Hydrodynamic forces exerted on an oscillating cylinder at its translational motion in water covered by compressed ice

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1 INTRODUCTION

According to available data, up to 12% of the oceans and seas are covered with ice. Most of the ice cover is found in the circumpolar regions, but during particularly cold winters the ice cover can deviate significantly south in the Northern Hemisphere or north in the Southern Hemisphere. In many cases, there are currents in the ocean beneath the ice cover. The currents can cause vibrations of pipelines leading to the wave motion in the ice. At certain conditions, the instability can arise resulting in the simultaneous growth of pipeline oscillations and flexural-gravity wave (FGW) amplitude in the ice cover. Therefore, the problem of description of such a phenomenon is topical and important from the practical point of view. It is also important to calculate the hydrodynamic loads exerting on the pipelines in the current in the seas or oceans covered by ice.

In this paper we examine in detail the effect of floating ice cover on the hydrodynamic loads exerting on the translationally moving and oscillating circular cylinder. In the linear approximation, we find a solution for the steady wave motion generated by the cylinder within the hydrodynamic set of equations for the incompressible ideal fluid of infinite depth. We show that depending on the rate of ice compression, the normal and anomalous dispersion can occur in the system. In the latter case, the group velocity can be opposite to the phase velocity in a certain range of wavenumbers. We investigate the dependences of hydrodynamic loads (the added mass, damping coefficients, wave resistance, and lift force) exerting on the cylinder on the translational velocity and frequency of oscillation. It is shown that there is a possibility of the appearance of negative values for the damping coefficients at the relatively big cylinder velocity; then the wave resistance decreases with increasing of the cylinder velocity. The negative damping coefficients, apparently, was obtained for a first time by Newman (1961) when he calculated the hydrodynamic loads exerting on an ellipsoid uniformly moving under the free surface of a homogeneous fluid and simultaneously oscillating along one of six possible degrees of freedom. The theoretical results are underpinned by the numerical calculations for the real parameters of ice and cylinder motion.

2 GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

We consider a circular cylinder of a radius a (a pipeline, for example) flowing around by a uniform current with the velocity $\mathbf{U} = -U\nabla x$ in an ideal incompressible fluid of the density ρ. The cylinder oscillates in the horizontal and vertical directions with the frequency Ω. We assume that the water is infinitely deep containing an ice cover on the top, can be modelled by a thin elastic plate. The main set of hydrodynamic equations and boundary conditions describing FGWs in the linear approximation is as follows. For the velocity potential $\Phi(x, y, t)$ we have the Laplace equation:

$$\Delta\Phi \equiv \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 0 \quad (|x| < \infty,\ -\infty < y \le 0), \tag{1}$$

where the fluid velocity $\mathbf{u} = \nabla\Phi$.

It is assumed that the lower boundary of the ice plate is always in contact with the water. By denoting through the $w(x,t)$ the vertical displacements of the ice cover from the undisturbed flat position, the kinematic and dynamic conditions at the upper boundary of the fluid (at $y=0$) can be written as:

$$\frac{\partial w}{\partial t}-\frac{\partial \Phi}{\partial y}=0, \qquad D\frac{\partial^4 w}{\partial x^4}+Q\frac{\partial^2 w}{\partial x^2}+M\frac{\partial^2 w}{\partial t^2}+\rho\, g\, w+\rho\frac{\partial \Phi}{\partial t}=0. \tag{2}$$

The notations used here are standard and can be found, for example, in the recent publication (Stepanyants & Sturova, 2021a). It is assumed that all perturbations vanish in the bulk of the fluid so that for both components of fluid velocity we have: $\nabla\Phi\to 0, \quad \text{when} \quad y\to-\infty$.
The dispersion relation $\omega(k)$, the phase $c_p(k)$, and group $c_g(k)$ velocities FGWs are as follows:

$$\omega(k)=\sqrt{\frac{k\,(Dk^4-Qk^2+\rho\, g)}{\rho+kM}}, \qquad c_p(k)=\frac{\omega(k)}{k}=\sqrt{\frac{Dk^4-Qk^2+\rho\, g}{k\,(\rho+kM)}}, \qquad c_g(k)=\frac{d\omega(k)}{dk}. \tag{3}$$

As well-known, the dispersion relation $\omega(k)$ imposes a restriction on the maximal value of the compression force. The stability of oscillations of a floating ice plate is guaranteed by the condition $Q<Q_*\equiv 2\sqrt{g\,\rho\, D}$, whereas at $Q>Q_*$ the buckling phenomenon occurs – the ice plate shatters. There is one more critical value of the parameter Q such that for $Q<Q_0<Q_*$ the group velocity of FGW is positive for all wavenumbers $k\ge 0$ (the case of the *normal dispersion*). In the interval $Q_0<Q<Q_*$, the group velocity is negative in a certain interval of wavenumbers (the case of the *anomalous dispersion*) – the details can be found in (Stepanyants & Sturova, 2021a, 2021b).
In this paper, we study a steady regime of fluid motion caused by the translationally moving and oscillating cylinder. In this case, the total potential of fluid velocity can be presented in the form:

$$\Phi(x,y,t)=-Ux+U\overline{\varphi}(x,y)+\mathrm{Re}\sum_{j=1}^{2}\eta_j\varphi_j(x,y)e^{i\,\Omega\, t}, \tag{4}$$

where $\overline{\varphi}$ is the velocity potential corresponding to the uniform motion of the body with the unit velocity, φ_j $(j=1,\ 2)$ are the radiation potentials due to the cylinder oscillation in the horizontal $(j=1)$ and vertical $(j=2)$ directions, η_j are the amplitudes of cylinder vibrations in these directions. Similarly to Eq. (4), the vertical displacements of the ice plate can be presented as:

$$w(x,t)=\overline{w}(x)+\mathrm{Re}\left[\sum_{j=1}^{2}\eta_j w_j(x)e^{i\,\Omega\, t}\right]. \tag{5}$$

The stationary part of the total potential $\overline{\varphi}$ satisfies the Laplace equation in the fluid: $\Delta\overline{\varphi}=0 \quad (|x|<\infty,\ -\infty<y\le 0)$ with the boundary conditions at $y=0$:

$$\frac{\partial \overline{w}}{\partial x}+\frac{\partial \overline{\varphi}}{\partial y}=0, \qquad \left(D\frac{\partial^4}{\partial x^4}+Q\frac{\partial^2}{\partial x^2}+MU^2\frac{\partial^2}{\partial x^2}+\rho g\right)\frac{\partial \overline{\varphi}}{\partial y}+\rho U^2\frac{\partial^2\overline{\varphi}}{\partial x^2}=0. \tag{6}$$

The boundary conditions far from the source are as follows:

$$\frac{\partial \overline{\varphi}}{\partial y}\to 0 \quad (y\to-\infty), \qquad \frac{\partial \overline{\varphi}}{\partial x}\to \psi_\pm \quad (x\to\pm\infty), \tag{7}$$

where functions $\psi_\pm(x,y)$ are equal to zero if the current speed U is less than the minimal phase speed of FGWs $(c_p)_{min}$. If $U>(c_p)_{min}$, than functions $\psi_\pm(x,y)$ represent stationary waves when $x\to\pm\infty$. On the circular contour $S:\ x^2+(y+h)^2=a^2$ which represents the surface of the rigid cylinder, the impermeability condition is posted: $\partial\overline{\varphi}/\partial n=n_1\ (x,y\in S)$, where $\mathbf{n}=(n_1,n_2)$ is the inner normal to the contour S, and h is the distance of the cylinder center from the upper boundary of the fluid.
The components of radiation potentials also satisfy the Laplace equation: $\Delta\varphi_j=0 \quad (|x|<\infty,\ -\infty<y\le 0)$ and the boundary conditions at $y=0$:

$$\Lambda w_j-\frac{\partial \varphi_j}{\partial y}=0, \qquad \left(D\frac{\partial^4}{\partial x^4}+Q\frac{\partial^2}{\partial x^2}+M\Lambda^2+\rho\, g\right)w_j+\rho\,\Lambda\varphi_j=0, \tag{8}$$

where the operator $\Lambda = i\Omega - U\partial/\partial x$. The boundary condition on the cylinder circular contour is:

$$\frac{\partial \varphi_j}{\partial n} = i\Omega n_j - U m_j \; (x, y \in S), \quad \text{where} \quad (m_1,\, m_2) = \nabla(\partial\overline{\varphi}/\partial n). \tag{9}$$

In the far-field zone when $x \to \pm\infty$, wave perturbations consist of superposition of several periodic waves [the details can be found in (Stepanyants & Sturova, 2021b)].
The hydrodynamic forces $\mathbf{F} = (F_1,\; F_2)$ exerting on the cylinder, are determined by integrating the fluid pressure (less the hydrostatic term) $p = -\rho\left(\partial\Phi/\partial t + |\nabla\Phi|^2/2\right)$ along the contour S. It is convenient to replace this integral by the sum $F_j = F_{sj} + \mathrm{Re}\left(F_{rj}e^{i\,\Omega\, t}\right)$ $(j = 1, 2)$, where F_{sj} are the stationary force components (the wave resistance and lift force) acting on a body in a stationary uniform flow, whereas F_{rj} are the radiation forces, which are usually written in the matrix form: $F_{rj} = \eta_1 T_{j1} + \eta_2 T_{j2}$. The quantities T_{jk} $(k = 1,\; 2)$ represent a complex force acting in the j-direction and caused by sinusoidal oscillations of a body with a unit amplitude in the k-direction; they can be represented as $T_{jk} = \Omega^2\mu_{jk} - i\Omega\lambda_{jk}$. The real quantities μ_{jk} and λ_{jk} are known as the added mass and damping coefficients, respectively.
By introducing polar coordinates with the origin in the centre of the sylinder contour, we obtain the expressions for the hydrodynamic forces (the symbol * stands for complex conjugate):

$$(F_{s1},\; F_{s2}) = \frac{\rho U^2}{2a}\int_0^{2\pi}\left[\frac{\partial(\overline{\varphi} - x)}{\partial\theta}\right]^2 (\sin\theta,\; \cos\theta)\, d\theta, \qquad T_{jk} = \rho a \int_0^{2\pi} \frac{\partial \varphi_j^*}{\partial n}\varphi_k\, d\theta. \tag{10}$$

3 NUMERICAL RESULTS

To investigate quantitatively the effect of cylinder oscillation on the ice plate covered an infinitely deep water and hydrodynamic forces exerting on the cylinder, we undertook numerical calculations with the following set of parameters:

$$E = 5{\cdot}10^9 \text{ Pa},\; \nu = 0.3,\; \rho_1 = 922.5 \text{ kg/m}^3,\; a = 5 \text{ m},\; h = 10 \text{ m},\; \rho = 1025 \text{ kg/m}^3,\; g = 9.81 \text{ m/s}^2. \tag{11}$$

Several ice-plate thickness was chosen to study its influence on FGWs and the hydrodynamic characteristics of the cylinder. The following dimensionless values of hydrodynamic forces were used: $\left(\overline{F}_{s1},\, \overline{F}_{s2}\right) = (F_{s1},\, F_{s2})\,/\,\left(\pi\rho\, g\, a^2\right)$. Figure 1 shows the dependence of stationary hydrodynamic forces on the Froude number $F = U/\sqrt{g\,a}$ for the several values of the normalised compression parameter $\widetilde{Q} \equiv Q/\sqrt{\rho\, gD} = 0,\; 1.2,\; 1.8,\; 1.95$.

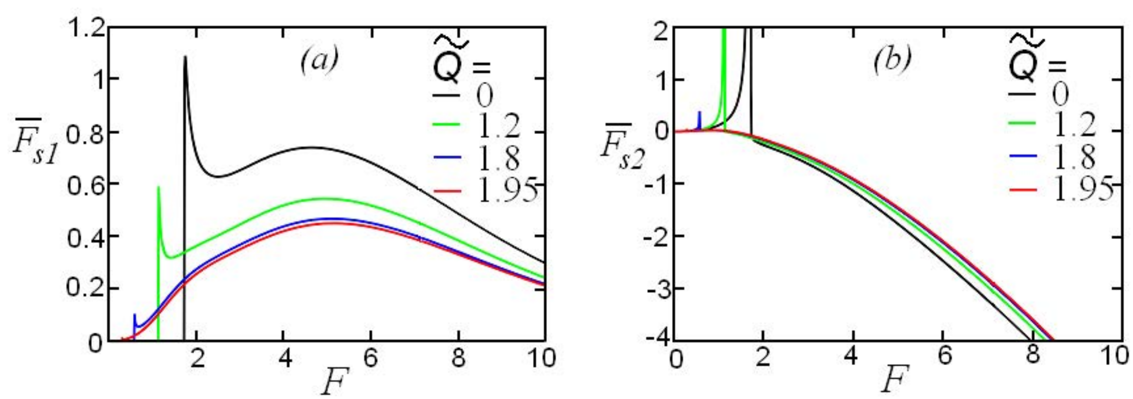

Figure 1: The wave resistance (a) and lift force (b) as functions of the Froude number F for different values of the compression parameter $\widetilde{Q} = 0, 1.2, 1.8, 1.95$.

The radiation loads (the added mass and damping coefficients) for the fixed Froude numbers and several compression parameters $\widetilde{Q}$ were calculated as functions of frequency of cylinder oscillation Ω. As the example Fig. 2 illustrates the dependence of the added mass $\overline{\mu}_{ij} = \mu_{ij}/\rho\, a^2$ $\quad (i, j) = 1, 2$ on the dimensionless frequency $\sigma = \Omega\sqrt{a/g}$. The hydrodynamic loads smoothly depend on the

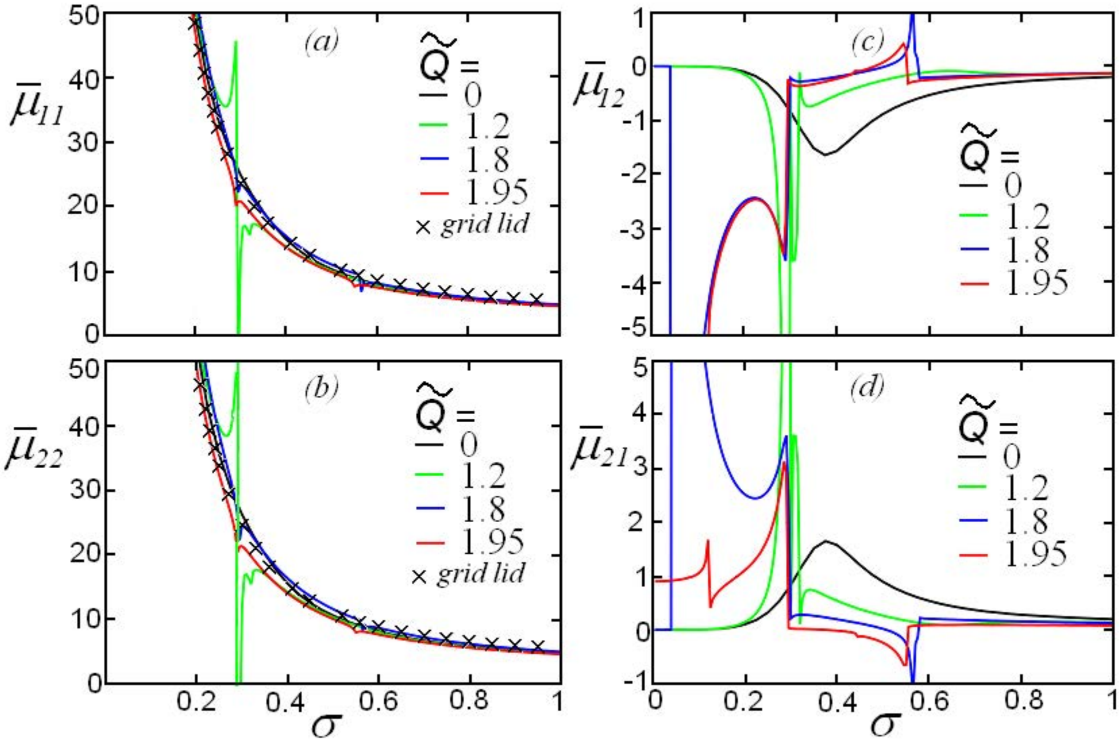

Figure 2: (Color online.) Dependence of the added mass coefficients: $\overline{\mu}_{ij}$ $(i, j) = 1, 2$ on the dimensionless frequency of cylinder oscillation for $\widetilde{Q} = 0, 1.2, 1.8, 1.95$ and fixed Froude number $F = 0.5$. Crosses show the values of $\overline{\mu}_{11}$ and $\overline{\mu}_{22}$ for the fluid covered by a rigid lid.

oscillation frequency only when $\widetilde{Q} = 0$. In all other investigated cases of the compression parameter $\widetilde{Q} = 1.2, 1.8, 1.95$, there are sharp changes in the values of the hydrodynamic loads in the vicinity of the frequencies where the regime of motion is changed. The hydrodynamic loads in the vicinity of these frequencies can significantly exceed the corresponding values for the uncompressed ice when $\widetilde{Q} = 0$. For a given Froude number $F = 0.5$, the diagonal coefficients of the added mass matrix in the cases of uncompressed ice and rigid cover practically coincide, they are indistinguishable in panels (a) and (b) of Fig. 2.

More detailed analysis of theoretical and numerical results will be presented at the Workshop.

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