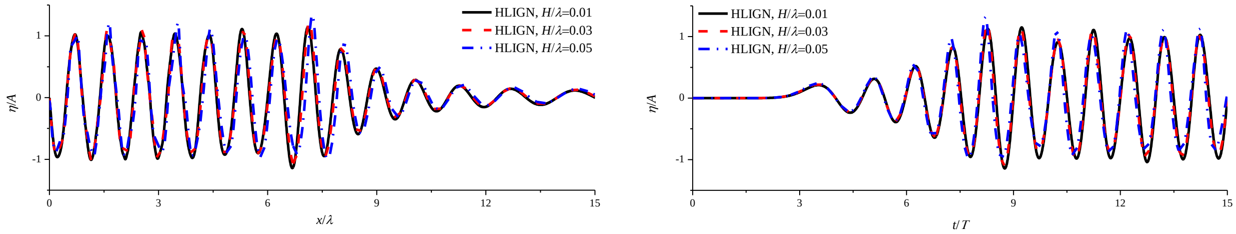
The nonlinearity effects on the transient water waves are studied in Figs. 4 and 5, in which the snapshot and the time history are displayed. The solid, dashed and dot dashed lines represent the HLIGN results with the different wave steepness (*H/λ*=0.01, 0.03, 0.05).



(a) Snapshot at *t/T*=15 (b) Time history at *x/λ*=4

Fig. 4 The HLIGN results with the different wave steepness (*d/λ*=0.50; *H/λ*=0.01, 0.03, 0.05)

If the water depth equals to *d/λ*=0.50, as shown in Fig.4, the nonlinearity will result in sharper wave crests and flatter wave troughs for the steady-state waves. There is a significant difference in wave phase for the steady-state waves as the increasing of nonlinearity because the speed of the strong nonlinear waves is getting faster. The maximum wave crest for the strong nonlinear wave group (*H/λ*=0.05) is significantly larger than that for the weakly nonlinear waves (*H/λ*=0.01). But the increasing nonlinearity does not affect so much on the wavefront, seeing the wavefront with the different wave steepness in Fig.4(a).



(a) Snapshot at t/T=15 (b) Time history at x/λ=4

Fig. 5 The HLIGN results with the different wave steepness (d/λ=0.25; H/λ=0.01, 0.03, 0.05)

If the water depth decreases to d/λ=0.25, as shown in Fig.5, the nonlinearity effects on the steady-state waves are getting more remarkable. And the differences in wavefront are still relatively small as the increasing of nonlinearity.

As stated above, the wavelength in wavefront is longer than the steady-state wavelength. The wavefront is thus not sensitive with the increasing of nonlinearity.

5 Conclusions

The effects of water depth and nonlinearity on the wavefront are investigated in this paper using the HLIGN equations for finite water depth.

The weakly nonlinear wavefront results obtained by HLIGN equations are compared with analytical solutions based on the linear wave theory in finite water depth. It is demonstrated that the HLIGN results agree well with the linear analytical solutions.

The wavefront is sensitive with the variation of water depth, but not sensitive with the nonlinearity. The decreasing water depth will shorten the wavelength in wavefront. And the combination effects of decreasing water depth and increasing nonlinearity contribute to a larger maximum wave crest for the transient regular waves.

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