Water wave scattering by a submerged inclined poroelastic plate

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1 Introduction

In recent years, water wave scattering by submerged structures having various structural properties and orientations have been a topic of great interest. In order to preserve the already exhausted natural resources researchers are focusing on extracting wave energy to meet the power requirements of the world. Elastic structures are quite efficient in reflecting back the waves but are not that impactful when it comes to reducing wave loads on the structure. Porous nature along with elasticity of the structure can serve this purpose by dissipating wave energy. The hydrodynamic behaviour of inclined barrier was studied by Parsons and Martin (1992) using the Green's theorem approach. Recently, Ashok *et al.* (2020) investigated water wave scattering by vertical flexible porous structure. To the authors' knowledge there is no work in literature on inclined *porous elastic* structures. Thus here we investigate the problem of water wave scattering by an inclined poroelastic thin plate submerged in deep water.

2 Mathematical Formulation

A rectangular Cartesian coordinate system is considered in which the positive y-axis is pointing vertically downwards in the fluid domain. A thin poroelastic plate Γ of length $2L$ and negligible thickness d is inclined at an angle θ with the positive y-axis. The midpoint of the plate is at the distance h from the mean free surface ($y = 0$). Assuming linear water wave theory and irrotational motion, a wave train described by $\mathrm{Re}\{\phi(x, y)\mathrm{e}^{-\mathrm{i}\sigma t'}\}$ is normally incident on the plate Γ.

Here $\phi(x, y)$ satisfies :

$$\phi_{xx} + \phi_{yy} = 0, \quad \text{in the fluid region,} \tag{1}$$

$$K\phi + \phi_y = 0, \quad \text{on } y = 0, -\infty < x < \infty, \tag{2}$$

$$\phi, \nabla\phi \to 0, \quad \text{as } y \to \infty, \tag{3}$$

$$r^{-1/2}\,\phi \ \text{ is bounded as } \ r \to 0, \tag{4}$$

where r is distance from any submerged end of the plate and the radiation conditions,

$$\phi(x, y) \to \begin{cases} \phi^{in}(x, y) + R\phi^{in}(-x, y) & \text{as } \ x \to -\infty, \\ T\phi^{in}(x, y) & \text{as } \ x \to \infty. \end{cases} \tag{5}$$

Here R and T denote the reflection and the transmission coefficients and $\phi^{in}(x, y)$ denotes the incident wave potential given by $\phi^{in}(x, y) = \mathrm{e}^{-Ky+\mathrm{i}Kx}$.

Let the displacement of the inclined plate be defined by $w(x, y, t') = \mathrm{Re}\{\chi(x, y)\mathrm{e}^{-\mathrm{i}\sigma t'}\}$, where $\chi(x, y)$ is displacement amplitude. The equation of motion due to fluid pressure is given by,

$$D\frac{\partial^4\chi}{\partial s^4} - \epsilon K\chi = -\frac{\mathrm{i}\sigma}{g}[\phi](q) \ \text{ on } \ \Gamma, \ \ q \in \Gamma, \tag{6}$$

where,

$$D = \frac{Ed^3}{12\rho_w(1-\nu^2)g}, \qquad \epsilon = \frac{\rho_p}{\rho_w}d.$$

Here, E represents the Young's modulus, ν denotes the Poisson's ratio, and ρ_p and ρ_w denote the material density of the plate and the density of water respectively. $[\phi](q) = \phi(q^+) - \phi(q^-)$ denotes the potential difference across the plate.

The boundary condition on the surface of the plate is given by:

$$\frac{\partial\phi}{\partial n_q} = -\mathrm{i}K\mathrm{G}[\phi](q) - \mathrm{i}\sigma\chi, \quad q \in \Gamma, \tag{7}$$

where, G represents the constant porosity parameter which varies along the length of the plate Γ. We consider the top end of the plate to be clamped and the bottom end to be moored. Thus we have,

$$\frac{\partial \chi}{\partial s} = 0 = \chi \text{ at top end of } \Gamma, \quad \text{and} \quad \frac{\partial^2 \chi}{\partial s^2} = 0, \; \frac{\partial^3 \chi}{\partial s^3} = M\chi \text{ at bottom end of } \Gamma. \tag{8}$$

Here $M = 2k_d \sin^2(\Theta)/EI\cos^3(\theta)$ where k_d is the spring constant and Θ denotes the mooring angle.

3 Method of Solution

We transform the (x, y) coordinate system to (u, v) coordinate system where the plate Γ can be treated as a vertical plate lying along the v-axis. From the origin of the new system the upper end of the plate is at a distance a and lower end is at a distance b. Thus $2L = b - a$ holds. In the (u, v) coordinate system equation (6) becomes,

$$\frac{\mathrm{d}^4\chi}{\mathrm{d}v^4} - \alpha^4\chi = -\frac{\mathrm{i}K}{\sigma D}[\phi](0, v), \quad a < v < b \tag{9}$$

where $\alpha^4 = \frac{\epsilon K}{D}$, along with end conditions,

$$\frac{\mathrm{d}\chi}{\mathrm{d}v} = 0 = \chi, \;\; \text{at } v = a, \quad \text{and} \quad \frac{\mathrm{d}^2\chi}{\mathrm{d}v^2} = 0, \; \frac{\mathrm{d}^3\chi}{\mathrm{d}v^3} = M\chi, \;\; \text{at } v = b. \tag{10}$$

We solve the above boundary value problem using the Green's function technique assuming that right hand side of equation (9) is known. Thus $g(\zeta, v)$ satisfies,

$$\frac{\mathrm{d}^4 g}{\mathrm{d}\zeta^4} - \alpha^4 g = \delta(\zeta - v), \qquad\qquad a < \zeta, v < b, \tag{11}$$

$$g = 0 = g_\zeta \;\; \text{at } \zeta = a, \quad \text{and} \quad g_{\zeta\zeta} = 0, \; g_{\zeta\zeta\zeta} = M\,g, \;\; \text{at } \zeta = b. \tag{12}$$

Along with continuity of $g, g_\zeta, g_{\zeta\zeta}$ at $\zeta = v$, and jump discontinuity $g_{\zeta\zeta\zeta}(v^+, v) - g_{\zeta\zeta\zeta}(v^-, v) = -1$. The general solution of equation (11) will be of the form

$$g(\zeta, v) = \begin{cases} A_1\cos(\alpha\zeta) + A_2\sin(\alpha\zeta) + A_3\cosh(\alpha\zeta) + A_4\sinh(\alpha\zeta), & a < \zeta < v < b, \\ B_1\cos(\alpha\zeta) + B_2\sin(\alpha\zeta) + B_3\cosh(\alpha\zeta) + B_4\sinh(\alpha\zeta), & a < v < \zeta < b \end{cases} \tag{13}$$

where A_i, B_i $(i = 1, 2, 3, 4)$ are all unknown functions of v only which are determined using the above equations satisfied by $g(\zeta, v)$. Once the form of g is found the expression for $\chi(0, v)$ is obtained as,

$$\chi(0, v) = -\frac{\mathrm{i}K}{\sigma D}\int_a^b g(\zeta, v)\,[\phi](\zeta)\,\mathrm{d}\zeta, \quad a < v < b. \tag{14}$$

By virtue of the above equation and the condition (7) on the plate $\frac{\partial\phi}{\partial n_{q_1}}$ can be expressed as,

$$\frac{\partial\phi}{\partial n_{q_1}} = -\mathrm{i}K\mathrm{G}[\phi](q_1) - \frac{K}{D}\int_\Gamma g(q_2, q_1)\,[\phi](q_2)\,\mathrm{d}s_{q_2}; \quad q_1 \in \Gamma. \tag{15}$$

We next apply Green's integral theorem on the scattered potential $(\phi - \phi^{in})(p_1)$ and the Green's function $\mathcal{G}(p_1, p_2)$ (cf. Mandal and Chakrabarti (2000)) due to a line source situated at the point p_2. Letting $p_i \to q_i$ and taking the normal derivative of the expression of ϕ thus obtained, at the point q_1 on Γ we get a second expression for the normal velocity on the plate as given by,

$$\frac{\partial\phi}{\partial n_{q_1}} = \frac{\partial}{\partial n_{q_1}}\left\{\phi^{in}\right\} - ⨎_\Gamma [\phi](q_2)\,\frac{\partial^2\mathcal{G}}{\partial n_{q_1}\partial n_{q_2}}(q_1, q_2)\,\mathrm{d}s_{q_2}; \quad q_1 \in \Gamma. \tag{16}$$

Comparing equations (15) and (16) we get a hypersingular integral equation of the second kind as,

$$⨎_\Gamma \Big[\frac{\partial^2\mathcal{G}}{\partial n_{q_1}\partial n_{q_2}}(q_1; q_2) - \frac{K}{D}\,g(q_2, q_1)\Big]\,[\phi](q_2)\,\mathrm{d}s_{q_2} - \mathrm{i}K\mathrm{G}[\phi](q_1) = \frac{\partial}{\partial n_{q_1}}\left\{\phi^{in}\right\}; \quad q_1 \in \Gamma. \tag{17}$$

Parametrizing the above equation we get,

$$\fint_{-1}^{1} f(t)\Big[-\frac{1}{(s-t)^2}+2\pi L^2\mathcal{L}(s,t)\Big]\,\mathrm{d}t - 2\pi\mathrm{i}KL\,Gf(s) = F(s), \quad -1<s<1 \tag{18}$$

where $f(t)(=[\phi](q_2))$ is an unknown function which must vanish at $t=\pm1$ and $\mathcal{L}$ and F are known bounded functions. To solve equation (18), $f(t)$ is approximated as

$$f(t) = (1-t^2)^{1/2}\sum_{n=1}^{M} a_n U_{n-1}(t), \tag{19}$$

where $U_n(t)$'s are the Chebyshev polynomial of the second kind and a_n's are unknown constants to be determined. Substituting (19) in (18) and collocating at M points $s=s_j$ gives,

$$\sum_{n=1}^{M} a_n B_n(s_j) = F(s_j), \quad j=1,...,M, \tag{20}$$

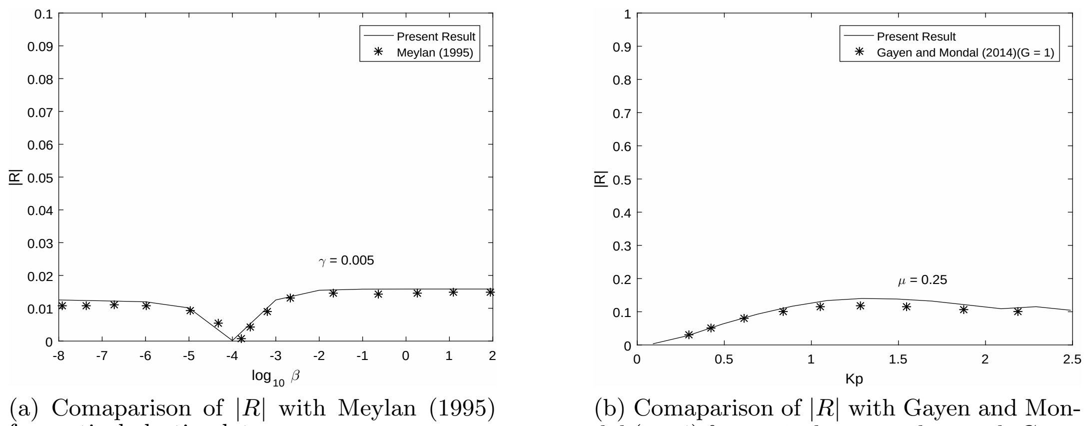
where $$B_n(s_j) = \Big[n - 2\mathrm{i}KL\,G\,(1-s_j^2)^{1/2}\Big]\pi U_{n-1}(s_j) + 2\pi L^2\int_{-1}^{1}(1-t^2)^{1/2}\mathcal{L}(s_j,t)\,U_{n-1}(t)\,\mathrm{d}t$$

and $$s_j = \cos\frac{2j-1}{2M}\pi, \; j=1,...,M.$$

Once a_n's are known, expression for reflection coefficient R, energy loss coefficient J and hydrodynamic force $\mathcal{F}$ can be determined analytically.

4 Numerical Results

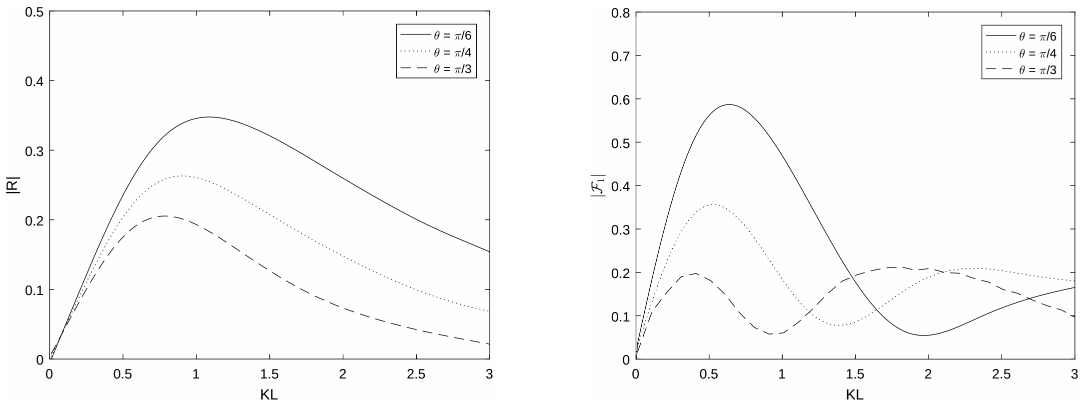
Non-dimensional hydrodynamic quantities are evaluated numerically. The obtained numerical results are depicted graphically against the non-dimensional wavenumber for varying parameters. In Figure 1, the validation of the present results are shown. Figure 1(a) depicts comparison with Meylan (1995) for a surface piercing vertical elastic plate taking porosity parameter $G=0$ here. Figure 1(b) compares present results with Gayen and Mondal (2014) for vertical porous plate with $G=1$ taking large value of flexural rigidity $D/p^4=1000$ where p is the depth of lower end of the plate. In Figure 2(a) and (b), absolute value of reflection coefficient $|R|$ and non-dimensional hydrodynamic force $|\mathcal{F}_1|$ respectively are plotted against the non-dimensional wavenumber KL. In both the graphs the angle of inclination θ is varied and the other parameters are non dimensionalised with respect to L. We choose $h/L=1.1$, $\epsilon/L=0.02$, $D/L^4=2$, $G=0.5$. It is observed that as θ increases, i.e. as the plate deviates from its vertical position $|R|$ decreases. Also for lower frequencies, hydrodynamic force decreases when θ increases but for higher frequencies the nature is not that clear. In Figure 3(a) and (b), energy loss coefficient J has been plotted for varying values of G and θ respectively. It is seen that as real part of G increases J increases, but J decreases as imaginary part of G increases when its real part is kept fixed. Similar behaviour of energy loss coefficient was also found in Ashok *et al.* (2020). Also increasing the value of θ decreases J.



(a) Comaparison of $|R|$ with Meylan (1995) for vertical elastic plate

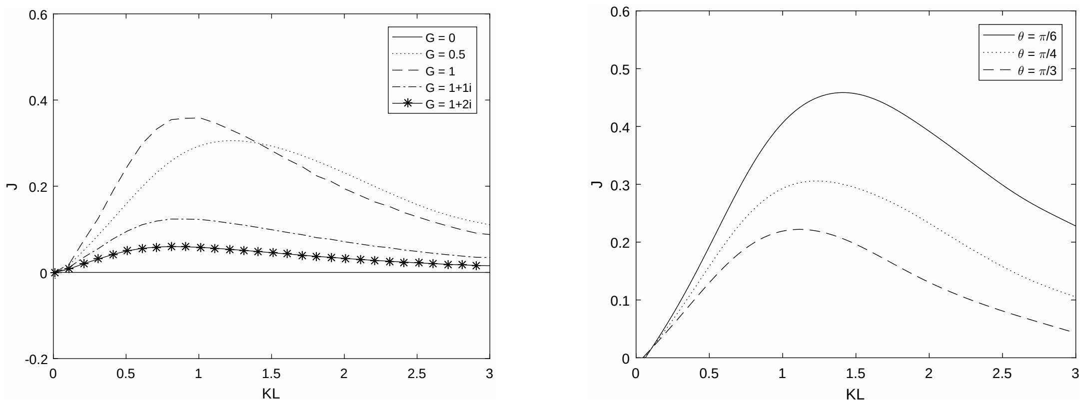
(b) Comaparison of $|R|$ with Gayen and Mondal (2014) for vertical porous plate with $G=1$

Figure 1: Validation of present results.



(a) $|R|$ vs KL (b) $|\mathcal{F}_1|$ vs KL

Figure 2: Reflection coefficient and hydrodynamic force for different θ and fixed $h/L = 1.1$, $\epsilon/L = 0.02$, $D/L^4 = 2$ and $G = 0.5$.



(a) J vs KL (b) J vs KL

Figure 3: Energy loss coefficient J for (a) different G, $\theta = \pi/4$ fixed, (b) different θ, $G = 0.5$ fixed and other parameters $h/L = 1.1$, $\epsilon/L = 0.02$ and $D/L^4 = 2$ are fixed for both.

5 Conclusion

Using hypersingular integral equations approach water wave scattering is studied by a poroelastic inclined plate submerged in deep water. It is observed that structural porosity along with elasticity aids in reducing hydrodynamic force on barrier by dissipating energy. Further it is seen that as inclination of the plate increases, both the amount of reflection and dissipated energy decrease.

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