Hydrodynamic pressure in hydroelastic impact problems

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1. Introduction. The present study is focused on hydrodynamic pressure and the total force acting on an elastic body entering water. The two-dimensional coupled problem is solved within the Wagner model of water impact using the normal mode method. This approach provides the amplitudes of the so-called dry modes of the elastic surface as functions of time for given initial shape of the structure, its elastic characteristics, and conditions of impact. The distribution of the hydrodynamic pressure along the wetted part of the body surface is not evaluated in this approach. This approach provides deflections of the body and elastic stresses in it. The pressure can be recovered *a posteriori* by using the structural model as an external load causing the obtained deflections. This method is reasonable and similar to the idea of the *equivalent static slamming pressure* for direct strength assessment and scantling determination, see ABS Guide(2020) and Zhu et al. (2020). A problem is that the normal mode method provides the pressure as a series with respect to the dry modes of the body, where the convergence of the series is poor. This approach is not practical for studies of the hydrodynamic loads during elastic impacts. The total force is also given by a series, convergence of which is better than the convergence of the series for the pressure, but still weak. If the stresses in the impacting structure due to water impact are known, one does not need pressures to access the structural integrity of the structure and possible damage to the structure.

Problems with hydrodynamic pressure due to elastic impacts occur also in experimental measurements. Faltinsen (1997) wrote "Kvalsvold et al. (1995) presented the measured peak pressures from all the different drop tests as a function of drop speed for different curvatures of the waves and impact positions. There was a tremendous scatter in the pressures at a given drop speed. The measured peak pressure results do not encourage the designer to treat the impact problem in a deterministic way even in deterministic environmental conditions. This study indicates that wetdeck slamming may be treated deterministically in deterministic environmental conditions as long as no attention is paid to the peak pressure."

However, in some problems, the impact pressure is needed to select a right model of elastic impact. Vibration of the elastic structure both during the impact stage, when the surface of the structure is wetted partly, and during the so-called free-vibration stage, when the structure is completely wetted, the loads are not as high but the structure continues to vibrate, may cause cavitation in the impact region, which requires another model of impact.

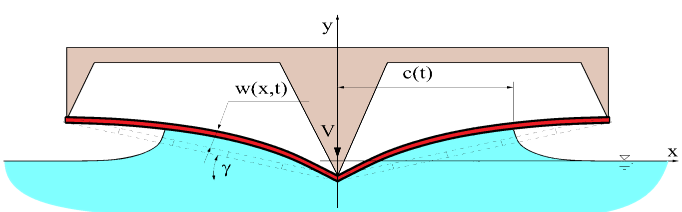

Fig.1. Elastic wedge impact onto water and notation.

In the present study, we do not include cavitation into our impact model as well. Our model is similar to that used by Kvalsvold (1994) and Faltinsen (1997). We introduce a technique which accurately describes the pressure distribution in the wetted part of the impacting structure during the initial impact stage, when the wetted region expands at a high rate. The technique is illustrated for the 2D symmetric problem of elastic wedge impact at a constant speed, see figure 1. The deflections of the wedge platings are obtained by the method presented in Khabakhpasheva & Korobkin (2013). The pressure decomposition into its singular and regular parts follows the paper by Korobkin(1998). Quick convergence of the regular part of the hydrodynamic pressure with the number of modes is demonstrated. This decomposition was found also helpful for prediction of the total force acting on the entering wedge.

2. Formulation of elastic wedge impact problem and the pressure singularity. An elastic wedge with small deadrise angle γ penetrates into water at constant speed V. The wedge plates are simply supported at their edges. The problem is formulated in dimensionless variables within the Wagner model, see Khabakhpasheva & Korobkin (2013) for full details. The length L of the side plate of the wedge is taken as the length scale, $(L/V)\sin\gamma$ as the time scale, $L\sin\gamma$ as the displacement scale, and $\rho V^2 \sin^{-1}\gamma$ as the pressure scale, where ρ is the liquid density. The dimensionless pressure in the flow region, $y < 0$, is given by the linearised Bernoulli equation, $p(x, y, t) = -\varphi_t(x, y, t)$. The velocity potential $\varphi(x, y, t)$ is the solution of the boundary value problem,

$$\nabla^2\varphi = 0 \quad (y < 0), \quad \varphi = 0 \quad (y = 0, |x| > c(t)), \quad \varphi_y = -1 + w_t(x, t) \quad (y = 0, |x| < c(t)), \tag{1}$$

where the potential decays at infinity, $\varphi \to 0$ as $x^2 + y^2 \to \infty$, and is continuous up to the boundary of the flow region. The dimensionless deflection $w(x, t)$ in (1) is governed by the equations

$$\alpha\frac{\partial^2 w}{\partial t^2} + \beta\frac{\partial^4 w}{\partial x^4} = p(x, 0, t) \quad (|x| < 1), \quad w = w_{xx} = 0 \quad (x = \pm 1, 0), \quad w = w_t = 0 \quad (t = 0), \tag{2}$$

where $\alpha = (\rho_b h)/(\rho L)$, $\beta = Eh^3 \sin^2\gamma/(12\rho L^3 V^2)$, ρ_b is the density of the wedge material, h is the plate thickness, and E is Young's modulus of the plate elasticity. The pressure in (2) is zero outside the wetted part of the wedge, $|x| < c(t)$, where the function $c(t)$ is related to the plate deflection by the Wagner condition,

$$c(t) = \frac{\pi}{2}t - \int_0^{\pi/2} w[c(t)\sin\theta, t]d\theta. \tag{3}$$

Within the method of normal modes, the plate deflection is sought in the form

$$w(x, t) = \sum_{n=1}^{\infty} a_n(t)\psi_n(x), \quad (|x| < 1), \quad \psi_n(x) = \sin(\lambda_n|x|), \quad \lambda_n = \pi n. \tag{4}$$

The dry modes, $\psi_n(x)$, of the simply supported plates are orthonormal. The coupled problem (1)-(3) is reduced to an infinite system of ordinary differential equations for the coefficients $a_n(t)$, which is truncated down to N_m modes, and integrated numerically by the fourth-order Runge-Kutta method, see Khabakhpasheva & Korobkin (2013) for full details.

When the deflection (4) has been obtained, the pressure can be approximately evaluated using the plate equation (2),

$$p(x, 0, t) = \alpha\sum_{n=1}^{Nm} \ddot{a}_n(t)\psi_n(x) + \beta\sum_{n=1}^{Nm} \lambda_n^4 a_n(t)\psi_n(x). \tag{P1}$$

The formula (P1) gives good approximation of the pressure during the free-vibration stage but it is not acceptable during the impact stage, when the theoretical pressure is square-root singular at the contact points $x = \pm c(t)$, see the numerical results below. To improve the convergence of (P1) during the impact stage, we use the following relation between the boundary values of the derivatives $\varphi_x(x, 0, t)$ and $\varphi_x(x, 0, t)$,

$$\varphi_x(x, 0, t) = -\frac{x}{\pi\sqrt{c^2 - x^2}}\int_{-c}^{c} \frac{\varphi_y(\tau, 0, t)d\tau}{\sqrt{c^2 - \tau^2}} + \frac{1}{\pi}\sqrt{c^2 - x^2}\ \text{P.v.}\int_{-c}^{c} \frac{\varphi_y(\tau, 0, t)d\tau}{(\tau - x)\sqrt{c^2 - \tau^2}}, \tag{5}$$

which follows from the Hilbert formula for analytical functions in $y < 0$. Using (1) and (4), we find

$$\varphi_x(x, 0, t) = \frac{A(t)x}{\sqrt{c^2 - x^2}} + O(\sqrt{c^2 - x^2}), \ \ A(t) = 1 - \frac{2}{\pi}\sum_{n=1}^{\infty} \dot{a}_n(t)D_n, \ D_n(c) = \int_0^{\pi/2} \psi_n[c(t)\sin\theta]d\theta. \tag{6}$$

where $-c < x < c$ and $|x| \to c(t)$. Therefore,

$$\varphi(x, 0, t) =-A(t)\sqrt{c^2 - x^2} + O([c^2 - x^2]^{\frac{3}{2}}), \ p(x, 0, t) = P_S(x, t) + O(\sqrt{c^2 - x^2}), P_S(x, t) = \frac{Ac\dot{c}}{\sqrt{c^2 - x^2}}$$

and equation (P1) can be written as

$$p(x, 0, t) = P_S(x, t) + P_R(x, t), \quad P_R(x, t) = \sum_{n=1}^{Nm} \Big(\alpha\ddot{a}_n + \beta\lambda_n^4 a_n - 2Ac\dot{c}D_n\Big)\psi_n(x). \tag{P2}$$

The formulae (3), (6) and (P2) with the singular $P_S(x,t)$ and regular $P_R(x,t)$ components of the pressure in the contact region can be used for any symmetric elastic body with proper orthonormal modes $\psi_n(x)$. The corresponding formulae for the total force $F(t)$ are

$$F(t) = \sum_{n=1}^{Nm} C_n\Big(\alpha \ddot{a}_n + \beta\lambda_n^4 a_n\Big), \qquad\qquad C_n(c) = 2\int_0^c \psi_n(x)dx, \tag{$F1$}$$

$$F(t) = \pi A(t) c\dot{c} + \sum_{n=1}^{Nm} C_n\Big(\alpha \ddot{a}_n + \beta\lambda_n^4 a_n - A(t)c\dot{c}D_n\Big). \tag{$F2$}$$

The formulae (P1) and (F1) are valid for both impact stage and free-vibration stage, however the formulae (P2) and (F2) are only for the impact stage.

3. Numerical results and conclusion. The results of numerical calculations are shown for elastic wedge with 10 degrees deadrise angle and aluminium platings of thickness 2 cm and length 80 cm impacting the initially calm water surface at speed 4 m/sec.

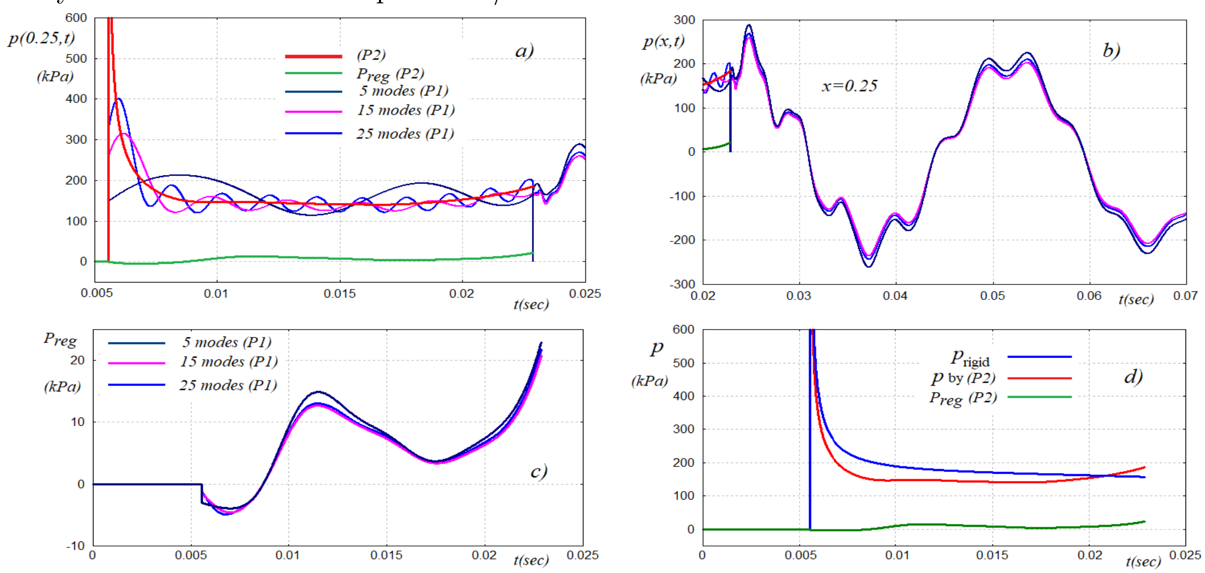

Fig.2. Pressure evolution at $x = 0.25$ by (P1) and (P2).

Fig.2a shows the pressure evolution at $x = 0.25$ by (P1) and (P2) during the impact stage. The pressures by (P1) converge to that by (P2) but very slow and with high-frequency oscillations. The regular component of the pressure $P_R(0.25,t)$ shown by the green line is much smaller than the singular component. During the free-vibration stage, see Fig.2b, formula (P1) well predicts the pressure even for small number of retained modes. Note that the pressure drops down to the vapour pressure at $t = 0.034$ sec, when another model including cavitation is required. Convergence of the regular component of the pressure $P_R(0.25,t)$ with the number of modes is shown in Fig.2c. Finally, Fig.2d compares the pressure by (P2)with the pressure for the equivalent rigid wedge. The pressure for elastic wedge is smaller than for the rigid wedge except the end of the impact stage.

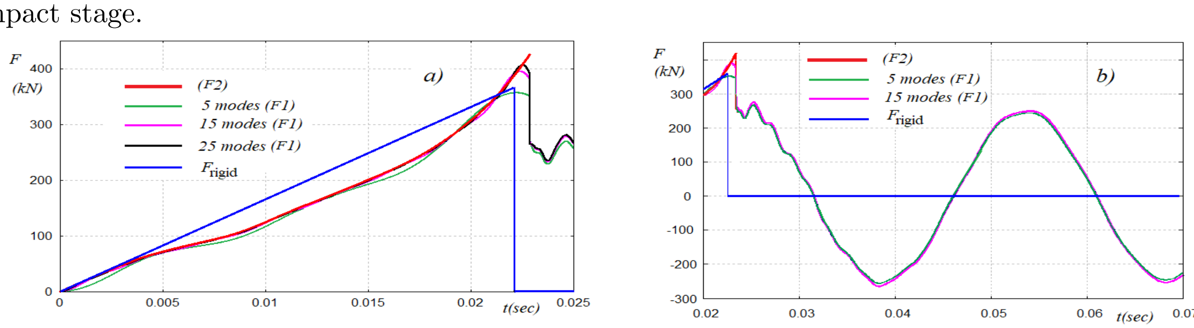

Fig.3. Total force evolution by (F1) and (F2).

Fig.3a shows that the total force (red line) is well predicted by the formula (F1) except the very end of the impact stage, when (F1) underpredicts the force. The force for the rigid wedge is higher than the force for the elastic wedge except the end of the impact stage. Fig.3b shows that the normal mode method accurately predicts the total force at the free-vibration stage even with 5 modes. Note that the force at this stage is zero for the rigid wedge.

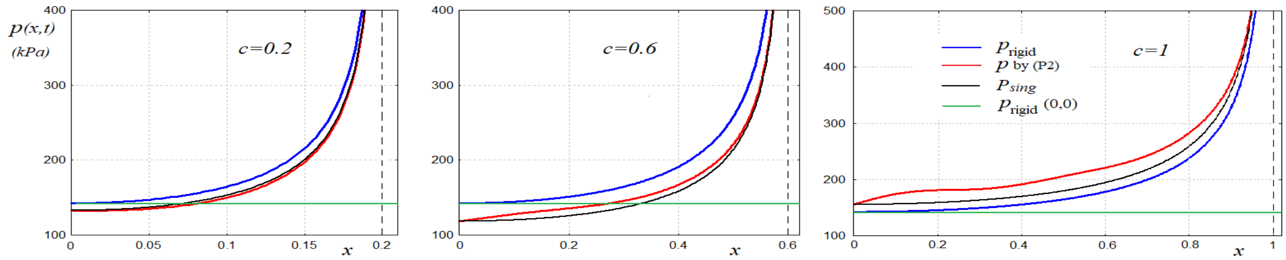

Fig.4. Distributions of the pressure along the wetted region for different time instants.

The pressures calculated by (P2) during the impact stage (red lines) are compared with the corresponding pressures for the rigid wedge (blue) and the singular component $P_S(x,t)$ (black) in Fig.4. The green line shows the pressure value at the centre of the wetted region. It is seen that the elastic pressure is higher than the rigid pressure everywhere in the wetted region at the end of the impact stage.

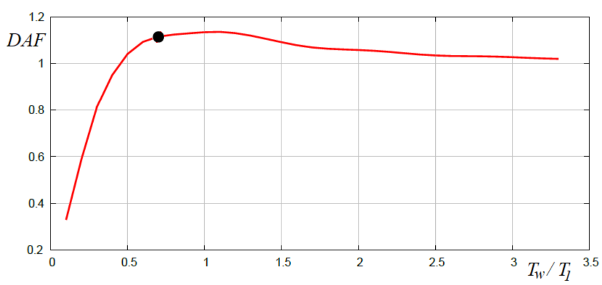

Fig.5. Dynamic Amplification Factor.

The derived accurate approximation of the hydrodynamic pressure during the impact stage for an elastic structure can be used to calibrate CFD results and control the hydrodynamic loads when they are taking their maximum values. Both the experimental measurements and the numerical simulations show that extremely large local pressures might occur during the impact. Due to the extreme sensitivity of the maximum impact pressures to the local details of the impact, the physical experiments show extreme variability and the numerical tools cannot properly reach the convergence whatever the mesh size. These facts do not necessarily represent big practical problem if the coupling with the structural response is properly taken into account. Indeed, the localized extreme pressures are usually accompanied with extremely short durations and the affected area is very small, so that the extreme pressure peaks are ultimately filtered by the structural dynamics. This means that the full hydroelastic coupling is absolutely necessary for the evaluation of the structural response. If applied directly on the structure in quasi static sense, these extreme pressures would induce severe structural failure in many conditions. The well-known dependency of the structural response on the ratio between the excitation time and the first structural natural period (Dynamic Amplification Factor – DAF) is shown in Fig.5 where the three different regimes are identified (impulsive, dynamic and quasi static) and the example case which is considered here is marked with bullet point.

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