Impulsive impact of a submerged body

Y.A. Semenov[1] yuriy.a.semenov@gmail.com
Y.N. Savchenko[1] hydro.ua@gmail.com
G.Y. Savchenko[1] lenchik123@ukr.net

[1]Institute of Hydromechanics of the NAS of Ukraine, Kyiv, Ukraine

Introduction

The concept of impulsive fluid/structure interaction widely used to study an initial stage of violent water impact flows, for which a strong couple between the nonlinear and unsteady effects may result in an extremely large hydrodynamic pressure and the force, respectively. Among the earliest work is that by Havelock (1949) in which he introduced a type of problem similar to that we are studying here: suddenly forced motion of a body with constant velocity at time zero. This concept has received much extension with application to aircrafts landing on water surface (von Karman, 1929), steep waves that suddenly hit coastal or marine structures (Cooker and Peregrine, 1995), impulsive motion of the submerged cylinder (Greenhow, 1987; Tyvand and Miloh, 1995) and impulsive sloshing in containers (Tyvand and Miloh, 2012), dam-break flows (Korobkin and Yilmaz, 2009). The solution based on the impulsive concept may contain a singularity at the three phase contact line. In such cases the solution is used as an outer solution which has to be matched with the inner solution following to the method of matched asymptotic expansion (Howison, Ockendon and Wilson, 1991).

In this study, we consider a body fully submerged into the liquid and subjected to impact such that the body suddenly set into motion at initial time. The motivation for this research comes from naval hydrodynamics of a high-speed hydrofoil craft whose foil system may experience sudden vertical motion caused by wave impacts onto the main body of the craft.

Boundary-value problem

A sketch of the physical domain is shown in figure 1(a). The body submerged below flat free surface is symmetric respect to Y-axis, therefore only half of the flow region is considered. Before the impact, $t = 0$, the body and the liquid are at rest. At time $t = 0^+$ the body suddenly set into motion with velocity U directed downward. The problem of a rigid body moving suddenly into a fluid body is dynamically equivalent to the problem of a fluid body moving suddenly around the rigid body with velocity U at infinity. We define a Cartesian system XY with its origin on the body surface at point A. The body is assumed to have an arbitrary shape which can be defined by the slope of the body as a function of the arc length coordinate S, $\beta_b = \beta_b(S)$. The liquid is assumed to be ideal and incompressible, and the flow is irrotational. The gravity and surface tension effects are ignored.

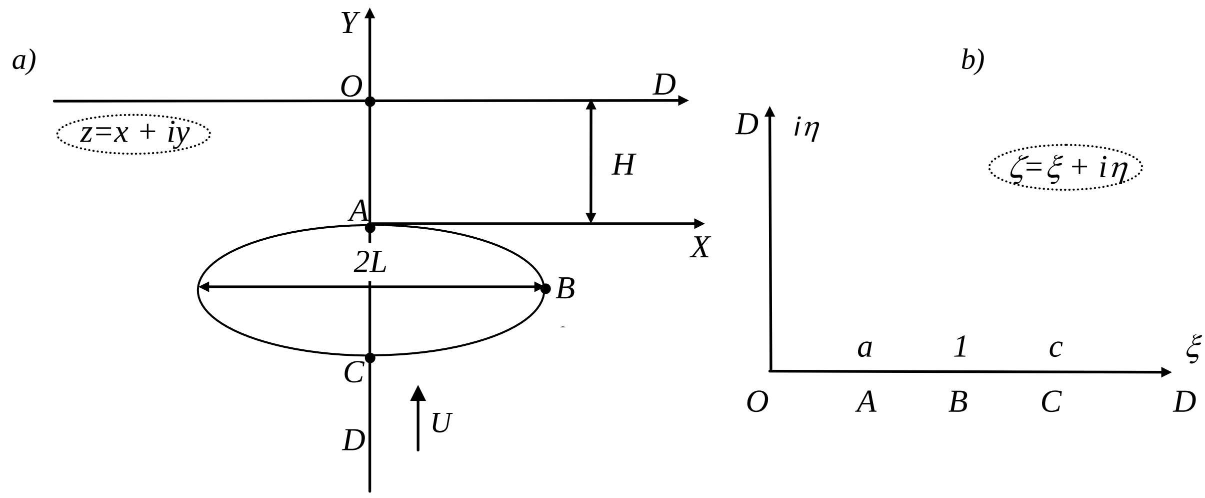

Figure 1. (a) The physical z-plane with the submerged body, (b) the parameter, or ζ–plane.

For two-dimensional inviscid, incompressible, irrotational flow we can introduce a complex potential $W(Z) = \Phi(x,y) + i\Psi(x,y)$ with $Z = X + iY$. We use non-dimensionalization based on U, L, ρ. Then, $v = |V|/U, x = X/L, y = Y/L, h = H/L$, $\phi(x,y) = \Phi(X,Y)/(LU)$, $\phi(s) = \Phi(S)/(LU)$.

By integrating Bernoulli's equation

$$\frac{\partial\Phi}{\partial t} + \frac{p}{\rho} + \frac{|V|^2}{2} = \frac{p_a}{\rho} + \frac{U^2}{2}. \tag{1}$$

over the infinitesimal time interval $\Delta t \to 0$ and taking into account that the integral of the third term tends to zero, one can obtain

$$P = \int_0^{\Delta t} p dt = -\rho\Phi, \tag{2}$$

where P is the impulsive pressure. Here, $|V|$ is the velocity magnitude, p and p_a are the hydrodynamic pressure and the pressure on the free surface, respectively.

The vertical impulse force F_y is obtained by integrating the impulse pressure over the body surface,

$$F_y = -2\rho \int_{S_A}^{S_C} \Phi(S) \cos{(n,y)}\, dS = \mathrm{mL^2U}, \tag{3}$$

where S is the arc length coordinate along the body surface; S_A and S_C are the arcthlength coordinates of points A and C; m is the coefficient of the added mass. The multiplier "2" appears to account force acting on the whole body. Since we consider the body symmetric respect to y-axis, the horizontal impulse force equals zero. As it follows from equation (3), the coefficient of the added mass is defined as

$$m = -2\rho \int_{s_A}^{s_C} \phi(s) \cos{(n,y)}\, ds. \tag{4}$$

The problem is to determine the velocity potential $\phi(x,y)$ immediately after the impact.

Conformal mapping

We introduce the complex potential $w(z) = \phi(x,y) + i\psi(x,y)$, where $\psi(x,y)$ is the stream function, and choose the first quadrant of the ζ-plane in figure 1b as the region corresponding to fluid region in the physical z-plane (figure 1a). The theorem on conformal mapping allows us to choose arbitrary locations of three points which are the points O at the origin ($\zeta = 0$), D (D') at infinity, and B at $\zeta = 1$ (see figure 1b). The position of points A ($\zeta = a$) and C ($\zeta = c$) has to be determined from physical considerations.

By applying integral hodograph method we can determine the complex velocity, dw/dz, and the derivative of the complex potential, $dw/d\zeta$, both defined *in* ζ-plane:

$$\frac{dw}{dz} = v_\infty \left(\frac{\zeta-a}{\zeta+a}\right)^{\frac{1}{2}} \left(\frac{\zeta-c}{\zeta+c}\right)^{\frac{1}{2}} exp\left[\frac{1}{\pi}\int_a^c \frac{d\beta_b}{d\xi} ln\left(\frac{\zeta-\xi}{\zeta+\xi}\right) d\xi - \frac{i}{\pi}\int_0^\infty \frac{dln\, v}{d\eta} ln\left(\frac{\zeta-i\eta}{\zeta+i\eta}\right) d\eta - i\frac{\pi}{2}\right], \tag{5}$$

$$\frac{dw}{d\zeta} = -K. \tag{6}$$

Here, $\beta_b(\xi)$ is the slope of the body as the function of the coordinate ξ, and $v(\eta)$ is the modulus of the velocity on the free surface just after the impact, K is a real constant. The derivative of the mapping function is obtained by dividing (6) by (5)

$$\frac{dz}{d\zeta} = -K \left(\frac{\zeta+a}{\zeta-a}\right)^{\frac{1}{2}} \left(\frac{\zeta+c}{\zeta-c}\right)^{\frac{1}{2}} exp\left[-\frac{1}{\pi}\int_a^c \frac{d\beta_b}{d\xi} ln\left(\frac{\zeta-\xi}{\zeta+\xi}\right) d\xi + \frac{i}{\pi}\int_0^\infty \frac{dln\, v}{d\eta} ln\left(\frac{\zeta-i\eta}{\zeta+i\eta}\right) d\eta + i\frac{\pi}{2}\right]. \tag{7}$$

Kinematic boundary condition on the free surface

The velocity generated by the impact is perpendicular to the free surface, or it is directed in y-direction. Accordingly, the argument of the complex velocity (5) is

$$arg\left(\frac{dw}{dz}\bigg|_{\zeta=i\eta}\right) = -\frac{\pi}{2}, \quad 0 \le \eta \le \infty. \tag{8}$$

Taking the argument of the complex velocity from (5), we obtain the following integral equation respect to the function $\frac{\mathrm{d}\ln v}{d\eta}$

$$\frac{1}{\pi}\int_0^\infty \frac{d\ln v}{d\eta'} ln\left|\frac{\eta'-\eta}{\eta'+\eta}\right| d\eta' + \ tan^{-1}\frac{\eta}{a} + tan^{-1}\frac{\eta}{c} + \frac{1}{\pi}\int_a^c \frac{d\beta_b}{d\xi} 2tan^{-1}\frac{\eta}{\xi} d\xi = 0, \tag{9}$$

Equation (9) is the Fredholm integral equation of the first kind with the logarithmic kernel. The solution takes the form

$$v(\eta) = \sqrt{\eta^2 + a^2}\sqrt{\eta^2 + c^2}\exp\left(\frac{1}{\pi}\int_a^c \frac{\mathrm{d}\beta_b}{d\xi} \ln\left(\eta^2 + \xi^2\right) d\xi\right). \tag{10}$$

For the case of a submerged flat plat, the solution can be simplified. The function $\beta_b(\xi)$ can be written explicitly

$$\beta_b(\xi) = \begin{cases} \pi, & a \le \xi < 1, \\ 0, & 1 < \xi \le c. \end{cases} \tag{11}$$

By substituting (11) into (5) and (10) and evaluating the integral over step change in the function $\beta_b(\xi)$ at $\xi = 1$, we obtain the expression for the complex velocity

$$\frac{dw}{dz} = v_\infty \left(\frac{\zeta-a}{\zeta+a}\right)^{\frac{1}{2}} \left(\frac{\zeta-c}{\zeta+c}\right)^{\frac{1}{2}} \left(\frac{\zeta+1}{\zeta-1}\right) exp\left[-\frac{i}{\pi}\int_0^\infty \frac{d\ln v}{d\eta} ln\left(\frac{\zeta-i\eta}{\zeta+i\eta}\right) d\eta - i\frac{\pi}{2}\right], \tag{12}$$

and for the velocity along the free surface

$$v(\eta) = \frac{\sqrt{\eta^2+a^2}\sqrt{\eta^2+c^2}}{\eta^2+1}. \tag{13}$$

The parameters a, c and K are determined using the depth of submergence h, length of the low side BC and upper side AB of the plate.

Results

The added mass coefficients according to definition (4) are shown the In table 1 for various shapes of the body. For the flat plate, $m \to \pi/2$ as $h \to 0$ that corresponds to the von Karman impact solution for the flat plate floating on the free surface. As $h = 50$ the effect of the free surface becomes negligible, and the coefficient of the added mass $m \to \pi$, that corresponds to the added mass in the unbounded fluid domain.

For the circular cylinder, we obtained value $m \to 2\pi$ for $h = 50$. This value is different from $m^* = \pi$ in Newman (1977, p.145). The difference occurs due to different definition of the added mass in (4) and in equation (114) in Newman (1977). It is found that they are related as

$$m^* = m - A/L^2, \tag{14}$$

where A is the area of the body. Since the area of the flat plate is zero, the values m and m^* coincides. For the cylinder with radius L, the area $A = \pi L^2$, and for the squire with the side length $2L$, the area $A = 4L^2$. At the depth $h = 50$, that is close to the impact in unbounded fluid domain, the values $m^* = 6.283 - \pi$ =3.141 and $m^* = 8.754 - 4$ = 4.754. They are coincide with the values from Newman (1977, p.145).

The fluid velocity on the free surface for the flat plate is shown in figure 2 for different depths of submergence. The minimum velocity occurs in the center of the plate, since the plate carries away the liquid down. At the edges of the plate the velocity of the liquid is infinite as it is seem from (12). We could expect the peak of the velocity on the free surface close to edges at $x = \pm L$. However, the location of the peak depends on the depth of submergence as it seen in figure 2.

H	0	0.02	0.05	0.1	0.3	0.5	1	5	50
Plate	1.571	1.624	1.735	1.876	2.265	2.516	2.835	3.108	3.137
Cylinder	-	5.232	5.253	5.304	5.512	5.673	5.919	6.246	6.283
Squire	-	6.993	7.011	7.048	7.292	7.552	8.024	8.667	8.754

Table 1. Added mass coefficient for various bodies and depths of submergence.

The fluid velocity along the free surface is shown in figure 3 for the square (2L× $2L$, blue lines), and for the rectangle (2L× L, red lines). The larger area of the body, the larger added mass, and the smaller peak of the velocity on the free surface.

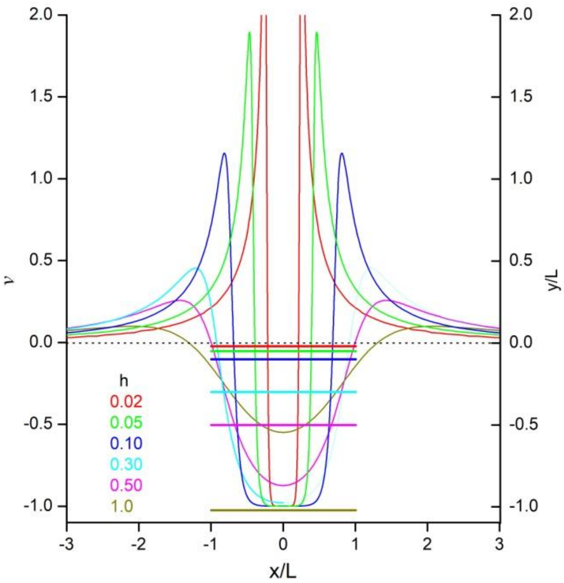

Figure 2. Velocity magnitude on the free surface

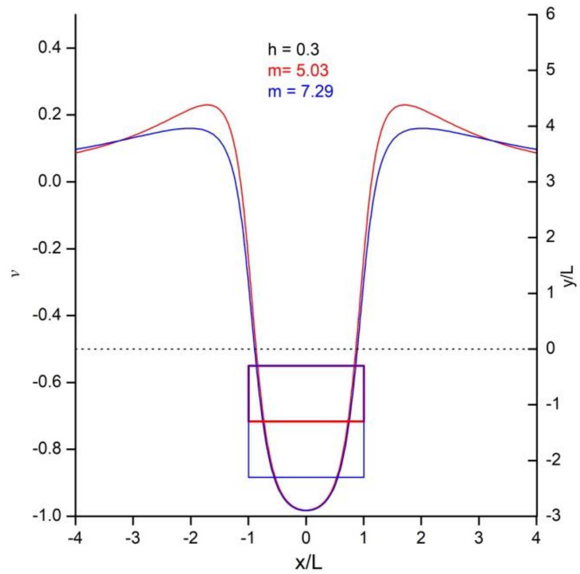

Figure 3. Velocity on the free surface for the square (2L× $2L$, blue lines), and for the rectangle (2L× L, red lines).

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